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On Replace Function and Swap Function for Finite Sequences

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Summary. In this article, we show the property of the Replace Function and the Swap Function of finite sequences. In the first section, we prepared some useful theorems for finite sequences. In the second section, we defined the Replace function and proved some theorems about the function. This function replaces an element of a sequence by another value. In the third section, we defined the Swap function and proved some theorems about the function. This function swaps two elements of a sequence. In the last section, we show the property of composed functions of the Replace Function and the Swap Function.

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The notation and terminology used here are introduced in the following papers: [7], [11], [2], [9], [3], [1], [5], [12], [6], [10], [8], and [4].

1. Some Basic Theorems

For simplicity, we adopt the following rules: D denotes a non empty set, f, g, h denote finite sequences of elements of D, p, p_1, p_2, p_3, q denote elements of D, and i, j, k, l, n denote natural numbers.

One can prove the following propositions:

(1) If $1 \leqslant i$ and $j \leqslant \operatorname{len} f$ and $i < j$, then $f = (f \restriction (i -' 1)) \frown \langle f(i) \rangle \frown (f_{\downharpoonright i} \restriction (j -' i -' 1)) \frown \langle f(j) \rangle \frown (f_{\downharpoonright j})$.

(2) If $\operatorname{len} g = \operatorname{len} h$ and $\operatorname{len} g < i$ and $i \leqslant \operatorname{len}(g \frown f)$, then $(g \frown f)(i) = (h \frown f)(i)$.

(3) If $1 \leqslant i$ and $i \leqslant \operatorname{len} f$, then $f(i) = (g \frown f)(\operatorname{len} g + i)$.

(4) If $i \in \operatorname{dom}(f_{\downharpoonright n})$, then $f_{\downharpoonright n}(i) = f(n + i)$.

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2. Definition of Replace Function and its Properties

Let D be a non empty set, let f be a finite sequence of elements of D, let i be a natural number, and let p be an element of D. Then $f +\cdot (i,p)$ is a finite sequence of elements of D and it can be characterized by the condition:

(Def. 1) $$f +\cdot (i,p) = \begin{cases} (f \restriction (i -' 1)) \frown \langle p \rangle \frown (f_{\mid i}), & \text{if } 1 \leqslant i \text{ and } i \leqslant \operatorname{len} f, \\ f, & \text{otherwise.} \end{cases}$$

We introduce $\operatorname{Replace}(f,i,p)$ as a synonym of $f +\cdot (i,p)$.

The following propositions are true:

(5) $\operatorname{Replace}(f,0,p) = f$.

(6) If $i > \operatorname{len} f$, then $\operatorname{Replace}(f,i,p) = f$.

(7) $\operatorname{len}\operatorname{Replace}(f,i,p) = \operatorname{len} f$.

(8) $\operatorname{rng}\operatorname{Replace}(f,i,p) \subseteq \operatorname{rng} f \cup \{p\}$.

(9) If $1 \leqslant i$ and $i \leqslant \operatorname{len} f$, then $p \in \operatorname{rng}\operatorname{Replace}(f,i,p)$.

(10) If $1 \leqslant i$ and $i \leqslant \operatorname{len} f$, then $(\operatorname{Replace}(f,i,p))_i = p$.

(11) If $1 \leqslant i$ and $i \leqslant \operatorname{len} f$, then for every k such that $0 < k$ and $k \leqslant \operatorname{len} f - i$ holds $(\operatorname{Replace}(f,i,p))(i+k) = f_{\mid i}(k)$.

(12) If $1 \leqslant k$ and $k \leqslant \operatorname{len} f$ and $k \neq i$, then $(\operatorname{Replace}(f,i,p))_k = f_k$.

(13) If $1 \leqslant i$ and $i < j$ and $j \leqslant \operatorname{len} f$, then $\operatorname{Replace}(\operatorname{Replace}(f,j,q),i,p) = (f \restriction (i -' 1)) \frown \langle p \rangle \frown (f_{\mid i} \restriction (j -' i -' 1)) \frown \langle q \rangle \frown (f_{\mid j})$.

(14) $\operatorname{Replace}(\langle p \rangle, 1, q) = \langle q \rangle$.

(15) $\operatorname{Replace}(\langle p_1, p_2 \rangle, 1, q) = \langle q, p_2 \rangle$.

(16) $\operatorname{Replace}(\langle p_1, p_2 \rangle, 2, q) = \langle p_1, q \rangle$.

(17) $\operatorname{Replace}(\langle p_1, p_2, p_3 \rangle, 1, q) = \langle q, p_2, p_3 \rangle$.

(18) $\operatorname{Replace}(\langle p_1, p_2, p_3 \rangle, 2, q) = \langle p_1, q, p_3 \rangle$.

(19) $\operatorname{Replace}(\langle p_1, p_2, p_3 \rangle, 3, q) = \langle p_1, p_2, q \rangle$.

3. Definition of Swap Function and its Properties

Let D be a non empty set, let f be a finite sequence of elements of D, and let i, j be natural numbers. The functor $\operatorname{Swap}(f,i,j)$ yields a finite sequence of elements of D and is defined as follows:

(Def. 2) $$\operatorname{Swap}(f,i,j) = \begin{cases} \operatorname{Replace}(\operatorname{Replace}(f,i,f_j),j,f_i), & \text{if } 1 \leqslant i \text{ and } i \leqslant \operatorname{len} f \\ \quad \text{and } 1 \leqslant j \text{ and } j \leqslant \operatorname{len} f, & \\ f, & \text{otherwise.} \end{cases}$$

Next we state a number of propositions:

(20) $\operatorname{len}\operatorname{Swap}(f,i,j) = \operatorname{len} f$.

(21) $\mathrm{Swap}(f, i, i) = f$.

(22) $\mathrm{Swap}(\mathrm{Swap}(f, i, j), j, i) = f$.

(23) $\mathrm{Swap}(f, i, j) = \mathrm{Swap}(f, j, i)$.

(24) $\mathrm{rng}\,\mathrm{Swap}(f, i, j) = \mathrm{rng}\, f$.

(25) $\mathrm{Swap}(\langle p_1, p_2\rangle, 1, 2) = \langle p_2, p_1\rangle$.

(26) $\mathrm{Swap}(\langle p_1, p_2, p_3\rangle, 1, 2) = \langle p_2, p_1, p_3\rangle$.

(27) $\mathrm{Swap}(\langle p_1, p_2, p_3\rangle, 1, 3) = \langle p_3, p_2, p_1\rangle$.

(28) $\mathrm{Swap}(\langle p_1, p_2, p_3\rangle, 2, 3) = \langle p_1, p_3, p_2\rangle$.

(29) If $1 \leqslant i$ and $i < j$ and $j \leqslant \mathrm{len}\, f$, then $\mathrm{Swap}(f, i, j) = (f \restriction (i -' 1)) \frown \langle f_j\rangle \frown (f_{\downharpoonright i} \restriction (j -' i -' 1)) \frown \langle f_i\rangle \frown (f_{\downharpoonright j})$.

(30) If $1 < i$ and $i \leqslant \mathrm{len}\, f$, then $\mathrm{Swap}(f, 1, i) = \langle f_i\rangle \frown (f_{\downharpoonright 1} \restriction (i -' 2)) \frown \langle f_1\rangle \frown (f_{\downharpoonright i})$.

(31) If $1 \leqslant i$ and $i < \mathrm{len}\, f$, then $\mathrm{Swap}(f, i, \mathrm{len}\, f) = (f \restriction (i -' 1)) \frown \langle f_{\mathrm{len}\, f}\rangle \frown (f_{\downharpoonright i} \restriction (\mathrm{len}\, f -' i -' 1)) \frown \langle f_i\rangle$.

(32) If $i \neq k$ and $j \neq k$ and $1 \leqslant k$ and $k \leqslant \mathrm{len}\, f$, then $(\mathrm{Swap}(f, i, j))_k = f_k$.

(33) If $1 \leqslant i$ and $i \leqslant \mathrm{len}\, f$ and $1 \leqslant j$ and $j \leqslant \mathrm{len}\, f$, then $(\mathrm{Swap}(f, i, j))_i = f_j$ and $(\mathrm{Swap}(f, i, j))_j = f_i$.

4. Properties of Combination Function of Replace Function and Swap Function

We now state four propositions:

(34) If $1 \leqslant i$ and $i \leqslant \mathrm{len}\, f$ and $1 \leqslant j$ and $j \leqslant \mathrm{len}\, f$, then $\mathrm{Replace}(\mathrm{Swap}(f, i, j), i, p) = \mathrm{Swap}(\mathrm{Replace}(f, j, p), i, j)$.

(35) If $i \neq k$ and $j \neq k$ and $1 \leqslant i$ and $i \leqslant \mathrm{len}\, f$ and $1 \leqslant j$ and $j \leqslant \mathrm{len}\, f$ and $1 \leqslant k$ and $k \leqslant \mathrm{len}\, f$, then $\mathrm{Swap}(\mathrm{Replace}(f, k, p), i, j) = \mathrm{Replace}(\mathrm{Swap}(f, i, j), k, p)$.

(36) If $i \neq k$ and $j \neq k$ and $1 \leqslant i$ and $i \leqslant \mathrm{len}\, f$ and $1 \leqslant j$ and $j \leqslant \mathrm{len}\, f$ and $1 \leqslant k$ and $k \leqslant \mathrm{len}\, f$, then $\mathrm{Swap}(\mathrm{Swap}(f, i, j), j, k) = \mathrm{Swap}(\mathrm{Swap}(f, i, k), i, j)$.

(37) Suppose $i \neq k$ and $j \neq k$ and $l \neq i$ and $l \neq j$ and $1 \leqslant i$ and $i \leqslant \mathrm{len}\, f$ and $1 \leqslant j$ and $j \leqslant \mathrm{len}\, f$ and $1 \leqslant k$ and $k \leqslant \mathrm{len}\, f$ and $1 \leqslant l$ and $l \leqslant \mathrm{len}\, f$. Then $\mathrm{Swap}(\mathrm{Swap}(f, i, j), k, l) = \mathrm{Swap}(\mathrm{Swap}(f, k, l), i, j)$.

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