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Schemes [1]

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Summary. Some basic schemes of quantifier calculus are proved.

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In the sequel a, b will be arbitrary. In this article we present several logical schemes. The scheme *Schemat0* concerns a unary predicate $\mathcal{P}$, and states that:

there exists a such that $\mathcal{P}[a]$

provided the parameter meets the following requirement:

- for every a holds $\mathcal{P}[a]$.

The scheme *Schemat1a* deals with $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:

for every a holds $\mathcal{P}[a]$ and $\mathcal{Q}[]$

provided the parameters meet the following requirement:

- for every a holds $\mathcal{P}[a]$ and $\mathcal{Q}[]$.

The scheme *Schemat1b* concerns $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:

for every a holds $\mathcal{P}[a]$ and $\mathcal{Q}[]$

provided the parameters have the following property:

- for every a holds $\mathcal{P}[a]$ and $\mathcal{Q}[]$.

The scheme *Schemat2a* concerns $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:

there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[]$

provided the parameters meet the following requirement:

- there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[]$.

The scheme *Schemat2b* deals with $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:

there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[]$

provided the following condition is met:

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- there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[]$.

The scheme *Schemat3* concerns a binary predicate $\mathcal{P}$, and states that:

for every b there exists a such that $\mathcal{P}[a, b]$

provided the parameter has the following property:

- there exists a such that for every b holds $\mathcal{P}[a, b]$.

The scheme *Schemat4a* concerns two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a such that $\mathcal{P}[a]$ or there exists a such that $\mathcal{Q}[a]$

provided the following condition is satisfied:

- there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[a]$.

The scheme *Schemat4b* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[a]$

provided the parameters meet the following requirement:

- there exists a such that $\mathcal{P}[a]$ or there exists a such that $\mathcal{Q}[a]$.

The scheme *Schemat5* concerns two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a such that $\mathcal{P}[a]$ and there exists a such that $\mathcal{Q}[a]$

provided the following condition is met:

- there exists a such that $\mathcal{P}[a]$ and $\mathcal{Q}[a]$.

The scheme *Schemat6a* concerns two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

for every a holds $\mathcal{P}[a]$ and for every a holds $\mathcal{Q}[a]$

provided the parameters satisfy the following condition:

- for every a holds $\mathcal{P}[a]$ and $\mathcal{Q}[a]$.

The scheme *Schemat6b* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

for every a holds $\mathcal{P}[a]$ and $\mathcal{Q}[a]$

provided the following requirement is met:

- for every a holds $\mathcal{P}[a]$ and for every a holds $\mathcal{Q}[a]$.

The scheme *Schemat7* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

for every a holds $\mathcal{P}[a]$ or $\mathcal{Q}[a]$

provided the following condition is satisfied:

- for every a holds $\mathcal{P}[a]$ or for every a holds $\mathcal{Q}[a]$.

The scheme *Schemat8* concerns two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

if for every a holds $\mathcal{P}[a]$, then for every a holds $\mathcal{Q}[a]$

provided the parameters satisfy the following condition:

- for every a such that $\mathcal{P}[a]$ holds $\mathcal{Q}[a]$.

The scheme *Schemat9* concerns two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

for every a holds $\mathcal{P}[a]$ if and only if for every a holds $\mathcal{Q}[a]$

provided the parameters have the following property:

- for every a holds $\mathcal{P}[a]$ if and only if $\mathcal{Q}[a]$.

The scheme *Schemat10a* concerns $\mathcal{P}$ and states that:
$\mathcal{P}[]$
provided the parameter satisfies the following condition:

- for every a holds $\mathcal{P}[]$.

The scheme *Schemat10b* concerns $\mathcal{P}$ and states that:
for every a holds $\mathcal{P}[]$
provided the parameter satisfies the following condition:

- $\mathcal{P}[]$.

The scheme *Schemat11a* concerns $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:
for every a holds $\mathcal{P}[a]$ or $\mathcal{Q}[]$
provided the following requirement is met:

- for every a holds $\mathcal{P}[a]$ or $\mathcal{Q}[]$.

The scheme *Schemat11b* deals with $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:
for every a holds $\mathcal{P}[a]$ or $\mathcal{Q}[]$
provided the parameters satisfy the following condition:

- for every a holds $\mathcal{P}[a]$ or $\mathcal{Q}[]$.

The scheme *Schemat12a* concerns $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:
there exists a such that $\mathcal{Q}[]$ and $\mathcal{P}[a]$
provided the following condition is satisfied:

- $\mathcal{Q}[]$ and there exists a such that $\mathcal{P}[a]$.

The scheme *Schemat12b* concerns $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:
$\mathcal{Q}[]$ and there exists a such that $\mathcal{P}[a]$
provided the following condition is satisfied:

- there exists a such that $\mathcal{Q}[]$ and $\mathcal{P}[a]$.

The scheme *Schemat13a* concerns $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:
for every a such that $\mathcal{Q}[]$ holds $\mathcal{P}[a]$
provided the parameters satisfy the following condition:

- if $\mathcal{Q}[]$, then for every a holds $\mathcal{P}[a]$.

The scheme *Schemat13b* deals with $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:
if $\mathcal{Q}[]$, then for every a holds $\mathcal{P}[a]$
provided the parameters satisfy the following condition:

- for every a such that $\mathcal{Q}[]$ holds $\mathcal{P}[a]$.

The scheme *Schemat14* concerns $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:
there exists a such that if $\mathcal{Q}[]$, then $\mathcal{P}[a]$
provided the parameters meet the following requirement:

- if $\mathcal{Q}[]$, then there exists a such that $\mathcal{P}[a]$.

The scheme *Schemat15* deals with $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:

for every a such that $\mathcal{P}[a]$ holds $\mathcal{Q}[]$
provided the following condition is met:

- if there exists a such that $\mathcal{P}[a]$, then $\mathcal{Q}[]$.

The scheme *Schemat16* deals with $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:

there exists a such that if $\mathcal{P}[a]$, then $\mathcal{Q}[]$
provided the parameters meet the following requirement:

- if for every a holds $\mathcal{P}[a]$, then $\mathcal{Q}[]$.

The scheme *Schemat17* concerns $\mathcal{Q}$, and a unary predicate $\mathcal{P}$, and states that:

if for every a holds $\mathcal{P}[a]$, then $\mathcal{Q}[]$
provided the parameters meet the following requirement:

- for every a such that $\mathcal{P}[a]$ holds $\mathcal{Q}[]$.

The scheme *Schemat18a* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a such that for every b holds $\mathcal{P}[a]$ or $\mathcal{Q}[b]$
provided the following condition is satisfied:

- there exists a such that $\mathcal{P}[a]$ or for every b holds $\mathcal{Q}[b]$.

The scheme *Schemat18b* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a such that $\mathcal{P}[a]$ or for every b holds $\mathcal{Q}[b]$
provided the parameters meet the following condition:

- there exists a such that for every b holds $\mathcal{P}[a]$ or $\mathcal{Q}[b]$.

The scheme *Schemat19a* concerns two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

for every b there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[b]$
provided the following condition is met:

- there exists a such that $\mathcal{P}[a]$ or for every b holds $\mathcal{Q}[b]$.

The scheme *Schemat19b* concerns two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a such that $\mathcal{P}[a]$ or for every b holds $\mathcal{Q}[b]$
provided the following condition is met:

- for every b there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[b]$.

The scheme *Schemat20a* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

for every b there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[b]$
provided the following condition is met:

- there exists a such that for every b holds $\mathcal{P}[a]$ or $\mathcal{Q}[b]$.

The scheme *Schemat20b* concerns two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a such that for every b holds $\mathcal{P}[a]$ or $\mathcal{Q}[b]$
provided the following requirement is met:

- for every b there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[b]$.

The scheme *Schemat21a* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a such that for every b holds $\mathcal{P}[a]$ and $\mathcal{Q}[b]$
provided the following condition is satisfied:

- there exists a such that $\mathcal{P}[a]$ and for every b holds $\mathcal{Q}[b]$.

The scheme *Schemat21b* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a such that $\mathcal{P}[a]$ and for every b holds $\mathcal{Q}[b]$
provided the following condition is satisfied:

- there exists a such that for every b holds $\mathcal{P}[a]$ and $\mathcal{Q}[b]$.

The scheme *Schemat22a* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

for every b there exists a such that $\mathcal{P}[a]$ and $\mathcal{Q}[b]$
provided the parameters meet the following condition:

- there exists a such that $\mathcal{P}[a]$ and for every b holds $\mathcal{Q}[b]$.

The scheme *Schemat22b* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a such that $\mathcal{P}[a]$ and for every b holds $\mathcal{Q}[b]$
provided the following requirement is met:

- for every b there exists a such that $\mathcal{P}[a]$ and $\mathcal{Q}[b]$.

The scheme *Schemat23a* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

for every b there exists a such that $\mathcal{P}[a]$ and $\mathcal{Q}[b]$
provided the following requirement is met:

- there exists a such that for every b holds $\mathcal{P}[a]$ and $\mathcal{Q}[b]$.

The scheme *Schemat23b* deals with two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a such that for every b holds $\mathcal{P}[a]$ and $\mathcal{Q}[b]$
provided the parameters satisfy the following condition:

- for every b there exists a such that $\mathcal{P}[a]$ and $\mathcal{Q}[b]$.

The scheme *Schemat24a* concerns a unary predicate $\mathcal{Q}$, and a binary predicate $\mathcal{P}$, and states that:

for every a there exists b such that if $\mathcal{P}[a, b]$, then $\mathcal{Q}[a]$
provided the parameters satisfy the following condition:

- for every a such that for every b holds $\mathcal{P}[a, b]$ holds $\mathcal{Q}[a]$.

The scheme *Schemat24b* deals with a unary predicate $\mathcal{Q}$, and a binary predicate $\mathcal{P}$, and states that:

for every a such that for every b holds $\mathcal{P}[a, b]$ holds $\mathcal{Q}[a]$
provided the following requirement is met:

- for every a there exists b such that if $\mathcal{P}[a, b]$, then $\mathcal{Q}[a]$.

The scheme *Schemat25a* concerns a unary predicate $\mathcal{Q}$, and a binary predicate $\mathcal{P}$, and states that:

for all a, b such that $\mathcal{P}[a, b]$ holds $\mathcal{Q}[a]$
provided the parameters have the following property:

- for every a such that there exists b such that $\mathcal{P}[a, b]$ holds $\mathcal{Q}[a]$.

The scheme *Schemat25b* concerns a unary predicate $\mathcal{Q}$, and a binary predicate $\mathcal{P}$, and states that:

for every a such that there exists b such that $\mathcal{P}[a,b]$ holds $\mathcal{Q}[a]$
provided the following condition is met:

- for all a, b such that $\mathcal{P}[a,b]$ holds $\mathcal{Q}[a]$.

The scheme *Schemat26* deals with a binary predicate $\mathcal{P}$, and states that:
there exists a such that for every b holds $\mathcal{P}[a,b]$
provided the following condition is met:

- for all a, b holds $\mathcal{P}[a,b]$.

The scheme *Schemat27* deals with a binary predicate $\mathcal{P}$, and states that:
for every a holds $\mathcal{P}[a,a]$
provided the parameter meets the following condition:

- for all a, b holds $\mathcal{P}[a,b]$.

The scheme *Schemat28* concerns a binary predicate $\mathcal{P}$, and states that:
there exists b such that for every a holds $\mathcal{P}[a,b]$
provided the following requirement is met:

- for all a, b holds $\mathcal{P}[a,b]$.

The scheme *Schemat29* deals with a binary predicate $\mathcal{P}$, and states that:
for every b there exists a such that $\mathcal{P}[a,b]$
provided the parameter has the following property:

- there exists a such that for every b holds $\mathcal{P}[a,b]$.

The scheme *Schemat30* deals with a binary predicate $\mathcal{P}$, and states that:
there exists a such that $\mathcal{P}[a,a]$
provided the parameter meets the following requirement:

- there exists a such that for every b holds $\mathcal{P}[a,b]$.

The scheme *Schemat31* concerns a binary predicate $\mathcal{P}$, and states that:
for every a there exists b such that $\mathcal{P}[b,a]$
provided the following condition is satisfied:

- for every a holds $\mathcal{P}[a,a]$.

The scheme *Schemat32* concerns a binary predicate $\mathcal{P}$, and states that:
there exists a such that $\mathcal{P}[a,a]$
provided the parameter meets the following condition:

- for every a holds $\mathcal{P}[a,a]$.

The scheme *Schemat33* deals with a binary predicate $\mathcal{P}$, and states that:
for every a there exists b such that $\mathcal{P}[a,b]$
provided the following condition is satisfied:

- for every a holds $\mathcal{P}[a,a]$.

The scheme *Schemat34* concerns a binary predicate $\mathcal{P}$, and states that:
there exists b such that $\mathcal{P}[b,b]$
provided the parameter meets the following requirement:

- there exists b such that for every a holds $\mathcal{P}[a,b]$.

The scheme *Schemat35* deals with a binary predicate $\mathcal{P}$, and states that:
for every a there exists b such that $\mathcal{P}[a,b]$
provided the parameter meets the following condition:

- there exists b such that for every a holds $\mathcal{P}[a,b]$.

The scheme *Schemat36* deals with a binary predicate $\mathcal{P}$, and states that:
there exist a, b such that $\mathcal{P}[a,b]$

provided the parameter meets the following requirement:

- for every b there exists a such that $\mathcal{P}[a, b]$.

The scheme *Schemat37* deals with a binary predicate $\mathcal{P}$, and states that:
there exist a, b such that $\mathcal{P}[a, b]$
provided the following condition is satisfied:

- there exists a such that $\mathcal{P}[a, a]$.

The scheme *Schemat38* concerns a binary predicate $\mathcal{P}$, and states that:
there exist a, b such that $\mathcal{P}[a, b]$
provided the parameter satisfies the following condition:

- for every a there exists b such that $\mathcal{P}[a, b]$.

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