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Some Facts about Union of Two Functions and Continuity of Union of Functions

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Summary. Proofs of two theorems connected with the union of any two functions and the proofs of two theorems on the continuity of the union of two continuous functions between topological spaces. The theorem stating that the union of two subsets of R^2, which are homeomorphic to unit interval and have only one terminal joined point, is also homeomorphic to unit interval is proved, too.

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The notation and terminology used in this paper have been introduced in the following papers: [14], [9], [15], [13], [2], [3], [4], [11], [7], [5], [12], [10], [1], [6], and [8]. In the sequel x, y, z are real numbers. Next we state the proposition

(1) If $x \leq y$ and $y \leq z$, then $[x, y] \cap [y, z] = \{y\}$.

In the sequel f, g will be functions and x_1, x_2 will be arbitrary. Next we state two propositions:

(2) If f is one-to-one and g is one-to-one and for all x_1, x_2 such that $x_1 \in \operatorname{dom} g$ and $x_2 \in \operatorname{dom} f \setminus \operatorname{dom} g$ holds $g(x_1) \neq f(x_2)$, then $f +\cdot g$ is one-to-one.

(3) If $f\,^\circ\,(\operatorname{dom} f \cap \operatorname{dom} g) \subseteq \operatorname{rng} g$, then $\operatorname{rng} f \cup \operatorname{rng} g = \operatorname{rng}(f +\cdot g)$.

We follow the rules: T, T_1, T_2, S will be topological spaces and p, p_1, p_2 will be points of T. Next we state two propositions:

(4) Let T_1, T_2 be subspaces of T. Let f be a map from T_1 into S. Let g be a map from T_2 into S. Suppose $\Omega_{T_1} \cup \Omega_{T_2} = \Omega_T$ and $\Omega_{T_1} \cap \Omega_{T_2} = \{p\}$ and T_1 is compact and T_2 is compact and T is a T_2 space and f is continuous and g is continuous and $f(p) = g(p)$. Then there exists a map h from T into S such that $h = f +\cdot g$ and h is continuous.

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(5) Let f be a map from T_1 into S. Let g be a map from T_2 into S. Suppose that

(i) T_1 is a subspace of T,
(ii) T_2 is a subspace of T,
(iii) $\Omega_{T_1} \cup \Omega_{T_2} = \Omega_T$,
(iv) $\Omega_{T_1} \cap \Omega_{T_2} = \{p_1, p_2\}$,
(v) T_1 is compact,
(vi) T_2 is compact,
(vii) T is a T_2 space,
(viii) f is continuous,
(ix) g is continuous,
(x) $f(p_1) = g(p_1)$,
(xi) $f(p_2) = g(p_2)$.

Then there exists a map h from T into S such that $h = f +\cdot\, g$ and h is continuous.

In the sequel P, Q denote subsets of $\mathcal{E}^2_{\mathrm{T}}$. One can prove the following proposition

(6) Let f be a map from $\mathbb{I}$ into $(\mathcal{E}^2_{\mathrm{T}}) \restriction P$. Let g be a map from $\mathbb{I}$ into $(\mathcal{E}^2_{\mathrm{T}}) \restriction Q$. Suppose $P \cap Q = \{p\}$ and f is a homeomorphism and $f(1) = p$ and g is a homeomorphism and $g(0) = p$. Then there exists a map h from $\mathbb{I}$ into $(\mathcal{E}^2_{\mathrm{T}}) \restriction P \cup Q$ **qua** a subset of $\mathcal{E}^2_{\mathrm{T}}$ such that h is a homeomorphism and $h(0) = f(0)$ and $h(1) = g(1)$.

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