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Extensions of Mappings on Generator Set

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Summary. The aim of the article is to prove the fact that if extensions of mappings on generator set are equal then these mappings are equal. The article contains the properties of epimorphisms & monomorphisms between Many Sorted Algebras.

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The articles [15], [17], [18], [6], [16], [8], [7], [1], [2], [3], [14], [5], [11], [13], [4], [10], [9], and [12] provide the terminology and notation for this paper.

1. Preliminaries

For simplicity we adopt the following convention: S will be a non void non empty many sorted signature, U_1, U_2, U_3 will be non-empty algebras over S, I will be a set, A will be a many sorted set indexed by I, and B, C will be non-empty many sorted sets indexed by I.

We now state four propositions:

(1) For every binary relation R and for all sets X, Y such that $X \subseteq Y$ holds $(R \restriction Y)^\circ X = R^\circ X$.

(2) Let A be a set, and let B, C be non empty sets, and let f be a function from A into B, and let g be a function from B into C, and let X be a subset of A. Then $(g \cdot f) \restriction X = g \cdot (f \restriction X)$.

(3) For every function yielding function f holds $\mathrm{dom}(\mathrm{dom}_\kappa f(\kappa)) = \mathrm{dom} f$.

(4) For every function yielding function f holds $\mathrm{dom}(\mathrm{rng}_\kappa f(\kappa)) = \mathrm{dom} f$.

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2. Facts about Many Sorted Functions

Next we state several propositions:

(5) Let F be a many sorted function from A into B and let X be a many sorted subset of A. If $A \subseteq X$, then $F \restriction X = F$.

(6) Let A, B be many sorted sets indexed by I, and let M be a many sorted subset of A, and let F be a many sorted function from A into B. Then $F \circ M \subseteq F \circ A$.

(7) Let F be a many sorted function from A into B and let M_1, M_2 be many sorted subsets of A. If $M_1 \subseteq M_2$, then $(F \restriction M_2) \circ M_1 = F \circ M_1$.

(8) Let F be a many sorted function from A into B, and let G be a many sorted function from B into C, and let X be a many sorted subset of A. Then $(G \circ F) \restriction X = G \circ (F \restriction X)$.

(9) Let A, B be many sorted sets indexed by I. Suppose A is transformable to B. Let F be a many sorted function from A into B and let C be a many sorted set indexed by I. Suppose B is a many sorted subset of C. Then F is a many sorted function from A into C.

(10) Let F be a many sorted function from A into B and let X be a many sorted subset of A. If F is "1-1", then $F \restriction X$ is "1-1".

3. dom's & rng's of Many Sorted Functions

Let us consider I and let F be a many sorted function of I. Then $\operatorname{dom}_\kappa F(\kappa)$ is a many sorted set indexed by I.

Let us consider I and let F be a many sorted function of I. Then $\operatorname{rng}_\kappa F(\kappa)$ is a many sorted set indexed by I.

We now state several propositions:

(11) For every many sorted function F from A into B and for every many sorted subset X of A holds $\operatorname{dom}_\kappa F \restriction X(\kappa) \subseteq \operatorname{dom}_\kappa F(\kappa)$.

(12) For every many sorted function F from A into B and for every many sorted subset X of A holds $\operatorname{rng}_\kappa F \restriction X(\kappa) \subseteq \operatorname{rng}_\kappa F(\kappa)$.

(13) Let A, B be many sorted sets indexed by I and let F be a many sorted function from A into B. Then F is "onto" if and only if $\operatorname{rng}_\kappa F(\kappa) = B$.

(14) For every non-empty many sorted set X indexed by the carrier of S holds $\operatorname{rng}_\kappa \operatorname{Reverse}(X)(\kappa) = X$.

(15) Let F be a many sorted function from A into B, and let G be a many sorted function from B into C, and let X be a non-empty many sorted subset of B. If $\operatorname{rng}_\kappa F(\kappa) \subseteq X$, then $(G \restriction X) \circ F = G \circ F$.

4. Other properties of "onto" & "1-1"

Next we state two propositions:

(16) Let F be a many sorted function from A into B. Then F is "onto" if and only if for every C and for all many sorted functions G, H from B into C such that $G \circ F = H \circ F$ holds $G = H$.

(17) Let F be a many sorted function from A into B. Suppose A is non-empty and B is non-empty. Then F is "1-1" if and only if for every many sorted set C indexed by I and for all many sorted functions G, H from C into A such that $F \circ G = F \circ H$ holds $G = H$.

5. Extensions of Mappings on Generator Set

We now state three propositions:

(18) Let X be a non-empty many sorted set indexed by the carrier of S and let h_1, h_2 be many sorted functions from $\mathrm{Free}(X)$ into U_1. Suppose h_1 is a homomorphism of $\mathrm{Free}(X)$ into U_1 and h_2 is a homomorphism of $\mathrm{Free}(X)$ into U_1 and $h_1 \restriction \mathrm{FreeGenerator}(X) = h_2 \restriction \mathrm{FreeGenerator}(X)$. Then $h_1 = h_2$.

(19) Let F be a many sorted function from U_1 into U_2. Suppose F is a homomorphism of U_1 into U_2. Suppose F is an epimorphism of U_1 onto U_2. Let U_3 be a non-empty algebra over S and let h_1, h_2 be many sorted functions from U_2 into U_3. Suppose h_1 is a homomorphism of U_2 into U_3 and h_2 is a homomorphism of U_2 into U_3. If $h_1 \circ F = h_2 \circ F$, then $h_1 = h_2$.

(20) Let F be a many sorted function from U_2 into U_3. Suppose F is a homomorphism of U_2 into U_3. Then F is a monomorphism of U_2 into U_3 if and only if for every non-empty algebra U_1 over S and for all many sorted functions h_1, h_2 from U_1 into U_2 such that h_1 is a homomorphism of U_1 into U_2 and h_2 is a homomorphism of U_1 into U_2 holds if $F \circ h_1 = F \circ h_2$, then $h_1 = h_2$.

Let us consider S, U_1. Note that there exists a generator set of U_1 which is non-empty.

We now state three propositions:

(21) For all non-empty subsets A, B of U_1 such that A is a many sorted subset of B holds $\mathrm{Gen}(A)$ is a subalgebra of $\mathrm{Gen}(B)$.

(22) Let U_2 be a non-empty subalgebra of U_1, and let B_1 be a non-empty subset of U_1, and let B_2 be a subset of U_2. If $B_1 = B_2$, then $\mathrm{Gen}(B_1) = \mathrm{Gen}(B_2)$.

(23) Let U_1 be a strict non-empty algebra over S, and let U_2 be a non-empty algebra over S, and let G_1 be a non-empty generator set of U_1, and let h_1, h_2 be many sorted functions from U_1 into U_2. Suppose h_1 is

a homomorphism of U_1 into U_2 and h_2 is a homomorphism of U_1 into U_2 and $h_1 \restriction G_1 = h_2 \restriction G_1$. Then $h_1 = h_2$.

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