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Institution of Many Sorted Algebras. Part I: Signature Reduct of an Algebra

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Summary. In the paper the notation necessary to construct the institution of many sorted algebras is introduced.

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The papers [23], [27], [16], [1], [28], [14], [9], [13], [2], [26], [17], [3], [4], [10], [6], [11], [20], [24], [25], [15], [12], [21], [19], [5], [22], [7], [18], and [8] provide the terminology and notation for this paper.

1. Preliminaries

One can prove the following propositions:

(1) Let I be a set, f be a function, and F, G be many sorted functions indexed by I. If $\operatorname{rng} f \subseteq I$, then $(G \circ F) \cdot f = (G \cdot f) \circ (F \cdot f)$.

(2) Let S be a non empty non void many sorted signature, o be an operation symbol of S, V be a non-empty many sorted set indexed by the carrier of S, and x be a set. Then x is an argument sequence of $\mathrm{Sym}(o, V)$ if and only if x is an element of $\mathrm{Args}(o, \mathrm{Free}(V))$.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, and let o be an operation symbol of S. Note that every element of $\mathrm{Args}(o, \mathrm{Free}(V))$ is decorated tree yielding.

Next we state two propositions:

(3) Let S be a non empty non void many sorted signature and A_1, A_2 be algebras over S. Suppose the sorts of A_1 are transformable to the sorts of A_2. Let o be an operation symbol of S. If $\mathrm{Args}(o, A_1) \neq \emptyset$, then $\mathrm{Args}(o, A_2) \neq \emptyset$.

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(4) Let S be a non empty non void many sorted signature, o be an operation symbol of S, V be a non-empty many sorted set indexed by the carrier of S, and x be an element of $\mathrm{Args}(o, \mathrm{Free}(V))$. Then $(\mathrm{Den}(o, \mathrm{Free}(V)))(x) = \langle o,$ the carrier of $S\rangle$-tree(x).

Let S be a non empty non void many sorted signature and let A be a non-empty algebra over S. One can check that the algebra of A is non-empty.

Next we state three propositions:

(5) Let S be a non empty non void many sorted signature and A, B be algebras over S. Suppose the algebra of $A =$ the algebra of B. Let o be an operation symbol of S. Then $\mathrm{Den}(o, A) = \mathrm{Den}(o, B)$.

(6) Let S be a non empty non void many sorted signature and A_1, A_2, B_1, B_2 be algebras over S. Suppose the algebra of $A_1 =$ the algebra of B_1 and the algebra of $A_2 =$ the algebra of B_2. Let f be a many sorted function from A_1 into A_2 and g be a many sorted function from B_1 into B_2. Suppose $f = g$. Let o be an operation symbol of S. Suppose $\mathrm{Args}(o, A_1) \neq \emptyset$ and $\mathrm{Args}(o, A_2) \neq \emptyset$. Let x be an element of $\mathrm{Args}(o, A_1)$ and y be an element of $\mathrm{Args}(o, B_1)$. If $x = y$, then $f\#x = g\#y$.

(7) Let S be a non empty non void many sorted signature and A_1, A_2, B_1, B_2 be algebras over S. Suppose that

(i) the algebra of $A_1 =$ the algebra of B_1,

(ii) the algebra of $A_2 =$ the algebra of B_2, and

(iii) the sorts of A_1 are transformable to the sorts of A_2.

Let h be a many sorted function from A_1 into A_2. Suppose h is a homomorphism of A_1 into A_2. Then there exists a many sorted function h' from B_1 into B_2 such that $h' = h$ and h' is a homomorphism of B_1 into B_2.

Let S be a many sorted signature. We say that S is feasible if and only if:

(Def. 1) If the carrier of $S = \emptyset$, then the operation symbols of $S = \emptyset$.

The following proposition is true

(8) Let S be a many sorted signature. Then S is feasible if and only if dom (the result sort of S) $=$ the operation symbols of S.

One can verify the following observations:

* every many sorted signature which is non empty is also feasible,
* every many sorted signature which is void is also feasible,
* every many sorted signature which is empty and feasible is also void, and
* every many sorted signature which is non void and feasible is also non empty.

Let us note that there exists a many sorted signature which is non void and non empty.

One can prove the following propositions:

(9) Let S be a feasible many sorted signature. Then $\mathrm{id}_{\text{the carrier of } S}$ and $\mathrm{id}_{\text{the operation symbols of } S}$ form morphism between S and S.

(10) Let S_1, S_2 be many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2. Then
(i) f is a function from the carrier of S_1 into the carrier of S_2, and
(ii) g is a function from the operation symbols of S_1 into the operation symbols of S_2.

2. Subsignatures

Let S be a feasible many sorted signature. A many sorted signature is said to be a subsignature of S if:

(Def. 2) $\mathrm{id}_{\text{the carrier of it}}$ and $\mathrm{id}_{\text{the operation symbols of it}}$ form morphism between it and S.

We now state the proposition

(11) Let S be a feasible many sorted signature and T be a subsignature of S. Then the carrier of $T \subseteq$ the carrier of S and the operation symbols of $T \subseteq$ the operation symbols of S.

Let S be a feasible many sorted signature. Note that every subsignature of S is feasible.

Next we state several propositions:

(12) Let S be a feasible many sorted signature and T be a subsignature of S. Then the result sort of $T \subseteq$ the result sort of S and the arity of $T \subseteq$ the arity of S.

(13) Let S be a feasible many sorted signature and T be a subsignature of S. Then
(i) the arity of $T =$ (the arity of S)$\restriction$(the operation symbols of T), and
(ii) the result sort of $T =$ (the result sort of S)$\restriction$(the operation symbols of T).

(14) Let S, T be feasible many sorted signatures. Suppose that
(i) the carrier of $T \subseteq$ the carrier of S,
(ii) the arity of $T \subseteq$ the arity of S, and
(iii) the result sort of $T \subseteq$ the result sort of S.
Then T is a subsignature of S.

(15) Let S, T be feasible many sorted signatures. Suppose that
(i) the carrier of $T \subseteq$ the carrier of S,
(ii) the arity of $T =$ (the arity of S)$\restriction$(the operation symbols of T), and
(iii) the result sort of $T =$ (the result sort of S)$\restriction$(the operation symbols of T).
Then T is a subsignature of S.

(16) Every feasible many sorted signature S is a subsignature of S.

(17) For every feasible many sorted signature S_1 and for every subsignature S_2 of S_1 holds every subsignature of S_2 is a subsignature of S_1.

(18) Let S_1 be a feasible many sorted signature and S_2 be a subsignature of S_1. Suppose S_1 is a subsignature of S_2. Then the many sorted signature of $S_1 =$ the many sorted signature of S_2.

Let S be a non empty many sorted signature. Observe that there exists a subsignature of S which is non empty.

Let S be a non void feasible many sorted signature. One can verify that there exists a subsignature of S which is non void.

One can prove the following three propositions:

(19) Let S be a feasible many sorted signature, S' be a subsignature of S, T be a many sorted signature, and f, g be functions. Suppose f and g form morphism between S and T. Then $f{\restriction}$the carrier of S' and $g{\restriction}$the operation symbols of S' form morphism between S' and T.

(20) Let S be a many sorted signature, T be a feasible many sorted signature, T' be a subsignature of T, and f, g be functions. Suppose f and g form morphism between S and T'. Then f and g form morphism between S and T.

(21) Let S be a many sorted signature, T be a feasible many sorted signature, T' be a subsignature of T, and f, g be functions. Suppose f and g form morphism between S and T and $\operatorname{rng} f \subseteq$ the carrier of T' and $\operatorname{rng} g \subseteq$ the operation symbols of T'. Then f and g form morphism between S and T'.

3. Signature reducts

Let S_1, S_2 be non empty many sorted signatures, let A be an algebra over S_2, and let f, g be functions. Let us assume that f and g form morphism between S_1 and S_2. The functor $A{\restriction}_{(f,g)}S_1$ yields a strict algebra over S_1 and is defined by the conditions (Def. 3).

(Def. 3)(i) The sorts of $A{\restriction}_{(f,g)}S_1 =$ (the sorts of A) $\cdot f$, and

(ii) the characteristics of $A{\restriction}_{(f,g)}S_1 =$ (the characteristics of A) $\cdot g$.

Let S_2, S_1 be non empty many sorted signatures and let A be an algebra over S_2. The functor $A{\restriction}S_1$ yields a strict algebra over S_1 and is defined as follows:

(Def. 4) $A{\restriction}S_1 = A{\restriction}_{(\mathrm{id}_{\text{the carrier of } S_1}, \mathrm{id}_{\text{the operation symbols of } S_1})}S_1$.

We now state two propositions:

(22) Let S_1, S_2 be non empty many sorted signatures and A, B be algebras over S_2. Suppose the algebra of $A =$ the algebra of B. Let f, g be functions. If f and g form morphism between S_1 and S_2, then $A{\restriction}_{(f,g)}S_1 = B{\restriction}_{(f,g)}S_1$.

(23) Let S_1, S_2 be non empty many sorted signatures, A be a non-empty algebra over S_2, and f, g be functions. If f and g form morphism between S_1 and S_2, then $A{\restriction}_{(f,g)}S_1$ is non-empty.

Let S_2 be a non empty many sorted signature, let S_1 be a non empty subsignature of S_2, and let A be a non-empty algebra over S_2. Observe that $A{\restriction}S_1$ is non-empty.

The following propositions are true:

(24) Let S_1, S_2 be non void non empty many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2. Let A be an algebra over S_2, o_1 be an operation symbol of S_1, and o_2 be an operation symbol of S_2. If $o_2 = g(o_1)$, then $\mathrm{Den}(o_1, A{\restriction}_{(f,g)}S_1) = \mathrm{Den}(o_2, A)$.

(25) Let S_1, S_2 be non void non empty many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2. Let A be an algebra over S_2, o_1 be an operation symbol of S_1, and o_2 be an operation symbol of S_2. If $o_2 = g(o_1)$, then $\mathrm{Args}(o_2, A) = \mathrm{Args}(o_1, A{\restriction}_{(f,g)}S_1)$ and $\mathrm{Result}(o_1, A{\restriction}_{(f,g)}S_1) = \mathrm{Result}(o_2, A)$.

(26) Let S be a non empty many sorted signature and A be an algebra over S. Then $A{\restriction}_{(\mathrm{id}_{\text{the carrier of } S}, \mathrm{id}_{\text{the operation symbols of } S})}S =$ the algebra of A.

(27) For every non empty many sorted signature S and for every algebra A over S holds $A{\restriction}S =$ the algebra of A.

(28) Let S_1, S_2, S_3 be non empty many sorted signatures and f_1, g_1 be functions. Suppose f_1 and g_1 form morphism between S_1 and S_2. Let f_2, g_2 be functions. Suppose f_2 and g_2 form morphism between S_2 and S_3. Let A be an algebra over S_3. Then $A{\restriction}_{(f_2 \cdot f_1, g_2 \cdot g_1)}S_1 = A{\restriction}_{(f_2,g_2)}S_2{\restriction}_{(f_1,g_1)}S_1$.

(29) Let S_1 be a non empty feasible many sorted signature, S_2 be a non empty subsignature of S_1, S_3 be a non empty subsignature of S_2, and A be an algebra over S_1. Then $A{\restriction}S_3 = A{\restriction}S_2{\restriction}S_3$.

(30) Let S_1, S_2 be non empty many sorted signatures, f be a function from the carrier of S_1 into the carrier of S_2, and g be a function. Suppose f and g form morphism between S_1 and S_2. Let A_1, A_2 be algebras over S_2 and h be a many sorted function from the sorts of A_1 into the sorts of A_2. Then $h \cdot f$ is a many sorted function from the sorts of $A_1{\restriction}_{(f,g)}S_1$ into the sorts of $A_2{\restriction}_{(f,g)}S_1$.

(31) Let S_1 be a non empty many sorted signature, S_2 be a non empty subsignature of S_1, A_1, A_2 be algebras over S_1, and h be a many sorted function from the sorts of A_1 into the sorts of A_2. Then $h{\restriction}$the carrier of S_2 is a many sorted function from the sorts of $A_1{\restriction}S_2$ into the sorts of $A_2{\restriction}S_2$.

(32) Let S_1, S_2 be non empty many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2. Let A be an algebra over S_2. Then $\mathrm{id}_{\text{the sorts of } A} \cdot f = \mathrm{id}_{\text{the sorts of } A{\restriction}_{(f,g)}S_1}$.

(33) Let S_1 be a non empty many sorted signature, S_2 be a non empty subsignature of S_1, and A be an algebra over S_1 Then $\mathrm{id}_{\text{the sorts of } A}{\restriction}$the carrier of $S_2 = \mathrm{id}_{\text{the sorts of } A{\restriction}S_2}$.

(34) Let S_1, S_2 be non void non empty many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2. Let A, B be algebras over S_2, h_2 be a many sorted function from A into B, and h_1 be a many sorted function from $A{\restriction}_{(f,g)}S_1$ into $B{\restriction}_{(f,g)}S_1$. Suppose $h_1 = h_2 \cdot f$. Let o_1 be an operation symbol of S_1 and o_2 be an operation symbol of S_2.

Suppose $o_2 = g(o_1)$ and $\mathrm{Args}(o_2, A) \neq \emptyset$ and $\mathrm{Args}(o_2, B) \neq \emptyset$. Let x_2 be an element of $\mathrm{Args}(o_2, A)$ and x_1 be an element of $\mathrm{Args}(o_1, A{\restriction}_{(f,g)}S_1)$. If $x_2 = x_1$, then $h_1 \# x_1 = h_2 \# x_2$.

(35) Let S, S' be non empty non void many sorted signatures and A_1, A_2 be algebras over S. Suppose the sorts of A_1 are transformable to the sorts of A_2. Let h be a many sorted function from A_1 into A_2. Suppose h is a homomorphism of A_1 into A_2. Let f be a function from the carrier of S' into the carrier of S and g be a function. Suppose f and g form morphism between S' and S. Then there exists a many sorted function h' from $A_1{\restriction}_{(f,g)}S'$ into $A_2{\restriction}_{(f,g)}S'$ such that $h' = h \cdot f$ and h' is a homomorphism of $A_1{\restriction}_{(f,g)}S'$ into $A_2{\restriction}_{(f,g)}S'$.

(36) Let S be a non void feasible many sorted signature, S' be a non void subsignature of S, and A_1, A_2 be algebras over S. Suppose the sorts of A_1 are transformable to the sorts of A_2. Let h be a many sorted function from A_1 into A_2. Suppose h is a homomorphism of A_1 into A_2. Then there exists a many sorted function h' from $A_1{\restriction}S'$ into $A_2{\restriction}S'$ such that $h' = h{\restriction}$the carrier of S' and h' is a homomorphism of $A_1{\restriction}S'$ into $A_2{\restriction}S'$.

(37) Let S, S' be non empty non void many sorted signatures, A be a non-empty algebra over S, f be a function from the carrier of S' into the carrier of S, and g be a function. Suppose f and g form morphism between S' and S. Let B be a non-empty algebra over S'. Suppose $B = A{\restriction}_{(f,g)}S'$. Let s_1, s_2 be sort symbols of S' and t be a function. Suppose t is an elementary translation in B from s_1 into s_2. Then t is an elementary translation in A from $f(s_1)$ into $f(s_2)$.

(38) Let S, S' be non empty non void many sorted signatures, f be a function from the carrier of S' into the carrier of S, and g be a function. Suppose f and g form morphism between S' and S. Let s_1, s_2 be sort symbols of S'. If $\mathrm{TranslRel}(S')$ reduces s_1 to s_2, then $\mathrm{TranslRel}(S)$ reduces $f(s_1)$ to $f(s_2)$.

(39) Let S, S' be non void non empty many sorted signatures, A be a non-empty algebra over S, f be a function from the carrier of S' into the carrier of S, and g be a function. Suppose f and g form morphism between S' and S. Let B be a non-empty algebra over S'. Suppose $B = A{\restriction}_{(f,g)}S'$. Let s_1, s_2 be sort symbols of S'. Suppose $\mathrm{TranslRel}(S')$ reduces s_1 to s_2. Then every translation in B from s_1 into s_2 is a translation in A from $f(s_1)$ into $f(s_2)$.

4. Translating homomorphisms

The scheme *GenFuncEx* concerns a non empty non void many sorted signature $\mathcal{A}$, a non-empty algebra $\mathcal{B}$ over $\mathcal{A}$, a non-empty many sorted set $\mathcal{C}$ indexed by the carrier of $\mathcal{A}$, and a binary functor $\mathcal{F}$ yielding a set, and states that:

There exists a many sorted function h from Free($\mathcal{C}$) into $\mathcal{B}$ such that

(i) h is a homomorphism of Free($\mathcal{C}$) into $\mathcal{B}$, and

(ii) for every sort symbol s of $\mathcal{A}$ and for every element x of $\mathcal{C}(s)$ holds $h(s)$(the root tree of $\langle x, s\rangle) = \mathcal{F}(x, s)$

provided the parameters meet the following requirement:

- For every sort symbol s of $\mathcal{A}$ and for every element x of $\mathcal{C}(s)$ holds $\mathcal{F}(x, s) \in$ (the sorts of $\mathcal{B})(s)$.

One can prove the following proposition

(40) Let I be a set, A, B be many sorted sets indexed by I, C be a many sorted subset of A, F be a many sorted function from A into B, and i be a set. Suppose $i \in I$. Let f, g be functions. Suppose $f = F(i)$ and $g = (F \restriction C)(i)$. Let x be a set. If $x \in C(i)$, then $g(x) = f(x)$.

Let S be a non void non empty many sorted signature and let X be a non-empty many sorted set indexed by the carrier of S. Note that FreeGenerator(X) is non-empty.

Let S_1, S_2 be non empty non void many sorted signatures, let X be a non-empty many sorted set indexed by the carrier of S_2, let f be a function from the carrier of S_1 into the carrier of S_2, and let g be a function. Let us assume that f and g form morphism between S_1 and S_2. The functor hom(f, g, X, S_1, S_2) yields a many sorted function from Free$(X \cdot f)$ into Free$(X)\restriction_{(f,g)}S_1$ and is defined by the conditions (Def. 5).

(Def. 5)(i) hom(f, g, X, S_1, S_2) is a homomorphism of Free$(X \cdot f)$ into Free$(X)\restriction_{(f,g)}S_1$, and

(ii) for every sort symbol s of S_1 and for every element x of $(X \cdot f)(s)$ holds $($hom$(f, g, X, S_1, S_2))(s)$(the root tree of $\langle x, s\rangle$) = the root tree of $\langle x, f(s)\rangle$.

We now state several propositions:

(41) Let S_1, S_2 be non void non empty many sorted signatures, X be a non-empty many sorted set indexed by the carrier of S_2, f be a function from the carrier of S_1 into the carrier of S_2, and g be a function. Suppose f and g form morphism between S_1 and S_2. Let o be an operation symbol of S_1, p be an element of Args$(o,$ Free$(X \cdot f))$, and q be a finite sequence. Suppose $q = $ hom$(f, g, X, S_1, S_2)\#p$. Then $($hom$(f, g, X, S_1, S_2))$(the result sort of $o)(\langle o,$ the carrier of $S_1\rangle$-tree$(p)) = \langle g(o),$ the carrier of $S_2\rangle$-tree(q).

(42) Let S_1, S_2 be non void non empty many sorted signatures, X be a non-empty many sorted set indexed by the carrier of S_2, f be a function from the carrier of S_1 into the carrier of S_2, and g be a function. Suppose f and g form morphism between S_1 and S_2. Let t be a term of S_1 over $X \cdot f$. Then $($hom$(f, g, X, S_1, S_2))$(the sort of $t)(t)$ is a compound term of S_2 over X if and only if t is a compound term of S_1 over $X \cdot f$.

(43) Let S_1, S_2 be non void non empty many sorted signatures, X be a non-empty many sorted set indexed by the carrier of S_2, f be a function from the carrier of S_1 into the carrier of S_2, and g be an one-to-

one function. Suppose f and g form morphism between S_1 and S_2. Then $\hom(f, g, X, S_1, S_2)$ is a monomorphism of $\mathrm{Free}(X \cdot f)$ into $\mathrm{Free}(X){\restriction}_{(f,g)}S_1$.

(44) Let S be a non void non empty many sorted signature and X be a non-empty many sorted set indexed by the carrier of S. Then $\hom(\mathrm{id}_{\text{the carrier of } S}, \mathrm{id}_{\text{the operation symbols of } S}, X, S, S) = \mathrm{id}_{\text{the sorts of } \mathrm{Free}(X)}$.

(45) Let S_1, S_2, S_3 be non void non empty many sorted signatures, X be a non-empty many sorted set indexed by the carrier of S_3, f_1 be a function from the carrier of S_1 into the carrier of S_2, and g_1 be a function. Suppose f_1 and g_1 form morphism between S_1 and S_2. Let f_2 be a function from the carrier of S_2 into the carrier of S_3 and g_2 be a function. Suppose f_2 and g_2 form morphism between S_2 and S_3. Then $\hom(f_2 \cdot f_1, g_2 \cdot g_1, X, S_1, S_3) = (\hom(f_2, g_2, X, S_2, S_3) \cdot f_1) \circ \hom(f_1, g_1, X \cdot f_2, S_1, S_2)$.

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