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Predicate Calculus for Boolean Valued Functions. Part VI

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The articles [4], [6], [1], [8], [7], [2], [3], [5], [11], [10], and [9] provide the terminology and notation for this paper.

1. Preliminaries

In this paper Y denotes a non empty set.

We now state several propositions:

(1) For every element z of Y and for all partitions P_1, P_2 of Y holds $\mathrm{EqClass}(z, P_1 \wedge P_2) = \mathrm{EqClass}(z, P_1) \cap \mathrm{EqClass}(z, P_2)$.

(2) Let G be a subset of PARTITIONS(Y) and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\bigwedge G = A \wedge B$.

(3) Let G be a subset of PARTITIONS(Y) and B, C, D be partitions of Y. Suppose G is a coordinate and $G = \{B, C, D\}$ and $B \neq C$ and $C \neq D$ and $D \neq B$. Then $\bigwedge G = B \wedge C \wedge D$.

(4) Let G be a subset of PARTITIONS(Y) and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\mathrm{CompF}(A, G) = B \wedge C$ and $\mathrm{CompF}(B, G) = C \wedge A$ and $\mathrm{CompF}(C, G) = A \wedge B$.

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(5) Let G be a subset of PARTITIONS(Y) and A, B, C, D be partitions of Y. Suppose $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$. Then $\mathrm{CompF}(A, G) = B \wedge C \wedge D$.

(6) Let G be a subset of PARTITIONS(Y) and A, B, C, D be partitions of Y. Suppose $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$. Then $\mathrm{CompF}(B, G) = A \wedge C \wedge D$.

(7) Let G be a subset of PARTITIONS(Y) and A, B, C, D be partitions of Y. Suppose $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$. Then $\mathrm{CompF}(C, G) = A \wedge B \wedge D$.

(8) Let G be a subset of PARTITIONS(Y) and A, B, C, D be partitions of Y. Suppose $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$. Then $\mathrm{CompF}(D, G) = A \wedge C \wedge B$.

2. Predicate Calculus

We adopt the following rules: a is an element of BVF(Y), G is a subset of PARTITIONS(Y), and A, B, C are partitions of Y.

One can prove the following propositions:

(9) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{a,A}G,B}G = \forall_{\forall_{a,B}G,A}G$.

(10) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\forall_{a,C}G,A}G,B}G = \forall_{\forall_{\forall_{a,C}G,B}G,A}G$.

(11) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\exists_{a,C}G,A}G,B}G = \forall_{\forall_{\exists_{a,C}G,B}G,A}G$.

(12) Let G be a subset of PARTITIONS(Y), B, C, D be partitions of Y, h be a function, and b, c, d be sets. Suppose $B \neq C$ and $C \neq D$ and $D \neq B$ and $h = (B \longmapsto b) +\cdot (C \longmapsto c) +\cdot (D \longmapsto d)$. Then $\mathrm{dom}\, h = \{B, C, D\}$ and $h(B) = b$ and $h(C) = c$ and $h(D) = d$ and $\mathrm{rng}\, h = \{h(B), h(C), h(D)\}$.

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