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Enumerated Sets

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Summary. We prove basic facts about enumerated sets: definitional theorems and their immediate consequences, some theorems related to the decomposition of an enumerated set into union of two sets, facts about removing elements that occur more than once, and facts about permutations of enumerated sets (with the length ≤ 4). The article includes also schemes enabling instantiation of up to nine universal quantifiers.

The terminology and notation used in this paper have been introduced in the papers [1] and [2]. For simplicity we adopt the following convention: x, $x1$, $x2$, $x3$, $x4$, $x5$, $x6$, $x7$, $x8$ have the type Any; X has the type set. In the article we present several logical schemes. The scheme *UI1* concerns a constant $\mathcal{A}$ and a unary predicate $\mathcal{P}$ and states that the following holds

$$\mathcal{P}[\mathcal{A}]$$

provided the parameters satisfy the following condition:

- **for** $x1$ **holds** $\mathcal{P}[x1]$.

The scheme *UI2* deals with a constant $\mathcal{A}$, a constant $\mathcal{B}$ and a binary predicate $\mathcal{P}$ and states that the following holds

$$\mathcal{P}[\mathcal{A},\mathcal{B}]$$

provided the parameters satisfy the following condition:

- **for** $x1$,$x2$ **holds** $\mathcal{P}[x1,x2]$.

The scheme *UI3* concerns a constant $\mathcal{A}$, a constant $\mathcal{B}$, a constant $\mathcal{C}$ and a ternary predicate $\mathcal{P}$ and states that the following holds

$$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C}]$$

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provided the parameters satisfy the following condition:

- **for** $x1$,$x2$,$x3$ **holds** $\mathcal{P}[x1,x2,x3]$.

The scheme *UI4* concerns a constant $\mathcal{A}$, a constant $\mathcal{B}$, a constant $\mathcal{C}$, a constant $\mathcal{D}$ and a 4-ary predicate $\mathcal{P}$ and states that the following holds

$$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}]$$

provided the parameters satisfy the following condition:

- **for** $x1$,$x2$,$x3$,$x4$ **holds** $\mathcal{P}[x1,x2,x3,x4]$.

The scheme *UI5* deals with a constant $\mathcal{A}$, a constant $\mathcal{B}$, a constant $\mathcal{C}$, a constant $\mathcal{D}$, a constant $\mathcal{E}$ and a 5-ary predicate $\mathcal{P}$ and states that the following holds

$$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E}]$$

provided the parameters satisfy the following condition:

- **for** $x1$,$x2$,$x3$,$x4$,$x5$ **holds** $\mathcal{P}[x1,x2,x3,x4,x5]$.

The scheme *UI6* deals with a constant $\mathcal{A}$, a constant $\mathcal{B}$, a constant $\mathcal{C}$, a constant $\mathcal{D}$, a constant $\mathcal{E}$, a constant $\mathcal{F}$ and a 6-ary predicate $\mathcal{P}$ and states that the following holds

$$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E},\mathcal{F}]$$

provided the parameters satisfy the following condition:

- **for** $x1$,$x2$,$x3$,$x4$,$x5$,$x6$ **holds** $\mathcal{P}[x1,x2,x3,x4,x5,x6]$.

The scheme *UI7* concerns a constant $\mathcal{A}$, a constant $\mathcal{B}$, a constant $\mathcal{C}$, a constant $\mathcal{D}$, a constant $\mathcal{E}$, a constant $\mathcal{F}$, a constant $\mathcal{G}$ and a 7-ary predicate $\mathcal{P}$ and states that the following holds

$$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E},\mathcal{F},\mathcal{G}]$$

provided the parameters satisfy the following condition:

- **for** $x1$,$x2$,$x3$,$x4$,$x5$,$x6$,$x7$ **holds** $\mathcal{P}[x1,x2,x3,x4,x5,x6,x7]$.

The scheme *UI8* concerns a constant $\mathcal{A}$, a constant $\mathcal{B}$, a constant $\mathcal{C}$, a constant $\mathcal{D}$, a constant $\mathcal{E}$, a constant $\mathcal{F}$, a constant $\mathcal{G}$, a constant $\mathcal{H}$ and a 8-ary predicate $\mathcal{P}$ and states that the following holds

$$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E},\mathcal{F},\mathcal{G},\mathcal{H}]$$

provided the parameters satisfy the following condition:

- **for** $x1$,$x2$,$x3$,$x4$,$x5$,$x6$,$x7$,$x8$ **holds** $\mathcal{P}[x1,x2,x3,x4,x5,x6,x7,x8]$.

The scheme *UI9* concerns a constant $\mathcal{A}$, a constant $\mathcal{B}$, a constant $\mathcal{C}$, a constant $\mathcal{D}$, a constant $\mathcal{E}$, a constant $\mathcal{F}$, a constant $\mathcal{G}$, a constant $\mathcal{H}$, a constant $\mathcal{I}$ and a 9-ary predicate $\mathcal{P}$ and states that the following holds

$$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E},\mathcal{F},\mathcal{G},\mathcal{H},\mathcal{I}]$$

provided the parameters satisfy the following condition:

- $$\textbf{for } x1,x2,x3,x4,x5,x6,x7,x8,x9 \textbf{ being } \mathrm{Any} \\ \textbf{holds } \mathcal{P}[x1,x2,x3,x4,x5,x6,x7,x8,x9].$$

We now state a number of propositions:

(1) $$\textbf{for } x1,X \textbf{ holds } X = \{x1\} \textbf{ iff for } x \textbf{ holds } x \in X \textbf{ iff } x = x1,$$

(2) $$\textbf{for } x1,x \textbf{ holds } x \in \{x1\} \textbf{ iff } x = x1,$$

(3) $$x \in \{x1\} \textbf{ implies } x = x1,$$

(4) $$x \in \{x\},$$

(5) $$\textbf{for } x1,X \textbf{ st for } x \textbf{ holds } x \in X \textbf{ iff } x = x1 \textbf{ holds } X = \{x1\},$$

(6) $$\textbf{for } x1,x2,X \textbf{ holds } X = \{x1,x2\} \textbf{ iff for } x \textbf{ holds } x \in X \textbf{ iff } x = x1 \textbf{ or } x = x2,$$

(7) $$\textbf{for } x1,x2 \textbf{ for } x \textbf{ holds } x \in \{x1,x2\} \textbf{ iff } x = x1 \textbf{ or } x = x2,$$

(8) $$x \in \{x1,x2\} \textbf{ implies } x = x1 \textbf{ or } x = x2,$$

(9) $$x = x1 \textbf{ or } x = x2 \textbf{ implies } x \in \{x1,x2\},$$

(10) $$\textbf{for } x1,x2,X \textbf{ st for } x \textbf{ holds } x \in X \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ holds } X = \{x1,x2\}.$$

Let us consider $x1$, $x2$, $x3$. The functor

$$\{x1,x2,x3\},$$

yields the type set and is defined by

$$x \in \textbf{it iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3.$$

One can prove the following propositions:

(11) $$\textbf{for } x1,x2,x3,X \\ \textbf{holds } X = \{x1,x2,x3\} \textbf{ iff for } x \textbf{ holds } x \in X \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3,$$

(12) $$\textbf{for } x1,x2,x3 \textbf{ for } x \textbf{ holds } x \in \{x1,x2,x3\} \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3,$$

(13) $$x \in \{x1,x2,x3\} \textbf{ implies } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3,$$

$$x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ implies } x \in \{x1,x2,x3\}, \tag{14}$$

$$\textbf{for } x1,x2,x3,X \tag{15}$$
$$\textbf{st for } x \textbf{ holds } x \in X \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ holds } X = \{x1,x2,x3\}.$$

Let us consider $x1$, $x2$, $x3$, $x4$. The functor

$$\{x1,x2,x3,x4\},$$

with values of the type set, is defined by

$$x \in \textbf{it iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4.$$

We now state several propositions:

$$\textbf{for } x1,x2,x3,x4,X \textbf{ holds } X = \{x1,x2,x3,x4\} \tag{16}$$
$$\textbf{iff for } x \textbf{ holds } x \in X \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4,$$

$$\textbf{for } x1,x2,x3,x4 \tag{17}$$
$$\textbf{for } x \textbf{ holds } x \in \{x1,x2,x3,x4\} \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4,$$

$$x \in \{x1,x2,x3,x4\} \textbf{ implies } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4, \tag{18}$$

$$x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ implies } x \in \{x1,x2,x3,x4\}, \tag{19}$$

$$\textbf{for } x1,x2,x3,x4,X \textbf{ st} \tag{20}$$
$$\textbf{for } x \textbf{ holds } x \in X \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4$$
$$\textbf{holds } X = \{x1,x2,x3,x4\}.$$

Let us consider $x1$, $x2$, $x3$, $x4$, $x5$. The functor

$$\{x1,x2,x3,x4,x5\},$$

yields the type set and is defined by

$$x \in \textbf{it iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5.$$

Next we state several propositions:

$$\textbf{for } x1,x2,x3,x4,x5 \textbf{ for } X \textbf{ being } \text{set} \textbf{ holds } X = \{x1,x2,x3,x4,x5\} \tag{21}$$
$$\textbf{iff for } x \textbf{ holds } x \in X \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5,$$

$$x \in \{x1,x2,x3,x4,x5\} \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5, \tag{22}$$

$$x \in \{x1,x2,x3,x4,x5\} \tag{23}$$
$$\textbf{implies } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5,$$

$$x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \tag{24}$$
$$\textbf{implies } x \in \{x1,x2,x3,x4,x5\},$$

(25) $$\textbf{for } X \textbf{ being } \mathrm{set} \textbf{ st}$$
$$\textbf{for } x \textbf{ holds } x \in X \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5$$
$$\textbf{holds } X = \{x1,x2,x3,x4,x5\}.$$

Let us consider $x1$, $x2$, $x3$, $x4$, $x5$, $x6$. The functor

$$\{x1,x2,x3,x4,x5,x6\},$$

with values of the type set, is defined by

$$x \in \textbf{it iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6.$$

We now state several propositions:

(26) $$\textbf{for } x1,x2,x3,x4,x5,x6 \textbf{ for } X \textbf{ being } \mathrm{set} \textbf{ holds } X = \{x1,x2,x3,x4,x5,x6\} \textbf{ iff}$$
$$\textbf{for } x$$
$$\textbf{holds } x \in X \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6,$$

(27) $$x \in \{x1,x2,x3,x4,x5,x6\}$$
$$\textbf{iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6,$$

(28) $$x \in \{x1,x2,x3,x4,x5,x6\}$$
$$\textbf{implies } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6,$$

(29) $$x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6$$
$$\textbf{implies } x \in \{x1,x2,x3,x4,x5,x6\},$$

(30) $$\textbf{for } X \textbf{ being } \mathrm{set} \textbf{ st}$$
$$\textbf{for } x$$
$$\textbf{holds } x \in X \textbf{ iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6$$
$$\textbf{holds } X = \{x1,x2,x3,x4,x5,x6\}.$$

Let us consider $x1$, $x2$, $x3$, $x4$, $x5$, $x6$, $x7$. The functor

$$\{x1,x2,x3,x4,x5,x6,x7\},$$

yields the type set and is defined by

$$x \in \textbf{it iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7.$$

The following propositions are true:

(31) $$\textbf{for } x1,x2,x3,x4,x5,x6,x7 \textbf{ for } X \textbf{ being } \mathrm{set} \textbf{ holds } X = \{x1,x2,x3,x4,x5,x6,x7\}$$
$$\textbf{iff for } x \textbf{ holds } x \in X$$
$$\textbf{iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7,$$

(32)
$$x \in \{x1,x2,x3,x4,x5,x6,x7\}$$
$$\textbf{iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7,$$

(33)
$$x \in \{x1,x2,x3,x4,x5,x6,x7\} \textbf{ implies}$$
$$x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7,$$

(34)
$$x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7$$
$$\textbf{implies } x \in \{x1,x2,x3,x4,x5,x6,x7\},$$

(35)
$$\textbf{for } X \textbf{ being } \text{set } \textbf{st}$$
$$\textbf{for } x \textbf{ holds } x \in X$$
$$\textbf{iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7$$
$$\textbf{holds } X = \{x1,x2,x3,x4,x5,x6,x7\}.$$

Let us consider $x1$, $x2$, $x3$, $x4$, $x5$, $x6$, $x7$, $x8$. The functor

$$\{x1,x2,x3,x4,x5,x6,x7,x8\},$$

with values of the type set, is defined by

$$x \in \textbf{it}$$
$$\textbf{iff } x = x1 \textbf{ or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7 \textbf{ or } x = x8.$$

Next we state a number of propositions:

(36)
$$\textbf{for } x1,x2,x3,x4,x5,x6,x7,x8 \textbf{ for } X \textbf{ being } \text{set } \textbf{holds}$$
$$X = \{x1,x2,x3,x4,x5,x6,x7,x8\} \textbf{ iff for } x \textbf{ holds } x \in X \textbf{ iff } x = x1$$
$$\textbf{or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7 \textbf{ or } x = x8,$$

(37)
$$x \in \{x1,x2,x3,x4,x5,x6,x7,x8\} \textbf{ iff } x = x1$$
$$\textbf{or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7 \textbf{ or } x = x8,$$

(38)
$$x \in \{x1,x2,x3,x4,x5,x6,x7,x8\} \textbf{ implies } x = x1$$
$$\textbf{or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7 \textbf{ or } x = x8,$$

(39)
$$x = x1$$
$$\textbf{or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7 \textbf{ or } x = x8$$
$$\textbf{implies } x \in \{x1,x2,x3,x4,x5,x6,x7,x8\},$$

(40)
$$\textbf{for } X \textbf{ being } \text{set } \textbf{st}$$
$$\textbf{for } x \textbf{ holds } x \in X \textbf{ iff } x = x1$$
$$\textbf{or } x = x2 \textbf{ or } x = x3 \textbf{ or } x = x4 \textbf{ or } x = x5 \textbf{ or } x = x6 \textbf{ or } x = x7 \textbf{ or } x = x8$$
$$\textbf{holds } X = \{x1,x2,x3,x4,x5,x6,x7,x8\},$$

$$\{x1,x2\} = \{x1\} \cup \{x2\}, \tag{41}$$

$$\{x1,x2,x3\} = \{x1\} \cup \{x2,x3\}, \tag{42}$$

$$\{x1,x2,x3\} = \{x1,x2\} \cup \{x3\}, \tag{43}$$

$$\{x1,x2,x3,x4\} = \{x1\} \cup \{x2,x3,x4\}, \tag{44}$$

$$\{x1,x2,x3,x4\} = \{x1,x2\} \cup \{x3,x4\}, \tag{45}$$

$$\{x1,x2,x3,x4\} = \{x1,x2,x3\} \cup \{x4\}, \tag{46}$$

$$\{x1,x2,x3,x4,x5\} = \{x1\} \cup \{x2,x3,x4,x5\}, \tag{47}$$

$$\{x1,x2,x3,x4,x5\} = \{x1,x2\} \cup \{x3,x4,x5\}, \tag{48}$$

$$\{x1,x2,x3,x4,x5\} = \{x1,x2,x3\} \cup \{x4,x5\}, \tag{49}$$

$$\{x1,x2,x3,x4,x5\} = \{x1,x2,x3,x4\} \cup \{x5\}, \tag{50}$$

$$\{x1,x2,x3,x4,x5,x6\} = \{x1\} \cup \{x2,x3,x4,x5,x6\}, \tag{51}$$

$$\{x1,x2,x3,x4,x5,x6\} = \{x1,x2\} \cup \{x3,x4,x5,x6\}, \tag{52}$$

$$\{x1,x2,x3,x4,x5,x6\} = \{x1,x2,x3\} \cup \{x4,x5,x6\}, \tag{53}$$

$$\{x1,x2,x3,x4,x5,x6\} = \{x1,x2,x3,x4\} \cup \{x5,x6\}, \tag{54}$$

$$\{x1,x2,x3,x4,x5,x6\} = \{x1,x2,x3,x4,x5\} \cup \{x6\}, \tag{55}$$

$$\{x1,x2,x3,x4,x5,x6,x7\} = \{x1\} \cup \{x2,x3,x4,x5,x6,x7\}, \tag{56}$$

$$\{x1,x2,x3,x4,x5,x6,x7\} = \{x1,x2\} \cup \{x3,x4,x5,x6,x7\}, \tag{57}$$

$$\{x1,x2,x3,x4,x5,x6,x7\} = \{x1,x2,x3\} \cup \{x4,x5,x6,x7\}, \tag{58}$$

$$\{x1,x2,x3,x4,x5,x6,x7\} = \{x1,x2,x3,x4\} \cup \{x5,x6,x7\}, \tag{59}$$

$$\{x1,x2,x3,x4,x5,x6,x7\} = \{x1,x2,x3,x4,x5\} \cup \{x6,x7\}, \tag{60}$$

$$\{x1,x2,x3,x4,x5,x6,x7\} = \{x1,x2,x3,x4,x5,x6\} \cup \{x7\}, \tag{61}$$

$$\{x1,x2,x3,x4,x5,x6,x7,x8\} = \{x1\} \cup \{x2,x3,x4,x5,x6,x7,x8\}, \tag{62}$$

$$\{x1,x2,x3,x4,x5,x6,x7,x8\} = \{x1,x2\} \cup \{x3,x4,x5,x6,x7,x8\}, \tag{63}$$

$$\{x1,x2,x3,x4,x5,x6,x7,x8\} = \{x1,x2,x3\} \cup \{x4,x5,x6,x7,x8\}, \tag{64}$$

$$\{x1,x2,x3,x4,x5,x6,x7,x8\} = \{x1,x2,x3,x4\} \cup \{x5,x6,x7,x8\}, \tag{65}$$

$$\{x1,x2,x3,x4,x5,x6,x7,x8\} = \{x1,x2,x3,x4,x5\} \cup \{x6,x7,x8\}, \tag{66}$$

$$\{x1,x2,x3,x4,x5,x6,x7,x8\} = \{x1,x2,x3,x4,x5,x6\} \cup \{x7,x8\}, \tag{67}$$

$$\{x1,x2,x3,x4,x5,x6,x7,x8\} = \{x1,x2,x3,x4,x5,x6,x7\} \cup \{x8\}, \tag{68}$$

$$\{x1,x1\} = \{x1\}, \tag{69}$$

$$\{x1,x1,x2\} = \{x1,x2\}, \tag{70}$$

$$\{x1,x1,x2,x3\} = \{x1,x2,x3\}, \tag{71}$$

$$\{x1,x1,x2,x3,x4\} = \{x1,x2,x3,x4\}, \tag{72}$$

$$\{x1,x1,x2,x3,x4,x5\} = \{x1,x2,x3,x4,x5\}, \tag{73}$$

$$\{x1,x1,x2,x3,x4,x5,x6\} = \{x1,x2,x3,x4,x5,x6\}, \tag{74}$$

$$\{x1,x1,x2,x3,x4,x5,x6,x7\} = \{x1,x2,x3,x4,x5,x6,x7\}, \tag{75}$$

$$\{x1,x1,x1\} = \{x1\}, \tag{76}$$

$$\{x1,x1,x1,x2\} = \{x1,x2\}, \tag{77}$$

$$\{x1,x1,x1,x2,x3\} = \{x1,x2,x3\}, \tag{78}$$

$$\{x1,x1,x1,x2,x3,x4\} = \{x1,x2,x3,x4\}, \tag{79}$$

$$\{x1,x1,x1,x2,x3,x4,x5\} = \{x1,x2,x3,x4,x5\}, \tag{80}$$

$$\{x1,x1,x1,x2,x3,x4,x5,x6\} = \{x1,x2,x3,x4,x5,x6\}, \tag{81}$$

$$\{x1,x1,x1,x1\} = \{x1\}, \tag{82}$$

$$\{x1,x1,x1,x1,x2\} = \{x1,x2\}, \tag{83}$$

$$\{x1,x1,x1,x1,x2,x3\} = \{x1,x2,x3\}, \tag{84}$$

$$\{x1,x1,x1,x1,x2,x3,x4\} = \{x1,x2,x3,x4\}, \tag{85}$$

$$\{x1,x1,x1,x1,x2,x3,x4,x5\} = \{x1,x2,x3,x4,x5\}, \tag{86}$$

$$\{x1,x1,x1,x1,x1\} = \{x1\}, \tag{87}$$

$$\{x1,x1,x1,x1,x1,x2\} = \{x1,x2\}, \tag{88}$$

$$\{x1,x1,x1,x1,x1,x2,x3\} = \{x1,x2,x3\}, \tag{89}$$

$$\{x1,x1,x1,x1,x1,x2,x3,x4\} = \{x1,x2,x3,x4\}, \tag{90}$$

$$\{x1,x1,x1,x1,x1,x1\} = \{x1\}, \tag{91}$$

$$\{x1,x1,x1,x1,x1,x1,x2\} = \{x1,x2\}, \tag{92}$$

$$\{x1,x1,x1,x1,x1,x1,x2,x3\} = \{x1,x2,x3\}, \tag{93}$$

$$\{x1,x1,x1,x1,x1,x1,x1\} = \{x1\}, \tag{94}$$

$$\{x1,x1,x1,x1,x1,x1,x1,x2\} = \{x1,x2\}, \tag{95}$$

$$\{x1,x1,x1,x1,x1,x1,x1,x1\} = \{x1\}, \tag{96}$$

$$\{x1,x2\} = \{x2,x1\}, \tag{97}$$

$$\{x1,x2,x3\} = \{x1,x3,x2\}, \tag{98}$$

$$\{x1,x2,x3\} = \{x2,x1,x3\}, \tag{99}$$

$$\{x1,x2,x3\} = \{x2,x3,x1\}, \tag{100}$$

$$\{x1,x2,x3\} = \{x3,x1,x2\}, \tag{101}$$

$$\{x1,x2,x3\} = \{x3,x2,x1\}, \tag{102}$$

$$\{x1,x2,x3,x4\} = \{x1,x2,x4,x3\}, \tag{103}$$

$$\{x1,x2,x3,x4\} = \{x1,x3,x2,x4\}, \tag{104}$$

$$\{x1,x2,x3,x4\} = \{x1,x3,x4,x2\}, \tag{105}$$

$$\{x1,x2,x3,x4\} = \{x1,x4,x2,x3\}, \tag{106}$$

$$\{x1,x2,x3,x4\} = \{x1,x4,x3,x2\}, \tag{107}$$

$$\{x1,x2,x3,x4\} = \{x2,x1,x3,x4\}, \tag{108}$$

$$\{x1,x2,x3,x4\} = \{x2,x1,x4,x3\}, \tag{109}$$

$$\{x1,x2,x3,x4\} = \{x2,x3,x1,x4\}, \tag{110}$$

$$\{x1,x2,x3,x4\} = \{x2,x3,x4,x1\}, \tag{111}$$

$$\{x1,x2,x3,x4\} = \{x2,x4,x1,x3\}, \tag{112}$$

$$\{x1,x2,x3,x4\} = \{x2,x4,x3,x1\}, \tag{113}$$

$$\{x1,x2,x3,x4\} = \{x3,x1,x2,x4\}, \tag{114}$$

$$\{x1,x2,x3,x4\} = \{x3,x1,x4,x2\}, \tag{115}$$

$$\{x1,x2,x3,x4\} = \{x3,x2,x1,x4\}, \tag{116}$$

$$\{x1,x2,x3,x4\} = \{x3,x2,x4,x1\}, \tag{117}$$

$$\{x1,x2,x3,x4\} = \{x3,x4,x1,x2\}, \tag{118}$$

$$\{x1,x2,x3,x4\} = \{x3,x4,x2,x1\}, \tag{119}$$

$$\{x1,x2,x3,x4\} = \{x4,x1,x2,x3\}, \tag{120}$$

$$\{x1,x2,x3,x4\} = \{x4,x1,x3,x2\}, \tag{121}$$

$$\{x1,x2,x3,x4\} = \{x4,x2,x1,x3\}, \tag{122}$$

$$\{x1,x2,x3,x4\} = \{x4,x2,x3,x1\}, \tag{123}$$

$$\{x1,x2,x3,x4\} = \{x4,x3,x1,x2\}, \tag{124}$$

$$\{x1,x2,x3,x4\} = \{x4,x3,x2,x1\}. \tag{125}$$

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