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Four Variable Predicate Calculus for Boolean Valued Functions. Part II

Shunichi Kobayashi
Ueda Multimedia Information Center
Nagano

Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of ordinary predicate logic.

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The notation and terminology used here have been introduced in the following papers: [1], [2], [4], [3], and [5].

For simplicity, we use the following convention: Y is a non empty set, a is an element of $\mathrm{BVF}(Y)$, G is a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C, D are partitions of Y.

Next we state a number of propositions:

(1) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg \exists_{\exists_{a,A}G,B}G \Subset \exists_{\forall_{\neg a,B}G,A}G$.

(2) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg \exists_{\exists_{a,A}G,B}G \Subset \forall_{\forall_{\neg a,B}G,A}G$.

(3) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\neg \exists_{a,A}G,B}G \Subset \neg \exists_{\forall_{a,B}G,A}G$.

(4) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg \exists_{a,A}G,B}G \Subset \neg \exists_{\forall_{a,B}G,A}G$.

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(5) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg \exists_{a,A} G, B} G \Subset \neg \forall_{\exists_{a,B} G, A} G$.

(6) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg \exists_{a,A} G, B} G \Subset \neg \exists_{\exists_{a,B} G, A} G$.

(7) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\neg \forall_{a,A} G, B} G \Subset \exists_{\neg \forall_{a,B} G, A} G$.

(8) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg \forall_{a,A} G, B} G \Subset \exists_{\neg \forall_{a,B} G, A} G$.

(9) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\neg \exists_{a,A} G, B} G \Subset \exists_{\neg \forall_{a,B} G, A} G$.

(10) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg \exists_{a,A} G, B} G \Subset \exists_{\neg \forall_{a,B} G, A} G$.

(11) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\neg \exists_{a,A} G, B} G \Subset \forall_{\neg \forall_{a,B} G, A} G$.

(12) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg \exists_{a,A} G, B} G \Subset \forall_{\neg \forall_{a,B} G, A} G$.

(13) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg \exists_{a,A} G, B} G \Subset \exists_{\neg \exists_{a,B} G, A} G$.

(14) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg \exists_{a,A} G, B} G \Subset \forall_{\neg \exists_{a,B} G, A} G$.

(15) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\neg \exists_{a,A} G, B} G \Subset \exists_{\exists_{\neg a,B} G, A} G$.

(16) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg \exists_{a,A} G, B} G \Subset \exists_{\exists_{\neg a,B} G, A} G$.

(17) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\neg \exists_{a,A} G, B} G \Subset \forall_{\exists_{\neg a,B} G, A} G$.

(18) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$

and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(19) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \exists_{\forall_{\neg a,B}G,A}G$.

(20) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \forall_{\forall_{\neg a,B}G,A}G$.

(21) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(22) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(23) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\forall_{\exists_{a,B}G,A}G$.

(24) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\exists_{a,B}G,A}G$.

(25) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\exists_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(26) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\exists_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(27) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(28) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(29) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(30) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(31) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset$

$\exists_{\neg\exists_{a,B}G,A}G$.

(32) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \forall_{\neg\exists_{a,B}G,A}G$.

(33) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\exists_{\neg a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(34) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\exists_{\neg a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(35) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(36) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(37) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(38) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(39) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \exists_{\forall_{\neg a,B}G,A}G$.

(40) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \forall_{\forall_{\neg a,B}G,A}G$.

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