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Weights of Continuous Lattices[1]

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Summary. This work is a continuation of formalization of [16]. Theorems from Chapter III, Section 4, pp. 170–171 are proved.

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The articles [24], [12], [28], [21], [15], [30], [27], [29], [10], [11], [9], [13], [2], [1], [22], [23], [26], [3], [4], [17], [31], [7], [5], [14], [6], [19], [18], [25], [8], and [20] provide the notation and terminology for this paper.

In this article we present several logical schemes. The scheme *UparrowUnion* deals with a relational structure $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

For every family S of subsets of $\mathcal{A}$ such that $S = \{X; X$ ranges over subsets of $\mathcal{A}$: $\mathcal{P}[X]\}$ holds $\uparrow\bigcup S = \bigcup\{\uparrow X; X$ ranges over subsets of $\mathcal{A} : \mathcal{P}[X]\}$

for all values of the parameters.

The scheme *DownarrowUnion* deals with a relational structure $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

For every family S of subsets of $\mathcal{A}$ such that $S = \{X; X$ ranges over subsets of $\mathcal{A}$: $\mathcal{P}[X]\}$ holds $\downarrow\bigcup S = \bigcup\{\downarrow X; X$ ranges over subsets of $\mathcal{A} : \mathcal{P}[X]\}$

for all values of the parameters.

Let L_1 be a lower-bounded continuous sup-semilattice and let B_1 be a CLbasis of L_1 with bottom. Observe that $\langle \mathrm{Ids}(\mathrm{sub}(B_1)), \subseteq\rangle$ is algebraic.

Let L_1 be a continuous sup-semilattice. The functor CLweight L_1 yields a cardinal number and is defined as follows:

(Def. 1) $\quad \mathrm{CLweight}\, L_1 = \bigcap\{\overline{\overline{B_1}} : B_1$ ranges over CLbasis of L_1 with bottom$\}$.

One can prove the following propositions:

(1) For every topological structure T and for every basis b of T holds $\mathrm{weight}\, T \subseteq \overline{\overline{b}}$.

(2) For every topological structure T there exists a basis b of T such that $\overline{\overline{b}} = \mathrm{weight}\, T$.

(3) For every continuous sup-semilattice L_1 and for every CLbasis B_1 of L_1 with bottom holds $\mathrm{CLweight}\, L_1 \subseteq \overline{\overline{B_1}}$.

(4) For every continuous sup-semilattice L_1 there exists a CLbasis B_1 of L_1 with bottom such that $\overline{\overline{B_1}} = \mathrm{CLweight}\, L_1$.

(5) For every algebraic lower-bounded lattice L_1 holds $\mathrm{CLweight}\, L_1 = \overline{\overline{\text{the carrier of CompactSublatt}(L_1)}}$.

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(6) Let T be a non empty topological space and L_1 be a continuous sup-semilattice. If $\langle$the topology of T, $\subseteq\rangle = L_1$, then every CLbasis of L_1 with bottom is a basis of T.

(7) Let T be a non empty topological space and L_1 be a continuous lower-bounded lattice. Suppose $\langle$the topology of T, $\subseteq\rangle = L_1$. Let B_1 be a basis of T and B_2 be a subset of L_1. If $B_1 = B_2$, then $\mathrm{finsups}(B_2)$ is a CLbasis of L_1 with bottom.

(8) Let T be a T_0 non empty topological space and L_1 be a continuous lower-bounded sup-semilattice. If $\langle$the topology of T, $\subseteq\rangle = L_1$, then if T is infinite, then $\mathrm{weight}\,T = \mathrm{CLweight}\,L_1$.

(9) Let T be a T_0 non empty topological space and L_1 be a continuous sup-semilattice. Suppose $\langle$the topology of T, $\subseteq\rangle = L_1$. Then $\overline{\overline{\text{the carrier of } T}} \subseteq \overline{\overline{\text{the carrier of } L_1}}$.

(10) For every T_0 non empty topological space T such that T is finite holds $\mathrm{weight}\,T = \overline{\overline{\text{the carrier of } T}}$.

(11) Let T be a topological structure and L_1 be a continuous lower-bounded lattice. Suppose $\langle$the topology of T, $\subseteq\rangle = L_1$ and T is finite. Then $\mathrm{CLweight}\,L_1 = \overline{\overline{\text{the carrier of } L_1}}$.

(12) Let L_1 be a continuous lower-bounded sup-semilattice, T_1 be a Scott topological augmentation of L_1, T_2 be a Lawson correct topological augmentation of L_1, and B_2 be a basis of T_2. Then $\{{\uparrow}V; V$ ranges over subsets of T_2: $V \in B_2\}$ is a basis of T_1.

(14)[1] For every up-complete non empty poset L_1 such that L_1 is finite and for every element x of L_1 holds $x \in \mathrm{compactbelow}(x)$.

(15) Every finite lattice is arithmetic.

One can verify that every lattice which is finite is also arithmetic.

Let us observe that there exists a relational structure which is trivial, reflexive, transitive, antisymmetric, lower-bounded, non empty, finite, and strict and has l.u.b.'s and g.l.b.'s.

We now state the proposition

(16) Let L_1 be a finite lattice and B_1 be a CLbasis of L_1 with bottom. Then $\overline{\overline{B_1}} = \mathrm{CLweight}\,L_1$ if and only if $B_1 =$ the carrier of $\mathrm{CompactSublatt}(L_1)$.

Let L_1 be a non empty reflexive relational structure, let A be a subset of L_1, and let a be an element of L_1. The functor $\mathrm{Way\_Up}(a,A)$ yields a subset of L_1 and is defined as follows:

(Def. 2) $\mathrm{Way\_Up}(a,A) = \mathord{\Uparrow} a \setminus {\uparrow}A$.

One can prove the following propositions:

(17) For every non empty reflexive relational structure L_1 and for every element a of L_1 holds $\mathrm{Way\_Up}(a, \emptyset_{(L_1)}) = \mathord{\Uparrow} a$.

(18) For every non empty poset L_1 and for every subset A of L_1 and for every element a of L_1 such that $a \in {\uparrow}A$ holds $\mathrm{Way\_Up}(a,A) = \emptyset$.

(19) For every non empty finite reflexive transitive relational structure L_1 holds $\mathrm{Ids}(L_1)$ is finite.

(20) For every continuous lower-bounded sup-semilattice L_1 such that L_1 is infinite holds every CLbasis of L_1 with bottom is infinite.

(23)[2] For every complete non empty poset L_1 and for every element x of L_1 such that x is compact holds $x = \inf \mathord{\Uparrow} x$.

(24) Let L_1 be a continuous lower-bounded sup-semilattice. Suppose L_1 is infinite. Let B_1 be a CLbasis of L_1 with bottom. Then $\overline{\overline{\{\mathrm{Way\_Up}(a,A); a \text{ ranges over elements of } L_1, A \text{ ranges over finite subsets of } L_1\colon a \in B_1}} = \overline{\overline{B_1}}$.

[1] The proposition (13) has been removed.

[2] The propositions (21) and (22) have been removed.

(25) For every Lawson complete top-lattice T and for every finite subset X of T holds $(\uparrow X)^{\rm c}$ is open and $(\downarrow X)^{\rm c}$ is open.

(26) Let L_1 be a continuous lower-bounded sup-semilattice, T be a Lawson correct topological augmentation of L_1, and B_1 be a CLbasis of L_1 with bottom. Then $\{\text{Way\_Up}(a,A); a$ ranges over elements of L_1, A ranges over finite subsets of L_1: $a \in B_1 \wedge A \subseteq B_1\}$ is a basis of T.

(27) Let L_1 be a continuous lower-bounded sup-semilattice, T be a Scott topological augmentation of L_1, and b be a basis of T. Then $\{\uparrow \inf u; u$ ranges over subsets of T: $u \in b\}$ is a basis of T.

(28) Let L_1 be a continuous lower-bounded sup-semilattice, T be a Scott topological augmentation of L_1, and B_1 be a basis of T. If B_1 is infinite, then $\{\inf u; u$ ranges over subsets of T: $u \in B_1\}$ is infinite.

(29) Let L_1 be a continuous lower-bounded sup-semilattice and T be a Scott topological augmentation of L_1. Then $\text{CLweight}\, L_1 = \text{weight}\, T$.

(30) Let L_1 be a continuous lower-bounded sup-semilattice and T be a Lawson correct topological augmentation of L_1. Then $\text{CLweight}\, L_1 = \text{weight}\, T$.

(31) Let L_1, L_2 be non empty relational structures. Suppose L_1 and L_2 are isomorphic. Then $\overline{\overline{\text{the carrier of } L_1}} = \overline{\overline{\text{the carrier of } L_2}}$.

(32) Let L_1 be a continuous lower-bounded sup-semilattice and B_1 be a CLbasis of L_1 with bottom. If $\overline{\overline{B_1}} = \text{CLweight}\, L_1$, then $\text{CLweight}\, L_1 = \text{CLweight}\langle \text{Ids}(\text{sub}(B_1)), \subseteq \rangle$.

Let L_1 be a continuous lower-bounded sup-semilattice. Observe that $\langle \sigma(L_1), \subseteq \rangle$ is continuous and has l.u.b.'s.

Next we state two propositions:

(33) For every continuous lower-bounded sup-semilattice L_1 holds $\text{CLweight}\, L_1 \subseteq \text{CLweight}\langle \sigma(L_1), \subseteq \rangle$.

(34) For every continuous lower-bounded sup-semilattice L_1 such that L_1 is infinite holds $\text{CLweight}\, L_1 = \text{CLweight}\langle \sigma(L_1), \subseteq \rangle$.

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