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Complex Sequences

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Summary. Definitions of complex sequence and operations on sequences (multiplication of sequences and multiplication by a complex number, addition, subtraction, division and absolute value of sequence) are given. We followed [3].

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The terminology and notation used here are introduced in the following articles: [5], [1], [2], [4], and [3].

For simplicity we follow a convention: f will denote a function, n will denote a natural number, r, p will denote elements of $\mathbb{C}$, and x will be arbitrary.

A complex sequence is a function from $\mathbb{N}$ into $\mathbb{C}$.

In the sequel s_1, s_2, s_3, s_4, s'_1, s'_2 denote complex sequences.

One can prove the following propositions:

(1) f is a complex sequence iff $\operatorname{dom} f = \mathbb{N}$ and for every x such that $x \in \mathbb{N}$ holds $f(x)$ is an element of $\mathbb{C}$.

(2) f is a complex sequence iff $\operatorname{dom} f = \mathbb{N}$ and for every n holds $f(n)$ is an element of $\mathbb{C}$.

Let us consider s_1, n. Then $s_1(n)$ is an element of $\mathbb{C}$.

The scheme *ExComplexSeq* deals with a unary functor $\mathcal{F}$ yielding an element of $\mathbb{C}$, and states that:

There exists s_1 such that for every n holds $s_1(n) = \mathcal{F}(n)$

for all values of the parameter.

A complex sequence is non-zero if:

(Def.1) $\operatorname{rng} \mathrm{it} \subseteq \mathbb{C} \setminus \{0_{\mathbb{C}}\}$.

One can prove the following proposition

(3) s_1 is non-zero iff for every x such that $x \in \mathbb{N}$ holds $s_1(x) \neq 0_{\mathbb{C}}$.

Let us mention that there exists a complex sequence which is non-zero.

Next we state four propositions:

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(4) s_1 is non-zero iff for every n holds $s_1(n) \neq 0_{\mathbb{C}}$.

(5) For all s_1, s_2 such that for every x such that $x \in \mathbb{N}$ holds $s_1(x) = s_2(x)$ holds $s_1 = s_2$.

(6) For all s_1, s_2 such that for every n holds $s_1(n) = s_2(n)$ holds $s_1 = s_2$.

(7) For every r there exists s_1 such that $\operatorname{rng} s_1 = \{r\}$.

Let us consider s_2, s_3. The functor $s_2 + s_3$ yielding a complex sequence is defined as follows:

(Def.2) For every n holds $(s_2 + s_3)(n) = s_2(n) + s_3(n)$.

The functor $s_2\, s_3$ yielding a complex sequence is defined by:

(Def.3) For every n holds $(s_2\, s_3)(n) = s_2(n) \cdot s_3(n)$.

Let us consider r, s_1. The functor $r\, s_1$ yielding a complex sequence is defined as follows:

(Def.4) For every n holds $(r\, s_1)(n) = r \cdot s_1(n)$.

Let us consider s_1. The functor $-s_1$ yielding a complex sequence is defined as follows:

(Def.5) For every n holds $(-s_1)(n) = -s_1(n)$.

Let us consider s_2, s_3. The functor $s_2 - s_3$ yields a complex sequence and is defined as follows:

(Def.6) $s_2 - s_3 = s_2 + -s_3$.

Let us consider s_1. The functor ${s_1}^{-1}$ yields a complex sequence and is defined as follows:

(Def.7) For every n holds ${s_1}^{-1}(n) = s_1(n)^{-1}$.

Let us consider s_2, s_1. The functor $\frac{s_2}{s_1}$ yielding a complex sequence is defined as follows:

(Def.8) $\frac{s_2}{s_1} = s_2\, {s_1}^{-1}$.

Let us consider s_1. The functor $|s_1|$ yields a sequence of real numbers and is defined by:

(Def.9) For every n holds $|s_1|(n) = |s_1(n)|$.

The following propositions are true:

(8) $s_2 + s_3 = s_3 + s_2$.

(9) $(s_2 + s_3) + s_4 = s_2 + (s_3 + s_4)$.

(10) $s_2\, s_3 = s_3\, s_2$.

(11) $(s_2\, s_3)\, s_4 = s_2\, (s_3\, s_4)$.

(12) $(s_2 + s_3)\, s_4 = s_2\, s_4 + s_3\, s_4$.

(13) $s_4\, (s_2 + s_3) = s_4\, s_2 + s_4\, s_3$.

(14) $-s_1 = (-1_{\mathbb{C}})\, s_1$.

(15) $r\, (s_2\, s_3) = (r\, s_2)\, s_3$.

(16) $r\, (s_2\, s_3) = s_2\, (r\, s_3)$.

(17) $(s_2 - s_3)\, s_4 = s_2\, s_4 - s_3\, s_4$.

(18) $s_4\, s_2 - s_4\, s_3 = s_4\, (s_2 - s_3)$.

(19) $r\,(s_2 + s_3) = r\,s_2 + r\,s_3$.

(20) $(r \cdot p)\,s_1 = r\,(p\,s_1)$.

(21) $r\,(s_2 - s_3) = r\,s_2 - r\,s_3$.

(22) If s_1 is non-zero, then $r\,\frac{s_2}{s_1} = \frac{r\,s_2}{s_1}$.

(23) $s_2 - (s_3 + s_4) = s_2 - s_3 - s_4$.

(24) $1_{\mathbb{C}}\,s_1 = s_1$.

(25) $--s_1 = s_1$.

(26) $s_2 - -s_3 = s_2 + s_3$.

(27) $s_2 - (s_3 - s_4) = (s_2 - s_3) + s_4$.

(28) $s_2 + (s_3 - s_4) = (s_2 + s_3) - s_4$.

(29) $(-s_2)\,s_3 = -s_2\,s_3$ and $s_2\,{-s_3} = -s_2\,s_3$.

(30) If s_1 is non-zero, then ${s_1}^{-1}$ is non-zero.

(31) If s_1 is non-zero, then $({s_1}^{-1})^{-1} = s_1$.

(32) s_1 is non-zero and s_2 is non-zero iff $s_1\,s_2$ is non-zero.

(33) If s_1 is non-zero and s_2 is non-zero, then ${s_1}^{-1}\,{s_2}^{-1} = (s_1\,s_2)^{-1}$.

(34) If s_1 is non-zero, then $\frac{s_2}{s_1}\,s_1 = s_2$.

(35) If s_1 is non-zero and s_2 is non-zero, then $\frac{s'_1}{s_1}\,\frac{s'_2}{s_2} = \frac{s'_1\,s'_2}{s_1\,s_2}$.

(36) If s_1 is non-zero and s_2 is non-zero, then $\frac{s_1}{s_2}$ is non-zero.

(37) If s_1 is non-zero and s_2 is non-zero, then $(\frac{s_1}{s_2})^{-1} = \frac{s_2}{s_1}$.

(38) If s_1 is non-zero, then $s_3\,\frac{s_2}{s_1} = \frac{s_3\,s_2}{s_1}$.

(39) If s_1 is non-zero and s_2 is non-zero, then $\frac{s_3}{\frac{s_1}{s_2}} = \frac{s_3\,s_2}{s_1}$.

(40) If s_1 is non-zero and s_2 is non-zero, then $\frac{s_3}{s_1} = \frac{s_3\,s_2}{s_1\,s_2}$.

(41) If $r \neq 0_{\mathbb{C}}$ and s_1 is non-zero, then $r\,s_1$ is non-zero.

(42) If s_1 is non-zero, then $-s_1$ is non-zero.

(43) If $r \neq 0_{\mathbb{C}}$ and s_1 is non-zero, then $(r\,s_1)^{-1} = r^{-1}\,{s_1}^{-1}$.

(44) If s_1 is non-zero, then $(-s_1)^{-1} = (-1_{\mathbb{C}})\,{s_1}^{-1}$.

(45) If s_1 is non-zero, then $-\frac{s_2}{s_1} = \frac{-s_2}{s_1}$ and $\frac{s_2}{-s_1} = -\frac{s_2}{s_1}$.

(46) If s_1 is non-zero, then $\frac{s_2}{s_1} + \frac{s'_2}{s_1} = \frac{s_2 + s'_2}{s_1}$ and $\frac{s_2}{s_1} - \frac{s'_2}{s_1} = \frac{s_2 - s'_2}{s_1}$.

(47) If s_1 is non-zero and s'_1 is non-zero, then $\frac{s_2}{s_1} + \frac{s'_2}{s'_1} = \frac{s_2\,s'_1 + s'_2\,s_1}{s_1\,s'_1}$ and $\frac{s_2}{s_1} - \frac{s'_2}{s'_1} = \frac{s_2\,s'_1 - s'_2\,s_1}{s_1\,s'_1}$.

(48) If s_1 is non-zero and s'_1 is non-zero and s_2 is non-zero, then $\frac{\frac{s'_2}{s_1}}{\frac{s'_1}{s_2}} = \frac{s'_2\,s_2}{s_1\,s'_1}$.

(49) $|s_1\,s'_1| = |s_1|\,|s'_1|$.

(50) If s_1 is non-zero, then $|s_1|$ is non-zero.

(51) If s_1 is non-zero, then $|s_1|^{-1} = |{s_1}^{-1}|$.

(52) If s_1 is non-zero, then $|\frac{s'_1}{s_1}| = \frac{|s'_1|}{|s_1|}$.

(53) $|r\, s_1| = |r|\, |s_1|$.

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