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Definitions and Basic Properties of Boolean & Union of Many Sorted Sets

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Summary. In the first part of this article I have proved theorems about boolean of many sorted sets which are corresponded to theorems about boolean of sets, whereas the second part of this article contains propositions about union of many sorted sets. Boolean as well as union of many sorted sets are defined as boolean and union on every sorts.

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The terminology and notation used here are introduced in the following articles: [11], [12], [13], [2], [3], [5], [9], [4], [1], [10], [7], [6], and [8].

1. Boolean of Many Sorted Sets

We follow a convention: I will denote a set, A, B, X, Y will denote many sorted sets indexed by I, and x, y will be arbitrary.

Let us consider I, A. The functor 2^A yielding a many sorted set indexed by I is defined as follows:

(Def.1) For arbitrary i such that $i \in I$ holds $2^A(i) = 2^{A(i)}$.

Let us consider I, A. Note that 2^A is non-empty.

One can prove the following propositions:

(1) $X = 2^Y$ iff for every A holds $A \in X$ iff $A \subseteq Y$.

(2) $2^{\emptyset_I} = I \longmapsto \{\emptyset\}$.

(3) $2^{I \longmapsto x} = I \longmapsto 2^x$.

(4) $2^{I \longmapsto \{x\}} = I \longmapsto \{\emptyset, \{x\}\}$.

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(5) $\emptyset_I \in 2^A$.

(6) If $A \subseteq B$, then $2^A \subseteq 2^B$.

(7) $2^A \cup 2^B \subseteq 2^{A \cup B}$.

(8) If $2^A \cup 2^B = 2^{A \cup B}$, then for arbitrary i such that $i \in I$ holds $A(i) \subseteq B(i)$ or $B(i) \subseteq A(i)$.

(9) $2^{A \cap B} = 2^A \cap 2^B$.

(10) $2^{A \setminus B} \subseteq (I \longmapsto \{\emptyset\}) \cup (2^A \setminus 2^B)$.

(11) $X \in 2^{A \setminus B}$ iff $X \subseteq A$ and X misses B.

(12) $2^{A \setminus B} \cup 2^{B \setminus A} \subseteq 2^{A \dot{-} B}$.

(13) $X \in 2^{A \dot{-} B}$ iff $X \subseteq A \cup B$ and X misses $A \cap B$.

(14) If $X \in 2^A$ and $Y \in 2^A$, then $X \cup Y \in 2^A$.

(15) If $X \in 2^A$ or $Y \in 2^A$, then $X \cap Y \in 2^A$.

(16) If $X \in 2^A$, then $X \setminus Y \in 2^A$.

(17) If $X \in 2^A$ and $Y \in 2^A$, then $X \dot{-} Y \in 2^A$.

(18) $[\![X, Y]\!] \subseteq 2^{2^{X \cup Y}}$.

(19) $X \subseteq A$ iff $X \in 2^A$.

(20) $\mathrm{MSFuncs}(A, B) \subseteq 2^{[\![A,B]\!]}$.

2. Union of Many Sorted Sets

Let us consider I, A. The functor $\bigcup A$ yields a many sorted set indexed by I and is defined as follows:

(Def.2) For arbitrary i such that $i \in I$ holds $(\bigcup A)(i) = \bigcup A(i)$.

Let us consider I. Observe that $\bigcup(\emptyset_I)$ is empty yielding.

We now state a number of propositions:

(21) $A \in \bigcup X$ iff there exists Y such that $A \in Y$ and $Y \in X$.

(22) $\bigcup(\emptyset_I) = \emptyset_I$.

(23) $\bigcup(I \longmapsto x) = I \longmapsto \bigcup x$.

(24) $\bigcup(I \longmapsto \{x\}) = I \longmapsto x$.

(25) $\bigcup(I \longmapsto \{\{x\}, \{y\}\}) = I \longmapsto \{x, y\}$.

(26) If $X \in A$, then $X \subseteq \bigcup A$.

(27) If $A \subseteq B$, then $\bigcup A \subseteq \bigcup B$.

(28) $\bigcup(A \cup B) = \bigcup A \cup \bigcup B$.

(29) $\bigcup(A \cap B) \subseteq \bigcup A \cap \bigcup B$.

(30) $\bigcup(2^A) = A$.

(31) $A \subseteq 2^{\bigcup A}$.

(32) If $\bigcup Y \subseteq A$ and $X \in Y$, then $X \subseteq A$.

(33) Let Z be a many sorted set indexed by I and let A be a non-empty many sorted set indexed by I. Suppose that for every many sorted set X indexed by I such that $X \in A$ holds $X \subseteq Z$. Then $\bigcup A \subseteq Z$.

(34) Let B be a many sorted set indexed by I and let A be a non-empty many sorted set indexed by I. Suppose that for every many sorted set X indexed by I such that $X \in A$ holds $X \cap B = \emptyset_I$. Then $\bigcup A \cap B = \emptyset_I$.

(35) Let A, B be many sorted sets indexed by I. Suppose $A \cup B$ is non-empty. Suppose that for all many sorted sets X, Y indexed by I such that $X \neq Y$ and $X \in A \cup B$ and $Y \in A \cup B$ holds $X \cap Y = \emptyset_I$. Then $\bigcup(A \cap B) = \bigcup A \cap \bigcup B$.

(36) Let A, X be many sorted sets indexed by I and let B be a non-empty many sorted set indexed by I. Suppose $X \subseteq \bigcup(A \cup B)$ and for every many sorted set Y indexed by I such that $Y \in B$ holds $Y \cap X = \emptyset_I$. Then $X \subseteq \bigcup A$.

(37) Let A be a locally-finite non-empty many sorted set indexed by I. Suppose that for all many sorted sets X, Y indexed by I such that $X \in A$ and $Y \in A$ holds $X \subseteq Y$ or $Y \subseteq X$. Then $\bigcup A \in A$.

References

[1] Czesław Byliński. A classical first order language. *Formalized Mathematics*, 1(**4**):669–676, 1990.
[2] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[3] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.
[4] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Formalized Mathematics*, 1(**3**):521–527, 1990.
[5] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.
[6] Artur Korniłowicz. On the group of automorphisms of universal algebra & many sorted algebra. *Formalized Mathematics*, 5(**2**):221–226, 1996.
[7] Beata Madras. Products of many sorted algebras. *Formalized Mathematics*, 5(**1**):55–60, 1996.
[8] Yatsuka Nakamura, Piotr Rudnicki, Andrzej Trybulec, and Pauline N. Kawamoto. Preliminaries to circuits, I. *Formalized Mathematics*, 5(**2**):167–172, 1996.
[9] Andrzej Trybulec. Binary operations applied to functions. *Formalized Mathematics*, 1(**2**):329–334, 1990.
[10] Andrzej Trybulec. Many-sorted sets. *Formalized Mathematics*, 4(**1**):15–22, 1993.
[11] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[12] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[13] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

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