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Basic Petri Net Concepts

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Summary. This article presents the basic place/transition net structure definition for building various types of Petri nets. The basic net structure fields include places, transitions, and arcs (place-transition, transition-place) which may be supplemented with other fields (e.g., capacity, weight, marking, etc.) as needed. The theorems included in this article are divided into the following categories: deadlocks, traps, and dual net theorems. Here, a dual net is taken as the result of inverting all arcs (place-transition arcs to transition-place arcs and vice-versa) in the original net.

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The papers [3], [5], [6], [7], [1], [4], and [2] provide the terminology and notation for this paper.

1. Basic Place/Transition Net Structure Definition

Let A, B be non-empty sets. Observe that there exists a non-empty relation between A and B.

Let A, B be non-empty sets, and let r be a non-empty relation between A and B. We see that the element of r is an element of $[: A, B :]$.

We consider place/transitions net structures which are systems
⟨places, transitions, S-T arcs, T-S arcs⟩,
where the places, the transitions constitute non-empty sets, the S-T arcs constitute a non-empty relation between the places and the transitions, and the T-S arcs constitute a non-empty relation between the transitions and the places.

In the sequel P_1 will denote a place/transitions net structure. We now define several new modes. Let us consider P_1. A place of P_1 is an element of the places of P_1.

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A transition of P_1 is an element of the transitions of P_1.

An S-T arc of P_1 is an element of the S-T arcs of P_1.

A T-S arc of P_1 is an element of the T-S arcs of P_1.

Let us consider P_1, and let x be an S-T arc of P_1. Then $x_{\mathbf{1}}$ is a place of P_1. Then $x_{\mathbf{2}}$ is a transition of P_1. Let us consider P_1, and let x be a T-S arc of P_1. Then $x_{\mathbf{1}}$ is a transition of P_1. Then $x_{\mathbf{2}}$ is a place of P_1.

The scheme *Set_of_Elements* deals with a non-empty set $\mathcal{A}$, and a unary predicate $\mathcal{P}$, and states that:

$\{x : \mathcal{P}[x]\}$, where x ranges over elements of $\mathcal{A}$, is a subset of $\mathcal{A}$

for all values of the parameters.

In the sequel S_0 will denote a set of places of P_1. We now define two new functors. Let us consider P_1, S_0. The functor ${}^*S_0$ yielding a set of transitions of P_1 is defined as follows:

(Def.1) ${}^*S_0 = \{t : \bigvee_f \bigvee_s [s \in S_0 \wedge f = \langle t, s\rangle]\}$, where t ranges over transitions of P_1, and f ranges over T-S arcs of P_1, and s ranges over places of P_1.

The functor ${S_0}^*$ yielding a set of transitions of P_1 is defined as follows:

(Def.2) ${S_0}^* = \{t : \bigvee_f \bigvee_s [s \in S_0 \wedge f = \langle s, t\rangle]\}$, where t ranges over transitions of P_1, and f ranges over S-T arcs of P_1, and s ranges over places of P_1.

Next we state four propositions:

(1) ${}^*S_0 = \{f_{\mathbf{1}} : f_{\mathbf{2}} \in S_0\}$, where f ranges over T-S arcs of P_1.

(2) For an arbitrary x holds $x \in {}^*S_0$ if and only if there exists a T-S arc f of P_1 and there exists a place s of P_1 such that $s \in S_0$ and $f = \langle x, s\rangle$.

(3) ${S_0}^* = \{f_{\mathbf{2}} : f_{\mathbf{1}} \in S_0\}$, where f ranges over S-T arcs of P_1.

(4) For an arbitrary x holds $x \in {S_0}^*$ if and only if there exists an S-T arc f of P_1 and there exists a place s of P_1 such that $s \in S_0$ and $f = \langle s, x\rangle$.

In the sequel T_0 is a set of transitions of P_1. We now define two new functors. Let us consider P_1, T_0. The functor ${}^*T_0$ yields a set of places of P_1 and is defined by:

(Def.3) ${}^*T_0 = \{s : \bigvee_f \bigvee_t [t \in T_0 \wedge f = \langle s, t\rangle]\}$, where s ranges over places of P_1, and f ranges over S-T arcs of P_1, and t ranges over transitions of P_1.

The functor ${T_0}^*$ yielding a set of places of P_1 is defined by:

(Def.4) ${T_0}^* = \{s : \bigvee_f \bigvee_t [t \in T_0 \wedge f = \langle t, s\rangle]\}$, where s ranges over places of P_1, and f ranges over T-S arcs of P_1, and t ranges over transitions of P_1.

Next we state several propositions:

(5) ${}^*T_0 = \{f_{\mathbf{1}} : f_{\mathbf{2}} \in T_0\}$, where f ranges over S-T arcs of P_1.

(6) For an arbitrary x holds $x \in {}^*T_0$ if and only if there exists an S-T arc f of P_1 and there exists a transition t of P_1 such that $t \in T_0$ and $f = \langle x, t\rangle$.

(7) ${T_0}^* = \{f_{\mathbf{2}} : f_{\mathbf{1}} \in T_0\}$, where f ranges over T-S arcs of P_1.

(8) For an arbitrary x holds $x \in {T_0}^*$ if and only if there exists a T-S arc f of P_1 and there exists a transition t of P_1 such that $t \in T_0$ and $f = \langle t, x\rangle$.

(9) $^*(\emptyset_{\text{the places of } P_1}) = \emptyset$.

(10) $(\emptyset_{\text{the places of } P_1})^* = \emptyset$.

(11) $^*(\emptyset_{\text{the transitions of } P_1}) = \emptyset$.

(12) $(\emptyset_{\text{the transitions of } P_1})^* = \emptyset$.

2. Deadlocks

We now define two new attributes. Let us consider P_1. A set of places of P_1 is deadlock-like if:

(Def.5) *it is a subset of it*.

A place/transitions net structure has deadlocks if:

(Def.6) there exists a set of places of it which is deadlock-like.

3. Traps

We now define two new attributes. Let us consider P_1. A set of places of P_1 is trap-like if:

(Def.7) it* is a subset of *it.

A place/transitions net structure has traps if:

(Def.8) there exists a set of places of it which is trap-like.

Let A, B be non-empty sets, and let r be a non-empty relation between A and B. Then $r^\smile$ is a non-empty relation between B and A.

4. Duality Theorems for Place/Transition Nets

Let us consider P_1. The functor $P_1{}^\circ$ yields a strict place/transitions net structure and is defined by:

(Def.9) $P_1{}^\circ = \langle$the places of P_1, the transitions of P_1, (the T-S arcs of $P_1)^\smile$, (the S-T arcs of $P_1)^\smile\rangle$.

One can prove the following propositions:

(13) $(P_1{}^\circ)^\circ =$ the place/transitions net structure of P_1.

(14) The places of P_1 = the places of $P_1{}^\circ$ and the transitions of P_1 = the transitions of $P_1{}^\circ$ and (the S-T arcs of $P_1)^\smile$ = the T-S arcs of $P_1{}^\circ$ and (the T-S arcs of $P_1)^\smile$ = the S-T arcs of $P_1{}^\circ$.

We now define several new functors. Let us consider P_1, and let S_0 be a set of places of P_1. The functor $S_0{}^\circ$ yields a set of places of $P_1{}^\circ$ and is defined as follows:

(Def.10) $S_0{}^\circ = S_0$.

Let us consider P_1, and let s be a place of P_1. The functor s° yields a place of ${P_1}^\circ$ and is defined by:

(Def.11) $s^\circ = s$.

Let us consider P_1, and let S_0 be a set of places of ${P_1}^\circ$. The functor ${}^\circ S_0$ yields a set of places of P_1 and is defined by:

(Def.12) ${}^\circ S_0 = S_0$.

Let us consider P_1, and let s be a place of ${P_1}^\circ$. The functor ${}^\circ s$ yields a place of P_1 and is defined by:

(Def.13) ${}^\circ s = s$.

Let us consider P_1, and let T_0 be a set of transitions of P_1. The functor ${T_0}^\circ$ yielding a set of transitions of ${P_1}^\circ$ is defined by:

(Def.14) ${T_0}^\circ = T_0$.

Let us consider P_1, and let t be a transition of P_1. The functor t° yields a transition of ${P_1}^\circ$ and is defined as follows:

(Def.15) $t^\circ = t$.

Let us consider P_1, and let T_0 be a set of transitions of ${P_1}^\circ$. The functor ${}^\circ T_0$ yielding a set of transitions of P_1 is defined by:

(Def.16) ${}^\circ T_0 = T_0$.

Let us consider P_1, and let t be a transition of ${P_1}^\circ$. The functor ${}^\circ t$ yielding a transition of P_1 is defined by:

(Def.17) ${}^\circ t = t$.

In the sequel S will denote a set of places of P_1. Next we state several propositions:

(15) $(S^\circ)^* = {}^*S$.

(16) ${}^*(S^\circ) = S^*$.

(17) S is deadlock-like if and only if S° is trap-like.

(18) S is trap-like if and only if S° is deadlock-like.

(19) For every P_1 being a place/transitions net structure and for every transition t of P_1 and for every S_0 being a set of places of P_1 holds $t \in {S_0}^*$ if and only if ${}^*\{t\} \cap S_0 \neq \emptyset$.

(20) For every P_1 being a place/transitions net structure and for every transition t of P_1 and for every S_0 being a set of places of P_1 holds $t \in {}^*S_0$ if and only if $\{t\}^* \cap S_0 \neq \emptyset$.

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