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# Scott Topology[1]

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**Summary.** In the article we continue the formalization in Mizar of [12, 98–105]. We work with structures of the form

$$L = \langle C, \leq, \tau \rangle,$$

where $C$ is the carrier of the structure, $\leq$ - an ordering relation on $C$ and $\tau$ a family of subsets of $C$. When $\langle C, \leq \rangle$ is a complete lattice we say that $L$ is Scott, if $\tau$ is the Scott topology of $\langle C, \leq \rangle$. We define the Scott convergence (lim inf convergence). Following [12] we prove that in the case of a continuous lattice $\langle C, \leq \rangle$ the Scott convergence is topological, i.e. enjoys the properties: (CONSTANTS), (SUBNETS), (DIVERGENCE), (ITERATED LIMITS). We formalize the theorem, that if the Scott convergence has the (ITERATED LIMITS) property, the $\langle C, \leq \rangle$ is continuous.

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The articles [23], [10], [29], [31], [11], [30], [7], [9], [8], [2], [28], [19], [21], [32], [22], [20], [34], [24], [1], [18], [27], [3], [4], [5], [13], [33], [14], [15], [16], [6], [25], [17], and [26] provide the notation and terminology for this paper.

## 1. Preliminaries

The scheme *Irrel* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding a set, a binary functor $\mathcal{F}$ yielding a set, and a unary predicate $\mathcal{P}$, and states that:

$\{\mathcal{F}(u); u$ ranges over elements of $\mathcal{A} : \mathcal{P}[u]\} = \{\mathcal{F}(i,v); i$ ranges over elements of $\mathcal{B}, v$ ranges over elements of $\mathcal{A} : \mathcal{P}[v]\}$

provided the following condition is met:

- For every element $i$ of $\mathcal{B}$ and for every element $u$ of $\mathcal{A}$ holds $\mathcal{F}(u) = \mathcal{F}(i,u)$.

We now state three propositions:

(1) For every complete lattice $L$ and for all subsets $X$, $Y$ of $L$ such that $Y$ is coarser than $X$ holds $\bigsqcap_L X \leq \bigsqcap_L Y$.

(2) For every complete lattice $L$ and for all subsets $X$, $Y$ of $L$ such that $X$ is finer than $Y$ holds $\bigsqcup_L X \leq \bigsqcup_L Y$.

(3) Let $T$ be a relational structure, $A$ be an upper subset of $T$, and $B$ be a directed subset of $T$. Then $A \cap B$ is directed.

Let $T$ be a reflexive non empty relational structure. One can verify that there exists a subset of $T$ which is non empty, directed, and finite.

We now state the proposition

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(4) For every poset $T$ with l.u.b.'s and for every non empty directed finite subset $D$ of $T$ holds $\sup D \in D$.

Let us observe that there exists a relational structure which is trivial, reflexive, transitive, non empty, antisymmetric, finite, and strict and has l.u.b.'s and g.l.b.'s.

One can check that there exists a 1-sorted structure which is finite, non empty, and strict.

Let $T$ be a finite 1-sorted structure. Observe that every subset of $T$ is finite.

Let $R$ be a relational structure. One can check that $\emptyset_R$ is lower and upper.

Let $R$ be a trivial non empty relational structure. One can verify that every subset of $R$ is upper.

Next we state two propositions:

(5) Let $T$ be a non empty relational structure, $x$ be an element of $T$, and $A$ be an upper subset of $T$. If $x \notin A$, then $A$ misses $\downarrow x$.

(6) Let $T$ be a non empty relational structure, $x$ be an element of $T$, and $A$ be a lower subset of $T$. If $x \in A$, then $\downarrow x \subseteq A$.

## 2. Scott Topology

Let $T$ be a non empty reflexive relational structure and let $S$ be a subset of $T$. We say that $S$ is inaccessible by directed joins if and only if:

(Def. 1) For every non empty directed subset $D$ of $T$ such that $\sup D \in S$ holds $D$ meets $S$.

We introduce $S$ is inaccessible as a synonym of $S$ is inaccessible by directed joins. We say that $S$ is closed under directed sups if and only if:

(Def. 2) For every non empty directed subset $D$ of $T$ such that $D \subseteq S$ holds $\sup D \in S$.

We introduce $S$ is directly closed as a synonym of $S$ is closed under directed sups. We say that $S$ is property(S) if and only if the condition (Def. 3) is satisfied.

(Def. 3) Let $D$ be a non empty directed subset of $T$. Suppose $\sup D \in S$. Then there exists an element $y$ of $T$ such that $y \in D$ and for every element $x$ of $T$ such that $x \in D$ and $x \geq y$ holds $x \in S$.

We introduce $S$ has the property (S) as a synonym of $S$ is property(S).

Let $T$ be a non empty reflexive relational structure. Observe that $\emptyset_T$ is property(S) and directly closed.

Let $T$ be a non empty reflexive relational structure. Note that there exists a subset of $T$ which is property(S) and directly closed.

Let $T$ be a non empty reflexive relational structure and let $S$ be a property(S) subset of $T$. Observe that $S^{c}$ is directly closed.

Let $T$ be a reflexive non empty FR-structure. We say that $T$ is Scott if and only if:

(Def. 4) For every subset $S$ of $T$ holds $S$ is open iff $S$ is inaccessible and upper.

Let $T$ be a reflexive transitive antisymmetric non empty finite relational structure with l.u.b.'s. One can verify that every subset of $T$ is inaccessible.

Let $T$ be a reflexive transitive antisymmetric non empty finite FR-structure with l.u.b.'s. Let us observe that $T$ is Scott if and only if:

(Def. 5) For every subset $S$ of $T$ holds $S$ is open iff $S$ is upper.

One can check that there exists a top-lattice which is trivial, complete, strict, non empty, and Scott.

Let $T$ be a non empty reflexive relational structure. One can verify that $\Omega_T$ is directly closed and inaccessible.

Let $T$ be a non empty reflexive relational structure. Note that there exists a subset of $T$ which is directly closed, lower, inaccessible, and upper.

Let $T$ be a non empty reflexive relational structure and let $S$ be an inaccessible subset of $T$. Note that $S^{\rm c}$ is directly closed.

Let $T$ be a non empty reflexive relational structure and let $S$ be a directly closed subset of $T$. Note that $S^{\rm c}$ is inaccessible.

Next we state several propositions:

(7) Let $T$ be an up-complete Scott non empty reflexive transitive FR-structure and $S$ be a subset of $T$. Then $S$ is closed if and only if $S$ is directly closed and lower.

(8) Let $T$ be an up-complete non empty reflexive transitive antisymmetric FR-structure and $x$ be an element of $T$. Then $\downarrow x$ is directly closed.

(9) For every complete Scott top-lattice $T$ and for every element $x$ of $T$ holds $\overline{\{x\}} = \downarrow x$.

(10) Every complete Scott top-lattice is a $T_0$-space.

(11) Let $T$ be a Scott up-complete non empty reflexive transitive antisymmetric FR-structure and $x$ be an element of $T$. Then $\downarrow x$ is closed.

(12) For every up-complete Scott top-lattice $T$ and for every element $x$ of $T$ holds $(\downarrow x)^{\rm c}$ is open.

(13) Let $T$ be an up-complete Scott top-lattice, $x$ be an element of $T$, and $A$ be an upper subset of $T$. If $x \notin A$, then $(\downarrow x)^{\rm c}$ is a neighbourhood of $A$.

(14) Let $T$ be a complete Scott top-lattice and $S$ be an upper subset of $T$. Then there exists a family $F$ of subsets of $T$ such that $S = \bigcap F$ and for every subset $X$ of $T$ such that $X \in F$ holds $X$ is a neighbourhood of $S$.

(15) Let $T$ be a Scott top-lattice and $S$ be a subset of $T$. Then $S$ is open if and only if $S$ is upper and property(S).

Let $T$ be a complete top-lattice. One can verify that every subset of $T$ which is lower is also property(S).

We now state the proposition

(16) Let $T$ be a non empty transitive reflexive FR-structure. Suppose the topology of $T = \{S; S$ ranges over subsets of $T$: $S$ has the property (S)$\}$. Then $T$ is topological space-like.

## 3. Scott Convergence

In the sequel $R$ denotes a non empty relational structure, $N$ denotes a net in $R$, and $i$ denotes an element of $N$.

Let us consider $R$, $N$. The functor $\liminf N$ yielding an element of $R$ is defined as follows:

(Def. 6) $\liminf N = \bigsqcup_R \{ \bigsqcap_R \{N(i) : i \geq j\} : j$ ranges over elements of $N\}$.

Let $R$ be a reflexive non empty relational structure, let $N$ be a net in $R$, and let $p$ be an element of $R$. We say that $p$ is S-limit of $N$ if and only if:

(Def. 7) $p \leq \liminf N$.

Let $R$ be a reflexive non empty relational structure. The Scott convergence of $R$ yields a convergence class of $R$ and is defined by the condition (Def. 8).

(Def. 8) Let $N$ be a strict net in $R$. Suppose $N \in \mathrm{NetUniv}(R)$. Let $p$ be an element of $R$. Then $\langle N, p\rangle \in$ the Scott convergence of $R$ if and only if $p$ is S-limit of $N$.

Next we state two propositions:

(17) Let $R$ be a complete lattice, $N$ be a net in $R$, and $p$, $q$ be elements of $R$. If $p$ is S-limit of $N$ and $N$ is eventually in $\downarrow q$, then $p \leq q$.

(18) Let $R$ be a complete lattice, $N$ be a net in $R$, and $p$, $q$ be elements of $R$. If $N$ is eventually in $\uparrow q$, then $\liminf N \geq q$.

Let $R$ be a reflexive non empty relational structure and let $N$ be a non empty net structure over $R$. Let us observe that $N$ is monotone if and only if:

(Def. 9) For all elements $i$, $j$ of $N$ such that $i \leq j$ holds $N(i) \leq N(j)$.

Let $R$ be a non empty relational structure, let $S$ be a non empty set, and let $f$ be a function from $S$ into the carrier of $R$. The functor $\mathrm{NetStr}(S, f)$ yielding a strict non empty net structure over $R$ is defined by the conditions (Def. 10).

(Def. 10)(i) The carrier of $\mathrm{NetStr}(S, f) = S$,

(ii) the mapping of $\mathrm{NetStr}(S, f) = f$, and

(iii) for all elements $i$, $j$ of $\mathrm{NetStr}(S, f)$ holds $i \leq j$ iff $(\mathrm{NetStr}(S, f))(i) \leq (\mathrm{NetStr}(S, f))(j)$.

The following propositions are true:

(19) Let $L$ be a non empty 1-sorted structure and $N$ be a non empty net structure over $L$. Then $\mathrm{rng}(\text{the mapping of } N) = \{N(i) : i \text{ ranges over elements of } N\}$.

(20) Let $R$ be a non empty relational structure, $S$ be a non empty set, and $f$ be a function from $S$ into the carrier of $R$. If $\mathrm{rng} f$ is directed, then $\mathrm{NetStr}(S, f)$ is directed.

Let $R$ be a non empty relational structure, let $S$ be a non empty set, and let $f$ be a function from $S$ into the carrier of $R$. One can check that $\mathrm{NetStr}(S, f)$ is monotone.

Let $R$ be a transitive non empty relational structure, let $S$ be a non empty set, and let $f$ be a function from $S$ into the carrier of $R$. Note that $\mathrm{NetStr}(S, f)$ is transitive.

Let $R$ be a reflexive non empty relational structure, let $S$ be a non empty set, and let $f$ be a function from $S$ into the carrier of $R$. Observe that $\mathrm{NetStr}(S, f)$ is reflexive.

Next we state the proposition

(21) Let $R$ be a non empty transitive relational structure, $S$ be a non empty set, and $f$ be a function from $S$ into the carrier of $R$. If $S \subseteq$ the carrier of $R$ and $\mathrm{NetStr}(S, f)$ is directed, then $\mathrm{NetStr}(S, f) \in \mathrm{NetUniv}(R)$.

Let $R$ be a lattice. Observe that there exists a net in $R$ which is monotone, reflexive, and strict.

Next we state three propositions:

(22) For every complete lattice $R$ and for every monotone reflexive net $N$ in $R$ holds $\liminf N = \sup N$.

(23) For every complete lattice $R$ and for every constant net $N$ in $R$ holds the value of $N = \liminf N$.

(24) For every complete lattice $R$ and for every constant net $N$ in $R$ holds the value of $N$ is S-limit of $N$.

Let $S$ be a non empty 1-sorted structure and let $e$ be an element of $S$. The functor $\mathrm{NetStr}(e)$ yielding a strict net structure over $S$ is defined as follows:

(Def. 11) The carrier of $\mathrm{NetStr}(e) = \{e\}$ and the internal relation of $\mathrm{NetStr}(e) = \{\langle e, e \rangle\}$ and the mapping of $\mathrm{NetStr}(e) = \mathrm{id}_{\{e\}}$.

Let $S$ be a non empty 1-sorted structure and let $e$ be an element of $S$. Note that $\mathrm{NetStr}(e)$ is non empty.

One can prove the following two propositions:

(25) For every non empty 1-sorted structure $S$ and for every element $e$ of $S$ and for every element $x$ of $\mathrm{NetStr}(e)$ holds $x = e$.

(26) For every non empty 1-sorted structure $S$ and for every element $e$ of $S$ and for every element $x$ of NetStr($e$) holds $(\mathrm{NetStr}(e))(x) = e$.

Let $S$ be a non empty 1-sorted structure and let $e$ be an element of $S$. Note that NetStr($e$) is reflexive, transitive, directed, and antisymmetric.

We now state several propositions:

(27) Let $S$ be a non empty 1-sorted structure, $e$ be an element of $S$, and $X$ be a set. Then NetStr($e$) is eventually in $X$ if and only if $e \in X$.

(28) For every reflexive antisymmetric non empty relational structure $S$ and for every element $e$ of $S$ holds $e = \liminf \mathrm{NetStr}(e)$.

(29) For every non empty reflexive relational structure $S$ and for every element $e$ of $S$ holds $\mathrm{NetStr}(e) \in \mathrm{NetUniv}(S)$.

(30) Let $R$ be a complete lattice, $Z$ be a net in $R$, and $D$ be a subset of $R$. Suppose $D = \{\bigsqcap_R\{Z(k); k$ ranges over elements of $Z$: $k \geq j\}$ : $j$ ranges over elements of $Z\}$. Then $D$ is non empty and directed.

(31) Let $L$ be a complete lattice and $S$ be a subset of $L$. Then $S \in$ the topology of ConvergenceSpace(the Scott convergence of $L$) if and only if $S$ is inaccessible and upper.

(32) For every complete Scott top-lattice $T$ holds the topological structure of $T =$ ConvergenceSpace(the Scott convergence of $T$).

(33) Let $T$ be a complete top-lattice. Suppose the topological structure of $T =$ ConvergenceSpace(the Scott convergence of $T$). Let $S$ be a subset of $T$. Then $S$ is open if and only if $S$ is inaccessible and upper.

(34) Let $T$ be a complete top-lattice. Suppose the topological structure of $T =$ ConvergenceSpace(the Scott convergence of $T$). Then $T$ is Scott.

Let $R$ be a complete lattice. Observe that the Scott convergence of $R$ has (CONSTANTS) property.

Let $R$ be a complete lattice. Note that the Scott convergence of $R$ has (SUBNETS) property.

Next we state the proposition

(35) Let $S$ be a non empty 1-sorted structure, $N$ be a net in $S$, $X$ be a set, and $M$ be a subnet of $N$. If $M = N^{-1}(X)$, then for every element $i$ of $M$ holds $M(i) \in X$.

Let $L$ be a non empty reflexive relational structure. The functor $\sigma(L)$ yielding a family of subsets of $L$ is defined as follows:

(Def. 12) $\sigma(L) =$ the topology of ConvergenceSpace(the Scott convergence of $L$).

One can prove the following propositions:

(36) For every continuous complete Scott top-lattice $L$ and for every element $x$ of $L$ holds $\twoheaduparrow x$ is open.

(37) For every complete top-lattice $T$ such that the topology of $T = \sigma(T)$ holds $T$ is Scott.

Let $R$ be a continuous complete lattice. One can verify that the Scott convergence of $R$ is topological.

Next we state a number of propositions:

(38) Let $T$ be a continuous complete Scott top-lattice, $x$ be an element of $T$, and $N$ be a net in $T$. If $N \in \mathrm{NetUniv}(T)$, then $x$ is S-limit of $N$ iff $x \in \mathrm{Lim}\,N$.

(39) Let $L$ be a complete non empty poset. Suppose the Scott convergence of $L$ has (ITERATED LIMITS) property. Then $L$ is continuous.

(40) For every complete Scott top-lattice $T$ holds $T$ is continuous iff Convergence$(T) =$ the Scott convergence of $T$.

(41) For every complete Scott top-lattice $T$ and for every upper subset $S$ of $T$ such that $S$ is open holds $S$ is open.

(42) Let $L$ be a non empty relational structure, $S$ be an upper subset of $L$, and $x$ be an element of $L$. If $x \in S$, then $\uparrow x \subseteq S$.

(43) Let $L$ be a complete continuous Scott top-lattice, $p$ be an element of $L$, and $S$ be a subset of $L$. If $S$ is open and $p \in S$, then there exists an element $q$ of $L$ such that $q \ll p$ and $q \in S$.

(44) Let $L$ be a complete continuous Scott top-lattice and $p$ be an element of $L$. Then $\{\Uparrow q; q$ ranges over elements of $L$: $q \ll p\}$ is a basis of $p$.

(45) For every complete continuous Scott top-lattice $T$ holds $\{\Uparrow x : x$ ranges over elements of $T\}$ is a basis of $T$.

(46) Let $T$ be a complete continuous Scott top-lattice and $S$ be an upper subset of $T$. Then $S$ is open if and only if $S$ is open.

(47) For every complete continuous Scott top-lattice $T$ and for every element $p$ of $T$ holds $\mathrm{Int}\uparrow p = \Uparrow p$.

(48) Let $T$ be a complete continuous Scott top-lattice and $S$ be a subset of $T$. Then $\mathrm{Int} S = \bigcup\{\Uparrow x; x$ ranges over elements of $T$: $\Uparrow x \subseteq S\}$.

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