FORMALIZED MATHEMATICS
Volume 9, Number 1, 2001
University of Białystok

Predicate Calculus for Boolean Valued Functions. Part III

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

MML Identifier: `BVFUNC11`.

The papers [8], [1], [3], [5], [2], [4], [7], and [6] provide the notation and terminology for this paper.

1. Preliminaries

In this paper Y is a non empty set.

We now state several propositions:

(1) For every element z of Y and for all partitions P_1, P_2 of Y such that $P_1 \Subset P_2$ holds $\mathrm{EqClass}(z, P_1) \subseteq \mathrm{EqClass}(z, P_2)$.

(2) For every element z of Y and for all partitions P_1, P_2 of Y holds $\mathrm{EqClass}(z, P_1) \subseteq \mathrm{EqClass}(z, P_1 \vee P_2)$.

(3) For every element z of Y and for all partitions P_1, P_2 of Y holds $\mathrm{EqClass}(z, P_1 \wedge P_2) \subseteq \mathrm{EqClass}(z, P_1)$.

(4) For every element z of Y and for every partition P_1 of Y holds $\mathrm{EqClass}(z, P_1) \subseteq \mathrm{EqClass}(z, \mathcal{O}(Y))$ and $\mathrm{EqClass}(z, \mathcal{I}(Y)) \subseteq \mathrm{EqClass}(z, P_1)$.

ISSN 1426–2630

(5) Let G be a subset of PARTITIONS(Y) and A, B be partitions of Y. Suppose G is an independent family of partitions and $G = \{A, B\}$ and $A \neq B$. Let a, b be sets. If $a \in A$ and $b \in B$, then $a \cap b \neq \emptyset$.

(6) Let G be a subset of PARTITIONS(Y) and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\bigwedge G = A \wedge B$.

(7) Let G be a subset of PARTITIONS(Y) and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\mathrm{CompF}(A, G) = B$ and $\mathrm{CompF}(B, G) = A$.

2. Predicate Calculus

One can prove the following propositions:

(8) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(9) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$, then $\forall_{\forall_{a,A}G,B}G = \forall_{\forall_{a,B}G,A}G$.

(10) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$, then $\exists_{\exists_{a,A}G,B}G = \exists_{\exists_{a,B}G,A}G$.

(11) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\forall_{a,B}G,A}G$.

(12) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(13) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(14) For every element a of BVF(Y) and for every subset G of PARTITIONS(Y) and for all partitions A, B of Y holds $\forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(15) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(16) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\neg\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(17) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\forall_{\forall_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(18) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(19) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(20) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,A}G,B}G$.

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Received July 14, 1999