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Some Facts about Union of Two Functions and Continuity of Union of Functions

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Summary. Proofs of two theorems connected with union of any two functions and the proofs of two theorems about continuity of the union of two continuous functions between topogical spaces. The theorem stating that union of two subsets of R^2, which are homeomorphic to unit interval and have only one terminal joined point is also homeomorphic to unit interval is proved, too.

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The articles [13], [15], [1], [14], [16], [3], [4], [5], [11], [7], [6], [12], [8], [10], [2], and [9] provide the notation and terminology for this paper.

One can prove the following proposition

(1) For all real numbers x, y, z such that $x \leq y$ and $y \leq z$ holds $[x,y] \cap [y,z] = \{y\}$.

In the sequel f, g denote functions and x_1, x_2 denote sets.

One can prove the following propositions:

(2) Suppose f is one-to-one and g is one-to-one and for all x_1, x_2 such that $x_1 \in \operatorname{dom} g$ and $x_2 \in \operatorname{dom} f \setminus \operatorname{dom} g$ holds $g(x_1) \neq f(x_2)$. Then $f+\cdot g$ is one-to-one.

(3) If $f^\circ(\operatorname{dom} f \cap \operatorname{dom} g) \subseteq \operatorname{rng} g$, then $\operatorname{rng} f \cup \operatorname{rng} g = \operatorname{rng}(f+\cdot g)$.

In the sequel T, S are non empty topological spaces and p is a point of T.

The following three propositions are true:

(4) Let T_1, T_2 be subspaces of T, f be a map from T_1 into S, and g be a map from T_2 into S. Suppose that $\Omega_{(T_1)} \cup \Omega_{(T_2)} = \Omega_T$ and $\Omega_{(T_1)} \cap \Omega_{(T_2)} = \{p\}$ and T_1 is compact and T_2 is compact and T is a T_2 space and f is continuous and g is continuous and $f(p) = g(p)$. Then there exists a map h from T into S such that $h = f+\cdot g$ and h is continuous.

(5) Let T be a non empty topological space, T_1, T_2 be subspaces of T, p_1, p_2 be points of T, f be a map from T_1 into S, and g be a map from T_2 into S. Suppose that $\Omega_{(T_1)} \cup \Omega_{(T_2)} = \Omega_T$ and $\Omega_{(T_1)} \cap \Omega_{(T_2)} = \{p_1, p_2\}$ and T_1 is compact and T_2 is compact and T is a T_2 space and f is continuous and g is continuous and $f(p_1) = g(p_1)$ and $f(p_2) = g(p_2)$. Then there exists a map h from T into S such that $h = f+\cdot g$ and h is continuous.

(6) Let n be a natural number, P, Q be subsets of $\mathcal{E}^n_{\mathrm{T}}$, p be a point of $\mathcal{E}^n_{\mathrm{T}}$, f be a map from $\mathbb{I}$ into $(\mathcal{E}^n_{\mathrm{T}})\restriction P$, and g be a map from $\mathbb{I}$ into $(\mathcal{E}^n_{\mathrm{T}})\restriction Q$. Suppose $P \cap Q = \{p\}$ and f is a homeomorphism and $f(1) = p$ and g is a homeomorphism and $g(0) = p$. Then there exists a map h from $\mathbb{I}$ into $(\mathcal{E}^n_{\mathrm{T}})\restriction(P \cup Q)$ such that h is a homeomorphism and $h(0) = f(0)$ and $h(1) = g(1)$.

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