JOURNAL OF FORMALIZED MATHEMATICS
Volume 13, Released 2001, Published 2003
Inst. of Computer Science, Univ. of Białystok

Miscellaneous Facts about Functors

Grzegorz Bancerek
University of Białystok
Shinshu University, Nagano

Summary. In the paper we show useful facts concerning reverse and inclusion functors and the restriction of functors. We also introduce a new notation for the intersection of categories and the isomorphism under arbitrary functors.

MML Identifier: YELLOW20.

WWW: http://mizar.org/JFM/Vol13/yellow20.html

The articles [13], [7], [20], [21], [22], [4], [5], [14], [3], [12], [8], [6], [15], [16], [11], [10], [2], [17], [18], [19], [9], and [1] provide the notation and terminology for this paper.

1. Reverse Functors

One can prove the following propositions:

(1) Let A, B be transitive non empty category structures with units and F be a feasible reflexive functor structure from A to B. Suppose F is coreflexive and bijective. Let a be an object of A and b be an object of B. Then $F(a) = b$ if and only if $F^{-1}(b) = a$.

(2) Let A, B be transitive non empty category structures with units, F be a precovariant feasible functor structure from A to B, and G be a precovariant feasible functor structure from B to A. Suppose F is bijective and $G = F^{-1}$. Let a_1, a_2 be objects of A. Suppose $\langle a_1, a_2 \rangle \neq \emptyset$. Let f be a morphism from a_1 to a_2 and g be a morphism from $F(a_1)$ to $F(a_2)$. Then $F(f) = g$ if and only if $G(g) = f$.

(3) Let A, B be transitive non empty category structures with units, F be a precontravariant feasible functor structure from A to B, and G be a precontravariant feasible functor structure from B to A. Suppose F is bijective and $G = F^{-1}$. Let a_1, a_2 be objects of A. Suppose $\langle a_1, a_2 \rangle \neq \emptyset$. Let f be a morphism from a_1 to a_2 and g be a morphism from $F(a_2)$ to $F(a_1)$. Then $F(f) = g$ if and only if $G(g) = f$.

(4) Let A, B be categories and F be a functor from A to B. Suppose F is bijective. Let G be a functor from B to A. If $F \cdot G = \mathrm{id}_B$, then the functor structure of $G = F^{-1}$.

(5) Let A, B be categories and F be a functor from A to B. Suppose F is bijective. Let G be a functor from B to A. If $G \cdot F = \mathrm{id}_A$, then the functor structure of $G = F^{-1}$.

(6) Let A, B be categories and F be a covariant functor from A to B. Suppose F is bijective. Let G be a covariant functor from B to A. Suppose that

(i) for every object b of B holds $F(G(b)) = b$, and

(ii) for all objects a, b of B such that $\langle a, b \rangle \neq \emptyset$ and for every morphism f from a to b holds $F(G(f)) = f$.

Then the functor structure of $G = F^{-1}$.

(7) Let A, B be categories and F be a contravariant functor from A to B. Suppose F is bijective. Let G be a contravariant functor from B to A. Suppose that

(i) for every object b of B holds $F(G(b)) = b$, and

(ii) for all objects a, b of B such that $\langle a,b\rangle \neq \emptyset$ and for every morphism f from a to b holds $F(G(f)) = f$.

Then the functor structure of $G = F^{-1}$.

(8) Let A, B be categories and F be a covariant functor from A to B. Suppose F is bijective. Let G be a covariant functor from B to A. Suppose that

(i) for every object a of A holds $G(F(a)) = a$, and

(ii) for all objects a, b of A such that $\langle a,b\rangle \neq \emptyset$ and for every morphism f from a to b holds $G(F(f)) = f$.

Then the functor structure of $G = F^{-1}$.

(9) Let A, B be categories and F be a contravariant functor from A to B. Suppose F is bijective. Let G be a contravariant functor from B to A. Suppose that

(i) for every object a of A holds $G(F(a)) = a$, and

(ii) for all objects a, b of A such that $\langle a,b\rangle \neq \emptyset$ and for every morphism f from a to b holds $G(F(f)) = f$.

Then the functor structure of $G = F^{-1}$.

2. Intersection of Categories

Let A, B be category structures. We say that A and B have the same composition if and only if:

(Def. 1) For all sets a_1, a_2, a_3 holds (the composition of A)($\langle\!\langle a_1, a_2, a_3\rangle\!\rangle$) $\approx$ (the composition of B)($\langle\!\langle a_1, a_2, a_3\rangle\!\rangle$).

Let us note that the predicate A and B have the same composition is symmetric.

Next we state three propositions:

(10) Let A, B be category structures. Then A and B have the same composition if and only if for all sets a_1, a_2, a_3, x such that $x \in \operatorname{dom}$(the composition of A)($\langle\!\langle a_1, a_2, a_3\rangle\!\rangle$) and $x \in \operatorname{dom}$(the composition of B)($\langle\!\langle a_1, a_2, a_3\rangle\!\rangle$) holds (the composition of A)($\langle\!\langle a_1, a_2, a_3\rangle\!\rangle$)$(x) =$ (the composition of B)($\langle\!\langle a_1, a_2, a_3\rangle\!\rangle$)$(x)$.

(11) Let A, B be transitive non empty category structures. Then A and B have the same composition if and only if for all objects a_1, a_2, a_3 of A such that $\langle a_1,a_2\rangle \neq \emptyset$ and $\langle a_2,a_3\rangle \neq \emptyset$ and for all objects b_1, b_2, b_3 of B such that $\langle b_1,b_2\rangle \neq \emptyset$ and $\langle b_2,b_3\rangle \neq \emptyset$ and $b_1 = a_1$ and $b_2 = a_2$ and $b_3 = a_3$ and for every morphism f_1 from a_1 to a_2 and for every morphism g_1 from b_1 to b_2 such that $g_1 = f_1$ and for every morphism f_2 from a_2 to a_3 and for every morphism g_2 from b_2 to b_3 such that $g_2 = f_2$ holds $f_2 \cdot f_1 = g_2 \cdot g_1$.

(12) For all para-functional semi-functional categories A, B holds A and B have the same composition.

Let f, g be functions. The functor $\operatorname{Intersect}(f,g)$ yields a function and is defined as follows:

(Def. 2) $\operatorname{dom}\operatorname{Intersect}(f,g) = \operatorname{dom} f \cap \operatorname{dom} g$ and for every set x such that $x \in \operatorname{dom} f \cap \operatorname{dom} g$ holds $(\operatorname{Intersect}(f,g))(x) = f(x) \cap g(x)$.

Let us notice that the functor $\operatorname{Intersect}(f,g)$ is commutative.

One can prove the following propositions:

(13) For every set I and for all many sorted sets A, B indexed by I holds $\operatorname{Intersect}(A,B) = A \cap B$.

(14) Let I, J be sets, A be a many sorted set indexed by I, and B be a many sorted set indexed by J. Then $\mathrm{Intersect}(A,B)$ is a many sorted set indexed by $I \cap J$.

(15) Let I, J be sets, A be a many sorted set indexed by I, B be a function, and C be a many sorted set indexed by J. If $C = \mathrm{Intersect}(A,B)$, then $C \dot{\subseteq} A$.

(16) Let A_1, A_2, B_1, B_2 be sets, f be a function from A_1 into A_2, and g be a function from B_1 into B_2. If $f \approx g$, then $f \cap g$ is a function from $A_1 \cap B_1$ into $A_2 \cap B_2$.

(17) Let I_1, I_2 be sets, A_1, B_1 be many sorted sets indexed by I_1, A_2, B_2 be many sorted sets indexed by I_2, and A, B be many sorted sets indexed by $I_1 \cap I_2$. Suppose $A = \mathrm{Intersect}(A_1,A_2)$ and $B = \mathrm{Intersect}(B_1,B_2)$. Let F be a many sorted function from A_1 into B_1 and G be a many sorted function from A_2 into B_2. Suppose that for every set x such that $x \in \mathrm{dom} F$ and $x \in \mathrm{dom} G$ holds $F(x) \approx G(x)$. Then $\mathrm{Intersect}(F,G)$ is a many sorted function from A into B.

(18) Let I, J be sets, F be a many sorted set indexed by $[\!: I, I :\!]$, and G be a many sorted set indexed by $[\!: J, J :\!]$. Then there exists a many sorted set H indexed by $[\!: I \cap J, I \cap J :\!]$ such that $H = \mathrm{Intersect}(F,G)$ and $\mathrm{Intersect}(\{|F|\},\{|G|\}) = \{|H|\}$.

(19) Let I, J be sets, F_1, F_2 be many sorted sets indexed by $[\!: I, I :\!]$, and G_1, G_2 be many sorted sets indexed by $[\!: J, J :\!]$. Then there exist many sorted sets H_1, H_2 indexed by $[\!: I \cap J, I \cap J :\!]$ such that $H_1 = \mathrm{Intersect}(F_1,G_1)$ and $H_2 = \mathrm{Intersect}(F_2,G_2)$ and $\mathrm{Intersect}(\{|F_1,F_2|\},\{|G_1,G_2|\}) = \{|H_1,H_2|\}$.

Let A, B be category structures. Let us assume that A and B have the same composition. The functor $\mathrm{Intersect}(A,B)$ yields a strict category structure and is defined by the conditions (Def. 3).

(Def. 3)(i) The carrier of $\mathrm{Intersect}(A,B) =$ (the carrier of A) $\cap$ (the carrier of B),

(ii) the arrows of $\mathrm{Intersect}(A,B) = \mathrm{Intersect}($the arrows of A, the arrows of $B)$, and

(iii) the composition of $\mathrm{Intersect}(A,B) = \mathrm{Intersect}($the composition of A, the composition of $B)$.

Next we state several propositions:

(20) For all category structures A, B such that A and B have the same composition holds $\mathrm{Intersect}(A,B) = \mathrm{Intersect}(B,A)$.

(21) Let A, B be category structures. Suppose A and B have the same composition. Then $\mathrm{Intersect}(A,B)$ is a substructure of A.

(22) Let A, B be category structures. Suppose A and B have the same composition. Let a_1, a_2 be objects of A, b_1, b_2 be objects of B, and o_1, o_2 be objects of $\mathrm{Intersect}(A,B)$. If $o_1 = a_1$ and $o_1 = b_1$ and $o_2 = a_2$ and $o_2 = b_2$, then $\langle o_1, o_2 \rangle = (\langle a_1, a_2 \rangle) \cap (\langle b_1, b_2 \rangle)$.

(23) Let A, B be transitive category structures. If A and B have the same composition, then $\mathrm{Intersect}(A,B)$ is transitive.

(24) Let A, B be category structures. Suppose A and B have the same composition. Let a_1, a_2 be objects of A, b_1, b_2 be objects of B, and o_1, o_2 be objects of $\mathrm{Intersect}(A,B)$. Suppose $o_1 = a_1$ and $o_1 = b_1$ and $o_2 = a_2$ and $o_2 = b_2$ and $\langle a_1, a_2 \rangle \neq \emptyset$ and $\langle b_1, b_2 \rangle \neq \emptyset$. Let f be a morphism from a_1 to a_2 and g be a morphism from b_1 to b_2. If $f = g$, then $f \in \langle o_1, o_2 \rangle$.

(25) Let A, B be non empty category structures with units. Suppose A and B have the same composition. Let a be an object of A, b be an object of B, and o be an object of $\mathrm{Intersect}(A,B)$. If $o = a$ and $o = b$ and $\mathrm{id}_a = \mathrm{id}_b$, then $\mathrm{id}_a \in \langle o, o \rangle$.

(26) Let A, B be categories. Suppose that

(i) A and B have the same composition,

(ii) $\mathrm{Intersect}(A,B)$ is non empty, and

(iii) for every object a of A and for every object b of B such that $a = b$ holds $\mathrm{id}_a = \mathrm{id}_b$.

Then $\mathrm{Intersect}(A,B)$ is a subcategory of A.

3. SUBCATEGORIES

The scheme *SubcategoryUniq* deals with a category $\mathcal{A}$, non empty subcategories $\mathcal{B}$, $\mathcal{C}$ of $\mathcal{A}$, a unary predicate $\mathcal{P}$, and a ternary predicate $\mathcal{Q}$, and states that:

The category structure of $\mathcal{B}$ = the category structure of $\mathcal{C}$

provided the parameters have the following properties:

- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{B}$ iff $\mathcal{P}[a]$,
- Let a, b be objects of $\mathcal{A}$ and a', b' be objects of $\mathcal{B}$. Suppose $a' = a$ and $b' = b$ and $\langle a,b \rangle \neq \emptyset$. Let f be a morphism from a to b. Then $f \in \langle a',b' \rangle$ if and only if $\mathcal{Q}[a,b,f]$,
- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{C}$ iff $\mathcal{P}[a]$, and
- Let a, b be objects of $\mathcal{A}$ and a', b' be objects of $\mathcal{C}$. Suppose $a' = a$ and $b' = b$ and $\langle a,b \rangle \neq \emptyset$. Let f be a morphism from a to b. Then $f \in \langle a',b' \rangle$ if and only if $\mathcal{Q}[a,b,f]$.

The following proposition is true

(27) Let A be a non empty category structure and B be a non empty substructure of A. Then B is full if and only if for all objects a_1, a_2 of A and for all objects b_1, b_2 of B such that $b_1 = a_1$ and $b_2 = a_2$ holds $\langle b_1, b_2 \rangle = \langle a_1, a_2 \rangle$.

Now we present two schemes. The scheme *FullSubcategoryEx* deals with a category $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

There exists a strict full non empty subcategory B of $\mathcal{A}$ such that for every object a of $\mathcal{A}$ holds a is an object of B if and only if $\mathcal{P}[a]$

provided the following requirement is met:

- There exists an object a of $\mathcal{A}$ such that $\mathcal{P}[a]$.

The scheme *FullSubcategoryUniq* deals with a category $\mathcal{A}$, full non empty subcategories $\mathcal{B}$, $\mathcal{C}$ of $\mathcal{A}$, and a unary predicate $\mathcal{P}$, and states that:

The category structure of $\mathcal{B}$ = the category structure of $\mathcal{C}$

provided the following requirements are met:

- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{B}$ iff $\mathcal{P}[a]$, and
- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{C}$ iff $\mathcal{P}[a]$.

4. INCLUSION FUNCTORS AND FUNCTOR RESTRICTIONS

Let f be a function yielding function and let x, y be sets. Observe that $f(x, y)$ is relation-like and function-like.

One can prove the following proposition

(28) Let A be a category, C be a non empty subcategory of A, and a, b be objects of C. If $\langle a,b \rangle \neq \emptyset$, then for every morphism f from a to b holds $(\overset{C}{\hookrightarrow})(f) = f$.

Let A be a category and let C be a non empty subcategory of A. Note that $\overset{C}{\hookrightarrow}$ is id-preserving and comp-preserving.

Let A be a category and let C be a non empty subcategory of A. Observe that $\overset{C}{\hookrightarrow}$ is precovariant.

Let A be a category and let C be a non empty subcategory of A. Then $\overset{C}{\hookrightarrow}$ is a strict covariant functor from C to A.

Let A, B be categories, let C be a non empty subcategory of A, and let F be a covariant functor from A to B. Then $F \restriction C$ is a strict covariant functor from C to B.

Let A, B be categories, let C be a non empty subcategory of A, and let F be a contravariant functor from A to B. Then $F \restriction C$ is a strict contravariant functor from C to B.

Next we state several propositions:

(29) Let A, B be categories, C be a non empty subcategory of A, F be a functor structure from A to B, a be an object of A, and c be an object of C. If $c = a$, then $(F \restriction C)(c) = F(a)$.

(30) Let A, B be categories, C be a non empty subcategory of A, F be a covariant functor from A to B, a, b be objects of A, and c, d be objects of C. Suppose $c = a$ and $d = b$ and $\langle c,d \rangle \neq \emptyset$. Let f be a morphism from a to b and g be a morphism from c to d. If $g = f$, then $(F \restriction C)(g) = F(f)$.

(31) Let A, B be categories, C be a non empty subcategory of A, F be a contravariant functor from A to B, a, b be objects of A, and c, d be objects of C. Suppose $c = a$ and $d = b$ and $\langle c, d\rangle \neq \emptyset$. Let f be a morphism from a to b and g be a morphism from c to d. If $g = f$, then $(F\restriction C)(g) = F(f)$.

(32) Let A, B be non empty graphs and F be a bimap structure from A into B. Suppose F is precovariant and one-to-one. Let a, b be objects of A. If $F(a) = F(b)$, then $a = b$.

(33) Let A, B be non empty reflexive graphs and F be a feasible precovariant functor structure from A to B. Suppose F is faithful. Let a, b be objects of A. Suppose $\langle a, b\rangle \neq \emptyset$. Let f, g be morphisms from a to b. If $F(f) = F(g)$, then $f = g$.

(34) Let A, B be non empty graphs and F be a precovariant functor structure from A to B. Suppose F is surjective. Let a, b be objects of B. Suppose $\langle a, b\rangle \neq \emptyset$. Let f be a morphism from a to b. Then there exist objects c, d of A and there exists a morphism g from c to d such that $a = F(c)$ and $b = F(d)$ and $\langle c, d\rangle \neq \emptyset$ and $f = F(g)$.

(35) Let A, B be non empty graphs and F be a bimap structure from A into B. Suppose F is precontravariant and one-to-one. Let a, b be objects of A. If $F(a) = F(b)$, then $a = b$.

(36) Let A, B be non empty reflexive graphs and F be a feasible precontravariant functor structure from A to B. Suppose F is faithful. Let a, b be objects of A. Suppose $\langle a, b\rangle \neq \emptyset$. Let f, g be morphisms from a to b. If $F(f) = F(g)$, then $f = g$.

(37) Let A, B be non empty graphs and F be a precontravariant functor structure from A to B. Suppose F is surjective. Let a, b be objects of B. Suppose $\langle a, b\rangle \neq \emptyset$. Let f be a morphism from a to b. Then there exist objects c, d of A and there exists a morphism g from c to d such that $b = F(c)$ and $a = F(d)$ and $\langle c, d\rangle \neq \emptyset$ and $f = F(g)$.

5. Isomorphisms under Arbitrary Functor

Let A, B be categories, let F be a functor structure from A to B, and let A', B' be categories. We say that A' and B' are isomorphic under F if and only if the conditions (Def. 4) are satisfied.

(Def. 4)(i) A' is a subcategory of A,

(ii) B' is a subcategory of B, and

(iii) there exists a covariant functor G from A' to B' such that G is bijective and for every object a' of A' and for every object a of A such that $a' = a$ holds $G(a') = F(a)$ and for all objects b', c' of A' and for all objects b, c of A such that $\langle b', c'\rangle \neq \emptyset$ and $b' = b$ and $c' = c$ and for every morphism f' from b' to c' and for every morphism f from b to c such that $f' = f$ holds $G(f') = (\text{Morph-Map}_F(b, c))(f)$.

We say that A' and B' are anti-isomorphic under F if and only if the conditions (Def. 5) are satisfied.

(Def. 5)(i) A' is a subcategory of A,

(ii) B' is a subcategory of B, and

(iii) there exists a contravariant functor G from A' to B' such that G is bijective and for every object a' of A' and for every object a of A such that $a' = a$ holds $G(a') = F(a)$ and for all objects b', c' of A' and for all objects b, c of A such that $\langle b', c'\rangle \neq \emptyset$ and $b' = b$ and $c' = c$ and for every morphism f' from b' to c' and for every morphism f from b to c such that $f' = f$ holds $G(f') = (\text{Morph-Map}_F(b, c))(f)$.

One can prove the following propositions:

(38) Let A, B, A_1, B_1 be categories and F be a functor structure from A to B. If A_1 and B_1 are isomorphic under F, then A_1 and B_1 are isomorphic.

(39) Let A, B, A_1, B_1 be categories and F be a functor structure from A to B. Suppose A_1 and B_1 are anti-isomorphic under F. Then A_1, B_1 are anti-isomorphic.

(40) Let A, B be categories and F be a covariant functor from A to B. If A and B are isomorphic under F, then F is bijective.

(41) Let A, B be categories and F be a contravariant functor from A to B. If A and B are anti-isomorphic under F, then F is bijective.

(42) Let A, B be categories and F be a covariant functor from A to B. If F is bijective, then A and B are isomorphic under F.

(43) Let A, B be categories and F be a contravariant functor from A to B. If F is bijective, then A and B are anti-isomorphic under F.

Now we present two schemes. The scheme *CoBijectRestriction* deals with non empty categories $\mathcal{A}$, $\mathcal{B}$, a covariant functor $\mathcal{C}$ from $\mathcal{A}$ to $\mathcal{B}$, a non empty subcategory $\mathcal{D}$ of $\mathcal{A}$, and a non empty subcategory $\mathcal{E}$ of $\mathcal{B}$, and states that:

$\mathcal{D}$ and $\mathcal{E}$ are isomorphic under $\mathcal{C}$

provided the parameters meet the following conditions:

- $\mathcal{C}$ is bijective,
- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{D}$ iff $\mathcal{C}(a)$ is an object of $\mathcal{E}$, and
- Let a, b be objects of $\mathcal{A}$. Suppose $\langle a,b\rangle \neq \emptyset$. Let a_1, b_1 be objects of $\mathcal{D}$. Suppose $a_1 = a$ and $b_1 = b$. Let a_2, b_2 be objects of $\mathcal{E}$. Suppose $a_2 = \mathcal{C}(a)$ and $b_2 = \mathcal{C}(b)$. Let f be a morphism from a to b. Then $f \in \langle a_1,b_1\rangle$ if and only if $\mathcal{C}(f) \in \langle a_2,b_2\rangle$.

The scheme *ContraBijectRestriction* deals with non empty categories $\mathcal{A}$, $\mathcal{B}$, a contravariant functor $\mathcal{C}$ from $\mathcal{A}$ to $\mathcal{B}$, a non empty subcategory $\mathcal{D}$ of $\mathcal{A}$, and a non empty subcategory $\mathcal{E}$ of $\mathcal{B}$, and states that:

$\mathcal{D}$ and $\mathcal{E}$ are anti-isomorphic under $\mathcal{C}$

provided the following conditions are satisfied:

- $\mathcal{C}$ is bijective,
- For every object a of $\mathcal{A}$ holds a is an object of $\mathcal{D}$ iff $\mathcal{C}(a)$ is an object of $\mathcal{E}$, and
- Let a, b be objects of $\mathcal{A}$. Suppose $\langle a,b\rangle \neq \emptyset$. Let a_1, b_1 be objects of $\mathcal{D}$. Suppose $a_1 = a$ and $b_1 = b$. Let a_2, b_2 be objects of $\mathcal{E}$. Suppose $a_2 = \mathcal{C}(a)$ and $b_2 = \mathcal{C}(b)$. Let f be a morphism from a to b. Then $f \in \langle a_1,b_1\rangle$ if and only if $\mathcal{C}(f) \in \langle b_2,a_2\rangle$.

We now state a number of propositions:

(44) For every category A and for every non empty subcategory B of A holds B and B are isomorphic under id_A.

(45) For all functions f, g such that $f \subseteq g$ holds $\curvearrowleft f \subseteq \curvearrowleft g$.

(46) For all functions f, g such that $\mathrm{dom} f$ is a binary relation and $\curvearrowleft f \subseteq \curvearrowleft g$ holds $f \subseteq g$.

(47) Let I, J be sets, A be a many sorted set indexed by $[: I, I :]$, and B be a many sorted set indexed by $[: J, J :]$. If $A \dot{\subseteq} B$, then $\curvearrowleft A \dot{\subseteq} \curvearrowleft B$.

(48) Let A be a transitive non empty category structure and B be a transitive non empty substructure of A. Then B^{op} is a substructure of A^{op}.

(49) For every category A and for every non empty subcategory B of A holds B^{op} is a subcategory of A^{op}.

(50) Let A be a category and B be a non empty subcategory of A. Then B and B^{op} are anti-isomorphic under the dualizing functor from A into A^{op}.

(51) Let A_1, A_2 be categories and F be a covariant functor from A_1 to A_2. Suppose F is bijective. Let B_1 be a non empty subcategory of A_1 and B_2 be a non empty subcategory of A_2. Suppose B_1 and B_2 are isomorphic under F. Then B_2 and B_1 are isomorphic under F^{-1}.

(52) Let A_1, A_2 be categories and F be a contravariant functor from A_1 to A_2. Suppose F is bijective. Let B_1 be a non empty subcategory of A_1 and B_2 be a non empty subcategory of A_2. Suppose B_1 and B_2 are anti-isomorphic under F. Then B_2 and B_1 are anti-isomorphic under F^{-1}.

(53) Let A_1, A_2, A_3 be categories, F be a covariant functor from A_1 to A_2, G be a covariant functor from A_2 to A_3, B_1 be a non empty subcategory of A_1, B_2 be a non empty subcategory of A_2, and B_3 be a non empty subcategory of A_3. Suppose B_1 and B_2 are isomorphic under F and B_2 and B_3 are isomorphic under G. Then B_1 and B_3 are isomorphic under $G \cdot F$.

(54) Let A_1, A_2, A_3 be categories, F be a contravariant functor from A_1 to A_2, G be a covariant functor from A_2 to A_3, B_1 be a non empty subcategory of A_1, B_2 be a non empty subcategory of A_2, and B_3 be a non empty subcategory of A_3. Suppose B_1 and B_2 are anti-isomorphic under F and B_2 and B_3 are isomorphic under G. Then B_1 and B_3 are anti-isomorphic under $G \cdot F$.

(55) Let A_1, A_2, A_3 be categories, F be a covariant functor from A_1 to A_2, G be a contravariant functor from A_2 to A_3, B_1 be a non empty subcategory of A_1, B_2 be a non empty subcategory of A_2, and B_3 be a non empty subcategory of A_3. Suppose B_1 and B_2 are isomorphic under F and B_2 and B_3 are anti-isomorphic under G. Then B_1 and B_3 are anti-isomorphic under $G \cdot F$.

(56) Let A_1, A_2, A_3 be categories, F be a contravariant functor from A_1 to A_2, G be a contravariant functor from A_2 to A_3, B_1 be a non empty subcategory of A_1, B_2 be a non empty subcategory of A_2, and B_3 be a non empty subcategory of A_3. Suppose B_1 and B_2 are anti-isomorphic under F and B_2 and B_3 are anti-isomorphic under G. Then B_1 and B_3 are isomorphic under $G \cdot F$.

REFERENCES

[1] Grzegorz Bancerek. Concrete categories. *Journal of Formalized Mathematics*, 13, 2001. http://mizar.org/JFM/Vol13/yellow18.html.

[2] Czesław Byliński. Basic functions and operations on functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_3.html.

[3] Czesław Byliński. Binary operations. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/binop_1.html.

[4] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[5] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[6] Czesław Byliński. Partial functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/partfun1.html.

[7] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[8] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/funct_4.html.

[9] Artur Korniłowicz. The composition of functors and transformations in alternative categories. *Journal of Formalized Mathematics*, 10, 1998. http://mizar.org/JFM/Vol10/functor3.html.

[10] Małgorzata Korolkiewicz. Homomorphisms of many sorted algebras. *Journal of Formalized Mathematics*, 6, 1994. http://mizar.org/JFM/Vol6/msualg_3.html.

[11] Beata Madras. Product of family of universal algebras. *Journal of Formalized Mathematics*, 5, 1993. http://mizar.org/JFM/Vol5/pralg_1.html.

[12] Michał Muzalewski and Wojciech Skaba. Three-argument operations and four-argument operations. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/multop_1.html.

[13] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[14] Andrzej Trybulec. Tuples, projections and Cartesian products. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/mcart_1.html.

[15] Andrzej Trybulec. Many-sorted sets. *Journal of Formalized Mathematics*, 5, 1993. http://mizar.org/JFM/Vol5/pboole.html.

[16] Andrzej Trybulec. Many sorted algebras. *Journal of Formalized Mathematics*, 6, 1994. http://mizar.org/JFM/Vol6/msualg_1.html.

[17] Andrzej Trybulec. Categories without uniqueness of cod and dom. *Journal of Formalized Mathematics*, 7, 1995. http://mizar.org/JFM/Vol7/altcat_1.html.

[18] Andrzej Trybulec. Examples of category structures. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/altcat_2.html.

[19] Andrzej Trybulec. Functors for alternative categories. *Journal of Formalized Mathematics*, 8, 1996. `http://mizar.org/JFM/Vol8/functor0.html`.

[20] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

[21] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

[22] Edmund Woronowicz. Relations defined on sets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relset_1.html`.

Received July 31, 2001

Published January 2, 2004