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On the Calculus of Binary Arithmetics

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Summary. In this paper, we have binary arithmetic and its related operations. We include some theorems concerning logical operators.

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The notation and terminology used in this paper have been introduced in the following articles: [3], [4], [2], and [1].

Let x, y be boolean sets. The functor x 'nand' y is defined as follows:

(Def. 1) x 'nand' $y = \neg(x \wedge y)$.

Let us note that the functor x 'nand' y is commutative.

Let x, y be boolean sets. Note that x 'nand' y is boolean.

Let x, y be elements of *Boolean*. Then x 'nand' y is an element of *Boolean*.

Let x, y be boolean sets. The functor x 'nor' y is defined by:

(Def. 2) x 'nor' $y = \neg(x \vee y)$.

Let us note that the functor x 'nor' y is commutative.

Let x, y be boolean sets. Note that x 'nor' y is boolean.

Let x, y be elements of *Boolean*. Then x 'nor' y is an element of *Boolean*.

Let x, y be boolean sets. The functor x 'xnor' y is defined as follows:

(Def. 3) x 'xnor' $y = \neg(x \oplus y)$.

Let us observe that the functor x 'xnor' y is commutative.

Let x, y be boolean sets. Note that x 'xnor' y is boolean.

Let x, y be elements of *Boolean*. Then x 'xnor' y is an element of *Boolean*.

In the sequel x, y, z, w are boolean sets.

The following propositions are true:

(1) *true* 'nand' $x = \neg x$.

(2) *false* 'nand' $x = $ *true*.

(3) x 'nand' $x = \neg x$ and $\neg(x$ 'nand' $x) = x$.

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(4) $\neg(x \text{ 'nand' } y) = x \wedge y$.

(5) $x \text{ 'nand' } \neg x = \mathit{true}$ and $\neg(x \text{ 'nand' } \neg x) = \mathit{false}$.

(6) $x \text{ 'nand' } y \wedge z = \neg(x \wedge y \wedge z)$.

(7) $x \text{ 'nand' } y \wedge z = x \wedge y \text{ 'nand' } z$.

(8) $x \text{ 'nand' } (y \vee z) = \neg(x \wedge y) \wedge \neg(x \wedge z)$.

(9) $x \text{ 'nand' } (y \oplus z) = x \wedge y \Leftrightarrow x \wedge z$.

(10) $\mathit{true} \text{ 'nor' } x = \mathit{false}$.

(11) $\mathit{false} \text{ 'nor' } x = \neg x$.

(12) $x \text{ 'nor' } x = \neg x$ and $\neg(x \text{ 'nor' } x) = x$.

(13) $\neg(x \text{ 'nor' } y) = x \vee y$.

(14) $x \text{ 'nor' } \neg x = \mathit{false}$ and $\neg(x \text{ 'nor' } \neg x) = \mathit{true}$.

(15) $x \text{ 'nor' } y \wedge z = \neg(x \vee y) \vee \neg(x \vee z)$.

(16) $x \text{ 'nor' } (y \vee z) = \neg(x \vee y \vee z)$.

(17) $\mathit{true} \text{ 'xnor' } x = x$.

(18) $\mathit{false} \text{ 'xnor' } x = \neg x$.

(19) $x \text{ 'xnor' } x = \mathit{true}$ and $\neg(x \text{ 'xnor' } x) = \mathit{false}$.

(20) $\neg(x \text{ 'xnor' } y) = x \oplus y$.

(21) $x \text{ 'xnor' } \neg x = \mathit{false}$ and $\neg(x \text{ 'xnor' } \neg x) = \mathit{true}$.

(22) $x \Subset y \Rightarrow z$ iff $x \wedge y \Subset z$.

(23) $x \Leftrightarrow y = (x \Rightarrow y) \wedge (y \Rightarrow x)$.

(24) $x \Leftrightarrow y = \mathit{true}$ iff $x \Rightarrow y = \mathit{true}$ and $y \Rightarrow x = \mathit{true}$.

(25) If $x \Rightarrow y = \mathit{true}$ and $y \Rightarrow x = \mathit{true}$, then $x = y$.

(26) If $x \Rightarrow y = \mathit{true}$ and $y \Rightarrow z = \mathit{true}$, then $x \Rightarrow z = \mathit{true}$.

(27) If $x \Leftrightarrow y = \mathit{true}$ and $y \Leftrightarrow z = \mathit{true}$, then $x \Leftrightarrow z = \mathit{true}$.

(28) $x \Rightarrow y = \neg y \Rightarrow \neg x$.

(29) $x \Leftrightarrow y = \neg x \Leftrightarrow \neg y$.

(30) If $x \Leftrightarrow y = \mathit{true}$ and $z \Leftrightarrow w = \mathit{true}$, then $x \wedge z \Leftrightarrow y \wedge w = \mathit{true}$.

(31) If $x \Leftrightarrow y = \mathit{true}$ and $z \Leftrightarrow w = \mathit{true}$, then $x \Rightarrow z \Leftrightarrow y \Rightarrow w = \mathit{true}$.

(32) If $x \Leftrightarrow y = \mathit{true}$ and $z \Leftrightarrow w = \mathit{true}$, then $x \vee z \Leftrightarrow y \vee w = \mathit{true}$.

(33) If $x \Leftrightarrow y = \mathit{true}$ and $z \Leftrightarrow w = \mathit{true}$, then $x \Leftrightarrow z \Leftrightarrow y \Leftrightarrow w = \mathit{true}$.

(34) If $x = \mathit{true}$ and $x \Rightarrow y = \mathit{true}$, then $y = \mathit{true}$.

(35) If $y = \mathit{true}$, then $x \Rightarrow y = \mathit{true}$.

(36) If $\neg x = \mathit{true}$, then $x \Rightarrow y = \mathit{true}$.

(37) $x \Rightarrow x = \mathit{true}$.

(38) If $x \Rightarrow y = \mathit{true}$ and $x \Rightarrow \neg y = \mathit{true}$, then $\neg x = \mathit{true}$.

(39) $\neg x \Rightarrow x \Rightarrow x = \mathit{true}$.

(40) $x \Rightarrow y \Rightarrow \neg(y \wedge z) \Rightarrow \neg(x \wedge z) = \mathit{true}$.

(41) $x \Rightarrow y \Rightarrow y \Rightarrow z \Rightarrow x \Rightarrow z = true$.

(42) If $x \Rightarrow y = true$, then $y \Rightarrow z \Rightarrow x \Rightarrow z = true$.

(43) $y \Rightarrow x \Rightarrow y = true$.

(44) $x \Rightarrow y \Rightarrow z \Rightarrow y \Rightarrow z = true$.

(45) $y \Rightarrow y \Rightarrow x \Rightarrow x = true$.

(46) $z \Rightarrow y \Rightarrow x \Rightarrow y \Rightarrow z \Rightarrow x = true$.

(47) $y \Rightarrow z \Rightarrow x \Rightarrow y \Rightarrow x \Rightarrow z = true$.

(48) $y \Rightarrow y \Rightarrow z \Rightarrow y \Rightarrow z = true$.

(49) $x \Rightarrow y \Rightarrow z \Rightarrow x \Rightarrow y \Rightarrow x \Rightarrow z = true$.

(50) If $x = true$, then $x \Rightarrow y \Rightarrow y = true$.

(51) If $z \Rightarrow y \Rightarrow x = true$, then $y \Rightarrow z \Rightarrow x = true$.

(52) If $z \Rightarrow y \Rightarrow x = true$ and $y = true$, then $z \Rightarrow x = true$.

(53) If $z \Rightarrow y \Rightarrow x = true$ and $y = true$ and $z = true$, then $x = true$.

(54) If $y \Rightarrow y \Rightarrow z = true$, then $y \Rightarrow z = true$.

(55) If $x \Rightarrow y \Rightarrow z = true$, then $x \Rightarrow y \Rightarrow x \Rightarrow z = true$.

Acknowledgments

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