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Introduction to Meet-Continuous Topological Lattices[1]

Artur Korniłowicz
University of Białystok

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The articles [21], [8], [26], [27], [6], [7], [11], [24], [19], [28], [25], [10], [15], [14], [1], [20], [4], [22], [5], [2], [3], [13], [12], [9], [29], [16], [17], [23], and [18] provide the notation and terminology for this paper.

1. Preliminaries

Let S be a finite 1-sorted structure. Note that the carrier of S is finite.

Let S be a trivial 1-sorted structure. Note that the carrier of S is trivial.

Let us mention that every set which is trivial is also finite.

Let us mention that every 1-sorted structure which is trivial is also finite.

Let us note that every 1-sorted structure which is non trivial is also non empty.

One can verify the following observations:

* there exists a 1-sorted structure which is strict, non empty, and trivial,
* there exists a relational structure which is strict, non empty, and trivial, and
* there exists a FR-structure which is strict, non empty, and trivial.

We now state the proposition

(1) For every T_1 non empty topological space T holds every finite subset of T is closed.

Let T be a T_1 non empty topological space. One can check that every subset of T which is finite is also closed.

Let T be a compact topological structure. Note that Ω_T is compact.

Let us note that there exists a topological space which is strict, non empty, and trivial.

Let us observe that every non empty topological space which is finite and T_1 is also discrete.

Let us mention that every topological space which is finite is also compact.

One can prove the following propositions:

(2) Every discrete non empty topological space is a T_4 space.

(3) Every discrete non empty topological space is a T_3 space.

(4) Every discrete non empty topological space is a T_2 space.

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(5) Every discrete non empty topological space is a T_1 space.

One can verify that every topological space which is discrete and non empty is also T_4, T_3, T_2, and T_1.

One can check that every non empty topological space which is T_4 and T_1 is also T_3.

Let us note that every non empty topological space which is T_3 and T_1 is also T_2.

Let us note that every topological space which is T_2 is also T_1.

Let us observe that every topological space which is T_1 is also T_0.

We now state three propositions:

(6) Let S be a reflexive relational structure, T be a reflexive transitive relational structure, f be a map from S into T, and X be a subset of S. Then $\downarrow(f^\circ X) \subseteq \downarrow(f^\circ \downarrow X)$.

(7) Let S be a reflexive relational structure, T be a reflexive transitive relational structure, f be a map from S into T, and X be a subset of S. If f is monotone, then $\downarrow(f^\circ X) = \downarrow(f^\circ \downarrow X)$.

(8) For every non empty poset N holds $\mathrm{IdsMap}(N)$ is one-to-one.

Let N be a non empty poset. Note that $\mathrm{IdsMap}(N)$ is one-to-one.

The following proposition is true

(9) For every finite lattice N holds $\mathrm{SupMap}(N)$ is one-to-one.

Let N be a finite lattice. Observe that $\mathrm{SupMap}(N)$ is one-to-one.

One can prove the following three propositions:

(10) For every finite lattice N holds N and $\langle \mathrm{Ids}(N), \subseteq \rangle$ are isomorphic.

(11) Let N be a complete non empty poset, x be an element of N, and X be a non empty subset of N. Then $x \sqcap \Box$ preserves inf of X.

(12) For every complete non empty poset N and for every element x of N holds $x \sqcap \Box$ is meet-preserving.

Let N be a complete non empty poset and let x be an element of N. Observe that $x \sqcap \Box$ is meet-preserving.

2. On the Basis of Topological Spaces

We now state several propositions:

(13) Let T be an anti-discrete non empty topological structure and p be a point of T. Then $\{$the carrier of $T\}$ is a basis of p.

(14) Let T be an anti-discrete non empty topological structure, p be a point of T, and D be a basis of p. Then $D = \{$the carrier of $T\}$.

(15) Let T be a non empty topological space, P be a basis of T, and p be a point of T. Then $\{A; A$ ranges over subsets of T: $A \in P \wedge p \in A\}$ is a basis of p.

(16) Let T be a non empty topological structure, A be a subset of T, and p be a point of T. Then $p \in \overline{A}$ if and only if for every basis K of p and for every subset Q of T such that $Q \in K$ holds A meets Q.

(17) Let T be a non empty topological structure, A be a subset of T, and p be a point of T. Then $p \in \overline{A}$ if and only if there exists a basis K of p such that for every subset Q of T such that $Q \in K$ holds A meets Q.

Let T be a topological structure and let p be a point of T. A family of subsets of T is said to be a generalized basis of p if:

(Def. 1) For every subset A of T such that $p \in \operatorname{Int} A$ there exists a subset P of T such that $P \in$ it and $p \in \operatorname{Int} P$ and $P \subseteq A$.

Let T be a non empty topological space and let p be a point of T. Let us note that the generalized basis of p can be characterized by the following (equivalent) condition:

(Def. 2) For every neighbourhood A of p there exists a neighbourhood P of p such that $P \in$ it and $P \subseteq A$.

Next we state two propositions:

(18) Let T be a topological structure and p be a point of T. Then $2^{\text{the carrier of } T}$ is a generalized basis of p.

(19) For every non empty topological space T and for every point p of T holds every generalized basis of p is non empty.

Let T be a non empty topological space and let p be a point of T. One can verify that every generalized basis of p is non empty.

Let T be a topological structure and let p be a point of T. Note that there exists a generalized basis of p which is non empty.

Let T be a topological structure, let p be a point of T, and let P be a generalized basis of p. We say that P is correct if and only if:

(Def. 3) For every subset A of T holds $A \in P$ iff $p \in \operatorname{Int} A$.

Let T be a topological structure and let p be a point of T. One can verify that there exists a generalized basis of p which is correct.

Next we state the proposition

(20) Let T be a topological structure and p be a point of T. Then $\{A; A$ ranges over subsets of T: $p \in \operatorname{Int} A\}$ is a correct generalized basis of p.

Let T be a non empty topological space and let p be a point of T. Observe that there exists a generalized basis of p which is non empty and correct.

The following propositions are true:

(21) Let T be an anti-discrete non empty topological structure and p be a point of T. Then $\{$the carrier of $T\}$ is a correct generalized basis of p.

(22) Let T be an anti-discrete non empty topological structure, p be a point of T, and D be a correct generalized basis of p. Then $D = \{$the carrier of $T\}$.

(23) For every non empty topological space T and for every point p of T holds every basis of p is a generalized basis of p.

Let T be a topological structure. A family of subsets of T is said to be a generalized basis of T if:

(Def. 4) For every point p of T holds it is a generalized basis of p.

One can prove the following two propositions:

(24) For every topological structure T holds $2^{\text{the carrier of } T}$ is a generalized basis of T.

(25) For every non empty topological space T holds every generalized basis of T is non empty.

Let T be a non empty topological space. Observe that every generalized basis of T is non empty.

Let T be a topological structure. Observe that there exists a generalized basis of T which is non empty.

Next we state two propositions:

(26) For every non empty topological space T and for every generalized basis P of T holds the topology of $T \subseteq \mathrm{UniCl}(\mathrm{Int}\,P)$.

(27) For every topological space T holds every basis of T is a generalized basis of T.

Let T be a non empty topological space-like FR-structure. We say that T satisfies conditions of topological semilattice if and only if:

(Def. 5) For every map f from $[\!: T, (T$ **qua** topological space$) :\!]$ into T such that $f = \sqcap_T$ holds f is continuous.

One can verify that every non empty topological space-like FR-structure which is reflexive and trivial satisfies also conditions of topological semilattice.

Let us mention that there exists a FR-structure which is reflexive, trivial, non empty, and topological space-like.

The following proposition is true

(28) Let T be a non empty topological space-like FR-structure satisfying conditions of topological semilattice and x be an element of T. Then $x \sqcap \square$ is continuous.

Let T be a non empty topological space-like FR-structure satisfying conditions of topological semilattice and let x be an element of T. Observe that $x \sqcap \square$ is continuous.

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