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Inverse Trigonometric Functions Arcsec and Arccosec

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Summary. This article describes definitions of inverse trigonometric functions arcsec and arccosec, as well as their main properties.

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The papers [1], [2], [16], [3], [12], [17], [13], [5], [8], [11], [14], [4], [6], [7], [10], [15], and [9] provide the notation and terminology for this paper.

In this paper x, r denote real numbers.

The following propositions are true:

(1) $[0, \frac{\pi}{2}[\subseteq \mathrm{dom}$ (the function sec).

(2) $]\frac{\pi}{2}, \pi] \subseteq \mathrm{dom}$ (the function sec).

(3) $[-\frac{\pi}{2}, 0[\subseteq \mathrm{dom}$ (the function cosec).

(4) $]0, \frac{\pi}{2}] \subseteq \mathrm{dom}$ (the function cosec).

(5) The function sec is differentiable on $]0, \frac{\pi}{2}[$ and for every x such that $x \in]0, \frac{\pi}{2}[$ holds (the function sec)$'(x) = \frac{\sin x}{(\cos x)^{\mathbf{2}}}$.

(6) The function sec is differentiable on $]\frac{\pi}{2}, \pi[$ and for every x such that $x \in]\frac{\pi}{2}, \pi[$ holds (the function sec)$'(x) = \frac{\sin x}{(\cos x)^{\mathbf{2}}}$.

(7)(i) The function cosec is differentiable on $]-\frac{\pi}{2}, 0[$, and

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(ii) for every x such that $x \in]-\frac{\pi}{2}, 0[$ holds (the function cosec)$'(x) = -\frac{\cos x}{(\sin x)^{\mathbf{2}}}$.

(8)(i) The function cosec is differentiable on $]0, \frac{\pi}{2}[$, and

(ii) for every x such that $x \in]0, \frac{\pi}{2}[$ holds (the function cosec)$'(x) = -\frac{\cos x}{(\sin x)^{\mathbf{2}}}$.

(9) The function sec is continuous on $]0, \frac{\pi}{2}[$.

(10) The function sec is continuous on $]\frac{\pi}{2}, \pi[$.

(11) The function cosec is continuous on $]-\frac{\pi}{2}, 0[$.

(12) The function cosec is continuous on $]0, \frac{\pi}{2}[$.

(13) The function sec is increasing on $]0, \frac{\pi}{2}[$.

(14) The function sec is increasing on $]\frac{\pi}{2}, \pi[$.

(15) The function cosec is decreasing on $]-\frac{\pi}{2}, 0[$.

(16) The function cosec is decreasing on $]0, \frac{\pi}{2}[$.

(17) The function sec is increasing on $[0, \frac{\pi}{2}[$.

(18) The function sec is increasing on $]\frac{\pi}{2}, \pi]$.

(19) The function cosec is decreasing on $[-\frac{\pi}{2}, 0[$.

(20) The function cosec is decreasing on $]0, \frac{\pi}{2}]$.

(21) (The function sec)$\restriction[0, \frac{\pi}{2}[$ is one-to-one.

(22) (The function sec)$\restriction]\frac{\pi}{2}, \pi]$ is one-to-one.

(23) (The function cosec)$\restriction[-\frac{\pi}{2}, 0[$ is one-to-one.

(24) (The function cosec)$\restriction]0, \frac{\pi}{2}]$ is one-to-one.

One can verify the following observations:

* (the function sec)$\restriction[0, \frac{\pi}{2}[$ is one-to-one,
* (the function sec)$\restriction]\frac{\pi}{2}, \pi]$ is one-to-one,
* (the function cosec)$\restriction[-\frac{\pi}{2}, 0[$ is one-to-one, and
* (the function cosec)$\restriction]0, \frac{\pi}{2}]$ is one-to-one.

The partial function the 1st part of arcsec from $\mathbb{R}$ to $\mathbb{R}$ is defined as follows:

(Def. 1) The 1st part of arcsec $=$ ((the function sec)$\restriction[0, \frac{\pi}{2}[)^{-1}$.

The partial function the 2nd part of arcsec from $\mathbb{R}$ to $\mathbb{R}$ is defined as follows:

(Def. 2) The 2nd part of arcsec $=$ ((the function sec)$\restriction]\frac{\pi}{2}, \pi])^{-1}$.

The partial function the 1st part of arccosec from $\mathbb{R}$ to $\mathbb{R}$ is defined by:

(Def. 3) The 1st part of arccosec $=$ ((the function cosec)$\restriction[-\frac{\pi}{2}, 0[)^{-1}$.

The partial function the 2nd part of arccosec from $\mathbb{R}$ to $\mathbb{R}$ is defined by:

(Def. 4) The 2nd part of arccosec $=$ ((the function cosec)$\restriction]0, \frac{\pi}{2}])^{-1}$.

Let r be a real number. The functor $\mathrm{arcsec}_1\, r$ is defined by:

(Def. 5) $\mathrm{arcsec}_1\, r =$ (the 1st part of arcsec)(r).

The functor $\mathrm{arcsec}_2\, r$ is defined as follows:

(Def. 6) $\operatorname{arcsec}_2 r = (\text{the 2nd part of arcsec})(r)$.

The functor $\operatorname{arccosec}_1 r$ is defined as follows:

(Def. 7) $\operatorname{arccosec}_1 r = (\text{the 1st part of arccosec})(r)$.

The functor $\operatorname{arccosec}_2 r$ is defined by:

(Def. 8) $\operatorname{arccosec}_2 r = (\text{the 2nd part of arccosec})(r)$.

Let r be a real number. Then $\operatorname{arcsec}_1 r$ is a real number. Then $\operatorname{arcsec}_2 r$ is a real number. Then $\operatorname{arccosec}_1 r$ is a real number. Then $\operatorname{arccosec}_2 r$ is a real number.

We now state four propositions:

(25) $\operatorname{rng}(\text{the 1st part of arcsec}) = [0, \frac{\pi}{2}[$.

(26) $\operatorname{rng}(\text{the 2nd part of arcsec}) =]\frac{\pi}{2}, \pi]$.

(27) $\operatorname{rng}(\text{the 1st part of arccosec}) = [-\frac{\pi}{2}, 0[$.

(28) $\operatorname{rng}(\text{the 2nd part of arccosec}) =]0, \frac{\pi}{2}]$.

One can check the following observations:

* the 1st part of arcsec is one-to-one,
* the 2nd part of arcsec is one-to-one,
* the 1st part of arccosec is one-to-one, and
* the 2nd part of arccosec is one-to-one.

Let t_1 be a real number. Then $\sec t_1$ is a real number. Then $\operatorname{cosec} t_1$ is a real number.

We now state a number of propositions:

(29) $\sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ and $\cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$.

(30) $\sin(-\frac{\pi}{4}) = -\frac{1}{\sqrt{2}}$ and $\cos(-\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ and $\sin(\frac{3}{4} \cdot \pi) = \frac{1}{\sqrt{2}}$ and $\cos(\frac{3}{4} \cdot \pi) = -\frac{1}{\sqrt{2}}$.

(31) $\sec 0 = 1$ and $\sec(\frac{\pi}{4}) = \sqrt{2}$ and $\sec(\frac{3}{4} \cdot \pi) = -\sqrt{2}$ and $\sec \pi = -1$.

(32) $\operatorname{cosec}(-\frac{\pi}{2}) = -1$ and $\operatorname{cosec}(-\frac{\pi}{4}) = -\sqrt{2}$ and $\operatorname{cosec}(\frac{\pi}{4}) = \sqrt{2}$ and $\operatorname{cosec}(\frac{\pi}{2}) = 1$.

(33) For every set x such that $x \in [0, \frac{\pi}{4}]$ holds $\sec x \in [1, \sqrt{2}]$.

(34) For every set x such that $x \in [\frac{3}{4} \cdot \pi, \pi]$ holds $\sec x \in [-\sqrt{2}, -1]$.

(35) For every set x such that $x \in [-\frac{\pi}{2}, -\frac{\pi}{4}]$ holds $\operatorname{cosec} x \in [-\sqrt{2}, -1]$.

(36) For every set x such that $x \in [\frac{\pi}{4}, \frac{\pi}{2}]$ holds $\operatorname{cosec} x \in [1, \sqrt{2}]$.

(37) The function sec is continuous on $[0, \frac{\pi}{2}[$.

(38) The function sec is continuous on $]\frac{\pi}{2}, \pi]$.

(39) The function cosec is continuous on $[-\frac{\pi}{2}, 0[$.

(40) The function cosec is continuous on $]0, \frac{\pi}{2}]$.

(41) $\operatorname{rng}((\text{the function sec}) \restriction [0, \frac{\pi}{4}]) = [1, \sqrt{2}]$.

(42) $\operatorname{rng}((\text{the function sec}) \restriction [\frac{3}{4} \cdot \pi, \pi]) = [-\sqrt{2}, -1]$.

(43) $\mathrm{rng}((\text{the function cosec})\restriction[-\frac{\pi}{2},-\frac{\pi}{4}]) = [-\sqrt{2},-1]$.

(44) $\mathrm{rng}((\text{the function cosec})\restriction[\frac{\pi}{4},\frac{\pi}{2}]) = [1,\sqrt{2}]$.

(45) $[1,\sqrt{2}] \subseteq \mathrm{dom}\,(\text{the 1st part of arcsec})$.

(46) $[-\sqrt{2},-1] \subseteq \mathrm{dom}\,(\text{the 2nd part of arcsec})$.

(47) $[-\sqrt{2},-1] \subseteq \mathrm{dom}\,(\text{the 1st part of arccosec})$.

(48) $[1,\sqrt{2}] \subseteq \mathrm{dom}\,(\text{the 2nd part of arccosec})$.

One can check the following observations:

* (the function sec)$\restriction[0,\frac{\pi}{4}]$ is one-to-one,
* (the function sec)$\restriction[\frac{3}{4}\cdot\pi,\pi]$ is one-to-one,
* (the function cosec)$\restriction[-\frac{\pi}{2},-\frac{\pi}{4}]$ is one-to-one, and
* (the function cosec)$\restriction[\frac{\pi}{4},\frac{\pi}{2}]$ is one-to-one.

One can prove the following propositions:

(49) (The 1st part of arcsec)$\restriction[1,\sqrt{2}] = ((\text{the function sec})\restriction[0,\frac{\pi}{4}])^{-1}$.

(50) (The 2nd part of arcsec)$\restriction[-\sqrt{2},-1] = ((\text{the function sec})\restriction[\frac{3}{4}\cdot\pi,\pi])^{-1}$.

(51) (The 1st part of arccosec)$\restriction[-\sqrt{2},-1] = ((\text{the function cosec})\restriction[-\frac{\pi}{2},-\frac{\pi}{4}])^{-1}$.

(52) (The 2nd part of arccosec)$\restriction[1,\sqrt{2}] = ((\text{the function cosec})\restriction[\frac{\pi}{4},\frac{\pi}{2}])^{-1}$.

(53) ((The function sec)$\restriction[0,\frac{\pi}{4}]$ **qua** function) $\cdot$((the 1st part of arcsec)$\restriction[1,\sqrt{2}]) = \mathrm{id}_{[1,\sqrt{2}]}$.

(54) ((The function sec)$\restriction[\frac{3}{4}\cdot\pi,\pi]$ **qua** function) $\cdot$((the 2nd part of arcsec)$\restriction[-\sqrt{2},-1]) = \mathrm{id}_{[-\sqrt{2},-1]}$.

(55) ((The function cosec)$\restriction[-\frac{\pi}{2},-\frac{\pi}{4}]$ **qua** function) $\cdot$((the 1st part of arccosec)$\restriction[-\sqrt{2},-1]) = \mathrm{id}_{[-\sqrt{2},-1]}$.

(56) ((The function cosec)$\restriction[\frac{\pi}{4},\frac{\pi}{2}]$ **qua** function) $\cdot$((the 2nd part of arccosec)$\restriction[1,\sqrt{2}]) = \mathrm{id}_{[1,\sqrt{2}]}$.

(57) ((The function sec)$\restriction[0,\frac{\pi}{4}]) \cdot$ ((the 1st part of arcsec)$\restriction[1,\sqrt{2}]) = \mathrm{id}_{[1,\sqrt{2}]}$.

(58) ((The function sec)$\restriction[\frac{3}{4}\cdot\pi,\pi]) \cdot$ ((the 2nd part of arcsec)$\restriction[-\sqrt{2},-1]) = \mathrm{id}_{[-\sqrt{2},-1]}$.

(59) ((The function cosec)$\restriction[-\frac{\pi}{2},-\frac{\pi}{4}])\cdot$((the 1st part of arccosec)$\restriction[-\sqrt{2},-1]) = \mathrm{id}_{[-\sqrt{2},-1]}$.

(60) ((The function cosec)$\restriction[\frac{\pi}{4},\frac{\pi}{2}]) \cdot$ ((the 2nd part of arccosec)$\restriction[1,\sqrt{2}]) = \mathrm{id}_{[1,\sqrt{2}]}$.

(61) (The 1st part of arcsec **qua** function) $\cdot$((the function sec)$\restriction[0,\frac{\pi}{2}[) = \mathrm{id}_{[0,\frac{\pi}{2}[}$.

(62) (The 2nd part of arcsec **qua** function) $\cdot$((the function sec)$\restriction]\frac{\pi}{2},\pi]) = \mathrm{id}_{]\frac{\pi}{2},\pi]}$.

(63) (The 1st part of arccosec **qua** function) $\cdot$((the function cosec)$\restriction[-\frac{\pi}{2},0[) = \mathrm{id}_{[-\frac{\pi}{2},0[}$.

(64) (The 2nd part of arccosec **qua** function) $\cdot$((the function cosec)$\restriction]0, \frac{\pi}{2}]) = \mathrm{id}_{]0, \frac{\pi}{2}]}$.
(65) (The 1st part of arcsec) $\cdot$((the function sec)$\restriction [0, \frac{\pi}{2}[) = \mathrm{id}_{[0, \frac{\pi}{2}[}$.
(66) (The 2nd part of arcsec) $\cdot$((the function sec)$\restriction]\frac{\pi}{2}, \pi]) = \mathrm{id}_{]\frac{\pi}{2}, \pi]}$.
(67) (The 1st part of arccosec) $\cdot$((the function cosec)$\restriction [-\frac{\pi}{2}, 0[) = \mathrm{id}_{[-\frac{\pi}{2}, 0[}$.
(68) (The 2nd part of arccosec) $\cdot$((the function cosec)$\restriction]0, \frac{\pi}{2}]) = \mathrm{id}_{]0, \frac{\pi}{2}]}$.
(69) If $0 \leq r < \frac{\pi}{2}$, then $\mathrm{arcsec}_1 \sec r = r$.
(70) If $\frac{\pi}{2} < r \leq \pi$, then $\mathrm{arcsec}_2 \sec r = r$.
(71) If $-\frac{\pi}{2} \leq r < 0$, then $\mathrm{arccosec}_1 \mathrm{cosec}\, r = r$.
(72) If $0 < r \leq \frac{\pi}{2}$, then $\mathrm{arccosec}_2 \mathrm{cosec}\, r = r$.
(73) $\mathrm{arcsec}_1 1 = 0$ and $\mathrm{arcsec}_1 \sqrt{2} = \frac{\pi}{4}$.
(74) $\mathrm{arcsec}_2(-\sqrt{2}) = \frac{3}{4} \cdot \pi$ and $\mathrm{arcsec}_2(-1) = \pi$.
(75) $\mathrm{arccosec}_1(-1) = -\frac{\pi}{2}$ and $\mathrm{arccosec}_1(-\sqrt{2}) = -\frac{\pi}{4}$.
(76) $\mathrm{arccosec}_2 \sqrt{2} = \frac{\pi}{4}$ and $\mathrm{arccosec}_2 1 = \frac{\pi}{2}$.
(77) The 1st part of arcsec is increasing on (the function sec) $^\circ[0, \frac{\pi}{2}[$.
(78) The 2nd part of arcsec is increasing on (the function sec) $^\circ]\frac{\pi}{2}, \pi]$.
(79) The 1st part of arccosec is decreasing on (the function cosec) $^\circ[-\frac{\pi}{2}, 0[$.
(80) The 2nd part of arccosec is decreasing on (the function cosec) $^\circ]0, \frac{\pi}{2}]$.
(81) The 1st part of arcsec is increasing on $[1, \sqrt{2}]$.
(82) The 2nd part of arcsec is increasing on $[-\sqrt{2}, -1]$.
(83) The 1st part of arccosec is decreasing on $[-\sqrt{2}, -1]$.
(84) The 2nd part of arccosec is decreasing on $[1, \sqrt{2}]$.
(85) For every set x such that $x \in [1, \sqrt{2}]$ holds $\mathrm{arcsec}_1 x \in [0, \frac{\pi}{4}]$.
(86) For every set x such that $x \in [-\sqrt{2}, -1]$ holds $\mathrm{arcsec}_2 x \in [\frac{3}{4} \cdot \pi, \pi]$.
(87) For every set x such that $x \in [-\sqrt{2}, -1]$ holds $\mathrm{arccosec}_1 x \in [-\frac{\pi}{2}, -\frac{\pi}{4}]$.
(88) For every set x such that $x \in [1, \sqrt{2}]$ holds $\mathrm{arccosec}_2 x \in [\frac{\pi}{4}, \frac{\pi}{2}]$.
(89) If $1 \leq r \leq \sqrt{2}$, then $\sec \mathrm{arcsec}_1 r = r$.
(90) If $-\sqrt{2} \leq r \leq -1$, then $\sec \mathrm{arcsec}_2 r = r$.
(91) If $-\sqrt{2} \leq r \leq -1$, then $\mathrm{cosec}\, \mathrm{arccosec}_1 r = r$.
(92) If $1 \leq r \leq \sqrt{2}$, then $\mathrm{cosec}\, \mathrm{arccosec}_2 r = r$.
(93) The 1st part of arcsec is continuous on $[1, \sqrt{2}]$.
(94) The 2nd part of arcsec is continuous on $[-\sqrt{2}, -1]$.
(95) The 1st part of arccosec is continuous on $[-\sqrt{2}, -1]$.
(96) The 2nd part of arccosec is continuous on $[1, \sqrt{2}]$.
(97) rng((the 1st part of arcsec)$\restriction [1, \sqrt{2}]) = [0, \frac{\pi}{4}]$.
(98) rng((the 2nd part of arcsec)$\restriction [-\sqrt{2}, -1]) = [\frac{3}{4} \cdot \pi, \pi]$.
(99) rng((the 1st part of arccosec)$\restriction [-\sqrt{2}, -1]) = [-\frac{\pi}{2}, -\frac{\pi}{4}]$.

(100) rng((the 2nd part of arccosec)$\restriction[1,\sqrt{2}]) = [\frac{\pi}{4},\frac{\pi}{2}]$.

(101) If $1 \leq r \leq \sqrt{2}$ and $\operatorname{arcsec}_1 r = 0$, then $r = 1$ and if $1 \leq r \leq \sqrt{2}$ and $\operatorname{arcsec}_1 r = \frac{\pi}{4}$, then $r = \sqrt{2}$.

(102) If $-\sqrt{2} \leq r \leq -1$ and $\operatorname{arcsec}_2 r = \frac{3}{4}\cdot\pi$, then $r = -\sqrt{2}$ and if $-\sqrt{2} \leq r \leq -1$ and $\operatorname{arcsec}_2 r = \pi$, then $r = -1$.

(103) If $-\sqrt{2} \leq r \leq -1$ and $\operatorname{arccosec}_1 r = -\frac{\pi}{2}$, then $r = -1$ and if $-\sqrt{2} \leq r \leq -1$ and $\operatorname{arccosec}_1 r = -\frac{\pi}{4}$, then $r = -\sqrt{2}$.

(104) If $1 \leq r \leq \sqrt{2}$ and $\operatorname{arccosec}_2 r = \frac{\pi}{4}$, then $r = \sqrt{2}$ and if $1 \leq r \leq \sqrt{2}$ and $\operatorname{arccosec}_2 r = \frac{\pi}{2}$, then $r = 1$.

(105) If $1 \leq r \leq \sqrt{2}$, then $0 \leq \operatorname{arcsec}_1 r \leq \frac{\pi}{4}$.

(106) If $-\sqrt{2} \leq r \leq -1$, then $\frac{3}{4}\cdot\pi \leq \operatorname{arcsec}_2 r \leq \pi$.

(107) If $-\sqrt{2} \leq r \leq -1$, then $-\frac{\pi}{2} \leq \operatorname{arccosec}_1 r \leq -\frac{\pi}{4}$.

(108) If $1 \leq r \leq \sqrt{2}$, then $\frac{\pi}{4} \leq \operatorname{arccosec}_2 r \leq \frac{\pi}{2}$.

(109) If $1 < r < \sqrt{2}$, then $0 < \operatorname{arcsec}_1 r < \frac{\pi}{4}$.

(110) If $-\sqrt{2} < r < -1$, then $\frac{3}{4}\cdot\pi < \operatorname{arcsec}_2 r < \pi$.

(111) If $-\sqrt{2} < r < -1$, then $-\frac{\pi}{2} < \operatorname{arccosec}_1 r < -\frac{\pi}{4}$.

(112) If $1 < r < \sqrt{2}$, then $\frac{\pi}{4} < \operatorname{arccosec}_2 r < \frac{\pi}{2}$.

(113) If $1 \leq r \leq \sqrt{2}$, then $\sin\operatorname{arcsec}_1 r = \frac{\sqrt{r^2-1}}{r}$ and $\cos\operatorname{arcsec}_1 r = \frac{1}{r}$.

(114) If $-\sqrt{2} \leq r \leq -1$, then $\sin\operatorname{arcsec}_2 r = -\frac{\sqrt{r^2-1}}{r}$ and $\cos\operatorname{arcsec}_2 r = \frac{1}{r}$.

(115) If $-\sqrt{2} \leq r \leq -1$, then $\sin\operatorname{arccosec}_1 r = \frac{1}{r}$ and $\cos\operatorname{arccosec}_1 r = -\frac{\sqrt{r^2-1}}{r}$.

(116) If $1 \leq r \leq \sqrt{2}$, then $\sin\operatorname{arccosec}_2 r = \frac{1}{r}$ and $\cos\operatorname{arccosec}_2 r = \frac{\sqrt{r^2-1}}{r}$.

(117) If $1 < r < \sqrt{2}$, then $\operatorname{cosec}\operatorname{arcsec}_1 r = \frac{r}{\sqrt{r^2-1}}$.

(118) If $-\sqrt{2} < r < -1$, then $\operatorname{cosec}\operatorname{arcsec}_2 r = -\frac{r}{\sqrt{r^2-1}}$.

(119) If $-\sqrt{2} < r < -1$, then $\sec\operatorname{arccosec}_1 r = -\frac{r}{\sqrt{r^2-1}}$.

(120) If $1 < r < \sqrt{2}$, then $\sec\operatorname{arccosec}_2 r = \frac{r}{\sqrt{r^2-1}}$.

(121) The 1st part of arcsec is differentiable on (the function sec) $^\circ]0,\frac{\pi}{2}[$.

(122) The 2nd part of arcsec is differentiable on (the function sec) $^\circ]\frac{\pi}{2},\pi[$.

(123) The 1st part of arccosec is differentiable on (the function cosec) $^\circ]-\frac{\pi}{2},0[$.

(124) The 2nd part of arccosec is differentiable on (the function cosec) $^\circ]0,\frac{\pi}{2}[$.

(125) (The function sec) $^\circ]0,\frac{\pi}{2}[$ is open.

(126) (The function sec) $^\circ]\frac{\pi}{2},\pi[$ is open.

(127) (The function cosec) $^\circ]-\frac{\pi}{2},0[$ is open.

(128) (The function cosec) $^\circ]0,\frac{\pi}{2}[$ is open.

(129) The 1st part of arcsec is continuous on (the function sec) $^\circ]0,\frac{\pi}{2}[$.

(130) The 2nd part of arcsec is continuous on (the function sec) $^\circ]\frac{\pi}{2},\pi[$.

(131) The 1st part of arccosec is continuous on (the function cosec) $^\circ]-\frac{\pi}{2},0[$.

(132) The 2nd part of arccosec is continuous on (the function cosec) $^\circ]0,\frac{\pi}{2}[$.

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