JOURNAL OF FORMALIZED MATHEMATICS
Volume 6, Released 1994, Published 2003
Inst. of Computer Science, Univ. of Białystok

Maximal Anti-Discrete Subspaces of Topological Spaces

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Summary. Let X be a topological space and let A be a subset of X. A is said to be *anti-discrete* provided for every open subset G of X either $A \cap G = \emptyset$ or $A \subseteq G$; equivalently, for every closed subset F of X either $A \cap F = \emptyset$ or $A \subseteq F$. An anti-discrete subset M of X is said to be *maximal anti-discrete* provided for every anti-discrete subset A of X if $M \subseteq A$ then $M = A$. A subspace of X is *maximal anti-discrete* iff its carrier is maximal anti-discrete in X. The purpose is to list a few properties of maximal anti-discrete sets and subspaces in Mizar formalism.

It is shown that every $x \in X$ is contained in a unique maximal anti-discrete subset $\mathrm{M}(x)$ of X, denoted in the text by MaxADSet(x). Such subset can be defined by

$$\mathrm{M}(x) = \bigcap \{S \subseteq X : x \in S, \text{ and } S \text{ is open or closed in } X\}.$$

It has the following remarkable properties: (1) $y \in \mathrm{M}(x)$ iff $\mathrm{M}(y) = \mathrm{M}(x)$, (2) either $\mathrm{M}(x) \cap \mathrm{M}(y) = \emptyset$ or $\mathrm{M}(x) = \mathrm{M}(y)$, (3) $\mathrm{M}(x) = \mathrm{M}(y)$ iff $\overline{\{x\}} = \overline{\{y\}}$, and (4) $\mathrm{M}(x) \cap \mathrm{M}(y) = \emptyset$ iff $\overline{\{x\}} \neq \overline{\{y\}}$. It follows from these properties that $\{\mathrm{M}(x) : x \in X\}$ is the T_0-partition of X defined by M.H. Stone in [8].

Moreover, it is shown that the operation M defined on all subsets of X by

$$\mathrm{M}(A) = \bigcup \{\mathrm{M}(x) : x \in A\},$$

denoted in the text by MaxADSet(A), satisfies the Kuratowski closure axioms (see e.g., [5]), i.e., (1) $\mathrm{M}(A \cup B) = \mathrm{M}(A) \cup \mathrm{M}(B)$, (2) $\mathrm{M}(A) = \mathrm{M}(\mathrm{M}(A))$, (3) $A \subseteq \mathrm{M}(A)$,and (4) $\mathrm{M}(\emptyset) = \emptyset$. Note that this operation commutes with the usual closure operation of X, and if A is an open (or a closed) subset of X, then $\mathrm{M}(A) = A$.

MML Identifier: `TEX_4`.

WWW: `http://mizar.org/JFM/Vol6/tex_4.html`

The articles [9], [1], [11], [6], [7], [12], [10], [3], [2], and [4] provide the notation and terminology for this paper.

1. Properties of the Closure and the Interior Operations

Let X be a non empty topological space and let A be a non empty subset of X. Observe that $\overline{A}$ is non empty.

Let X be a non empty topological space and let A be an empty subset of X. Note that $\overline{A}$ is empty.

Let X be a non empty topological space and let A be a non proper subset of X. Observe that $\overline{A}$ is non proper.

Let X be a non trivial non empty topological space and let A be a non trivial non empty subset of X. Note that $\overline{A}$ is non trivial.

In the sequel X denotes a non empty topological space.

One can prove the following propositions:

(1) For every subset A of X holds $\overline{A} = \bigcap\{F; F$ ranges over subsets of X: F is closed $\wedge A \subseteq F\}$.

(2) For every point x of X holds $\overline{\{x\}} = \bigcap\{F; F$ ranges over subsets of X: F is closed $\wedge x \in F\}$.

(3) For all subsets A, B of X such that $B \subseteq \overline{A}$ holds $\overline{B} \subseteq \overline{A}$.

Let X be a non empty topological space and let A be a non proper subset of X. Note that Int$A$ is non proper.

Let X be a non empty topological space and let A be a proper subset of X. Observe that Int$A$ is proper.

Let X be a non empty topological space and let A be an empty subset of X. One can check that Int$A$ is empty.

The following two propositions are true:

(4) For every subset A of X holds Int$A = \bigcup\{G; G$ ranges over subsets of X: G is open $\wedge G \subseteq A\}$.

(5) For all subsets A, B of X such that Int$A \subseteq B$ holds Int$A \subseteq$ Int$B$.

2. Anti-Discrete Subsets of Topological Structures

Let Y be a topological structure and let I_1 be a subset of Y. We say that I_1 is anti-discrete if and only if:

(Def. 1) For every point x of Y and for every subset G of Y such that G is open and $x \in G$ holds if $x \in I_1$, then $I_1 \subseteq G$.

Let Y be a non empty topological structure and let A be a subset of Y. Let us observe that A is anti-discrete if and only if:

(Def. 2) For every point x of Y and for every subset F of Y such that F is closed and $x \in F$ holds if $x \in A$, then $A \subseteq F$.

Let Y be a topological structure and let A be a subset of Y. Let us observe that A is anti-discrete if and only if:

(Def. 3) For every subset G of Y such that G is open holds A misses G or $A \subseteq G$.

Let Y be a topological structure and let A be a subset of Y. Let us observe that A is anti-discrete if and only if:

(Def. 4) For every subset F of Y such that F is closed holds A misses F or $A \subseteq F$.

The following proposition is true

(6) Let Y_0, Y_1 be topological structures, D_0 be a subset of Y_0, and D_1 be a subset of Y_1. Suppose the topological structure of $Y_0 =$ the topological structure of Y_1 and $D_0 = D_1$. If D_0 is anti-discrete, then D_1 is anti-discrete.

In the sequel Y denotes a non empty topological structure.

We now state three propositions:

(7) For all subsets A, B of Y such that $B \subseteq A$ holds if A is anti-discrete, then B is anti-discrete.

(8) For every point x of Y holds $\{x\}$ is anti-discrete.

(9) Every empty subset of Y is anti-discrete.

Let Y be a topological structure and let I_1 be a family of subsets of Y. We say that I_1 is anti-discrete-set-family if and only if:

(Def. 5) For every subset A of Y such that $A \in I_1$ holds A is anti-discrete.

One can prove the following propositions:

(10) Let F be a family of subsets of Y. Suppose F is anti-discrete-set-family. If $\bigcap F \neq \emptyset$, then $\bigcup F$ is anti-discrete.

(11) For every family F of subsets of Y such that F is anti-discrete-set-family holds $\bigcap F$ is anti-discrete.

Let Y be a topological structure and let x be a point of Y. The functor MaxADSF(x) yields a family of subsets of Y and is defined by:

(Def. 6) $\mathrm{MaxADSF}(x) = \{A; A$ ranges over subsets of Y: A is anti-discrete $\wedge\, x \in A\}$.

Let Y be a non empty topological structure and let x be a point of Y. Observe that MaxADSF(x) is non empty.

In the sequel x denotes a point of Y.

Next we state four propositions:

(12) MaxADSF(x) is anti-discrete-set-family.

(13) $\{x\} = \bigcap \mathrm{MaxADSF}(x)$.

(14) $\{x\} \subseteq \bigcup \mathrm{MaxADSF}(x)$.

(15) $\bigcup \mathrm{MaxADSF}(x)$ is anti-discrete.

3. Maximal Anti-Discrete Subsets of Topological Structures

Let Y be a topological structure and let I_1 be a subset of Y. We say that I_1 is maximal anti-discrete if and only if:

(Def. 7) I_1 is anti-discrete and for every subset D of Y such that D is anti-discrete and $I_1 \subseteq D$ holds $I_1 = D$.

The following proposition is true

(16) Let Y_0, Y_1 be topological structures, D_0 be a subset of Y_0, and D_1 be a subset of Y_1. Suppose the topological structure of Y_0 = the topological structure of Y_1 and $D_0 = D_1$. If D_0 is maximal anti-discrete, then D_1 is maximal anti-discrete.

In the sequel Y is a non empty topological structure.

One can prove the following propositions:

(17) For every empty subset A of Y holds A is not maximal anti-discrete.

(18) For every non empty subset A of Y such that A is anti-discrete and open holds A is maximal anti-discrete.

(19) For every non empty subset A of Y such that A is anti-discrete and closed holds A is maximal anti-discrete.

Let Y be a topological structure and let x be a point of Y. The functor MaxADSet(x) yielding a subset of Y is defined as follows:

(Def. 8) $\mathrm{MaxADSet}(x) = \bigcup \mathrm{MaxADSF}(x)$.

Let Y be a non empty topological structure and let x be a point of Y. One can verify that MaxADSet(x) is non empty.

The following propositions are true:

(20) For every point x of Y holds $\{x\} \subseteq \mathrm{MaxADSet}(x)$.

(21) For every subset D of Y and for every point x of Y such that D is anti-discrete and $x \in D$ holds $D \subseteq \mathrm{MaxADSet}(x)$.

(22) For every point x of Y holds $\mathrm{MaxADSet}(x)$ is maximal anti-discrete.

(23) For all points x, y of Y holds $y \in \mathrm{MaxADSet}(x)$ iff $\mathrm{MaxADSet}(y) = \mathrm{MaxADSet}(x)$.

(24) For all points x, y of Y holds $\mathrm{MaxADSet}(x)$ misses $\mathrm{MaxADSet}(y)$ or $\mathrm{MaxADSet}(x) = \mathrm{MaxADSet}(y)$.

(25) For every subset F of Y and for every point x of Y such that F is closed and $x \in F$ holds $\mathrm{MaxADSet}(x) \subseteq F$.

(26) For every subset G of Y and for every point x of Y such that G is open and $x \in G$ holds $\mathrm{MaxADSet}(x) \subseteq G$.

(27) Let x be a point of Y. Suppose $\{F; F$ ranges over subsets of Y: F is closed $\wedge\, x \in F\} \neq \emptyset$. Then $\mathrm{MaxADSet}(x) \subseteq \bigcap\{F; F$ ranges over subsets of Y: F is closed $\wedge\, x \in F\}$.

(28) Let x be a point of Y. Suppose $\{G; G$ ranges over subsets of Y: G is open $\wedge\, x \in G\} \neq \emptyset$. Then $\mathrm{MaxADSet}(x) \subseteq \bigcap\{G; G$ ranges over subsets of Y: G is open $\wedge\, x \in G\}$.

Let Y be a non empty topological structure and let A be a subset of Y. Let us observe that A is maximal anti-discrete if and only if:

(Def. 9) There exists a point x of Y such that $x \in A$ and $A = \mathrm{MaxADSet}(x)$.

One can prove the following proposition

(29) For every subset A of Y and for every point x of Y such that $x \in A$ holds if A is maximal anti-discrete, then $A = \mathrm{MaxADSet}(x)$.

Let Y be a non empty topological structure and let A be a non empty subset of Y. Let us observe that A is maximal anti-discrete if and only if:

(Def. 10) For every point x of Y such that $x \in A$ holds $A = \mathrm{MaxADSet}(x)$.

Let Y be a non empty topological structure and let A be a subset of Y. The functor $\mathrm{MaxADSet}(A)$ yielding a subset of Y is defined by:

(Def. 11) $\mathrm{MaxADSet}(A) = \bigcup\{\mathrm{MaxADSet}(a); a$ ranges over points of Y: $a \in A\}$.

We now state a number of propositions:

(30) For every point x of Y holds $\mathrm{MaxADSet}(x) = \mathrm{MaxADSet}(\{x\})$.

(31) For every subset A of Y and for every point x of Y such that $\mathrm{MaxADSet}(x)$ meets $\mathrm{MaxADSet}(A)$ holds $\mathrm{MaxADSet}(x)$ meets A.

(32) For every subset A of Y and for every point x of Y such that $\mathrm{MaxADSet}(x)$ meets $\mathrm{MaxADSet}(A)$ holds $\mathrm{MaxADSet}(x) \subseteq \mathrm{MaxADSet}(A)$.

(33) For all subsets A, B of Y such that $A \subseteq B$ holds $\mathrm{MaxADSet}(A) \subseteq \mathrm{MaxADSet}(B)$.

(34) For every subset A of Y holds $A \subseteq \mathrm{MaxADSet}(A)$.

(35) For every subset A of Y holds $\mathrm{MaxADSet}(A) = \mathrm{MaxADSet}(\mathrm{MaxADSet}(A))$.

(36) For all subsets A, B of Y such that $A \subseteq \mathrm{MaxADSet}(B)$ holds $\mathrm{MaxADSet}(A) \subseteq \mathrm{MaxADSet}(B)$.

(37) For all subsets A, B of Y holds $B \subseteq \mathrm{MaxADSet}(A)$ and $A \subseteq \mathrm{MaxADSet}(B)$ iff $\mathrm{MaxADSet}(A) = \mathrm{MaxADSet}(B)$.

(38) For all subsets A, B of Y holds $\mathrm{MaxADSet}(A \cup B) = \mathrm{MaxADSet}(A) \cup \mathrm{MaxADSet}(B)$.

(39) For all subsets A, B of Y holds $\mathrm{MaxADSet}(A \cap B) \subseteq \mathrm{MaxADSet}(A) \cap \mathrm{MaxADSet}(B)$.

Let Y be a non empty topological structure and let A be a non empty subset of Y. One can verify that $\mathrm{MaxADSet}(A)$ is non empty.

Let Y be a non empty topological structure and let A be an empty subset of Y. Observe that $\mathrm{MaxADSet}(A)$ is empty.

Let Y be a non empty topological structure and let A be a non proper subset of Y. Note that $\mathrm{MaxADSet}(A)$ is non proper.

Let Y be a non trivial non empty topological structure and let A be a non trivial non empty subset of Y. Observe that $\mathrm{MaxADSet}(A)$ is non trivial.

The following four propositions are true:

(40) For every subset G of Y and for every subset A of Y such that G is open and $A \subseteq G$ holds $\mathrm{MaxADSet}(A) \subseteq G$.

(41) Let A be a subset of Y. Suppose $\{G; G$ ranges over subsets of Y: G is open $\wedge\ A \subseteq G\} \neq \emptyset$. Then $\mathrm{MaxADSet}(A) \subseteq \bigcap\{G; G$ ranges over subsets of Y: G is open $\wedge\ A \subseteq G\}$.

(42) For every subset F of Y and for every subset A of Y such that F is closed and $A \subseteq F$ holds $\mathrm{MaxADSet}(A) \subseteq F$.

(43) Let A be a subset of Y. Suppose $\{F; F$ ranges over subsets of Y: F is closed $\wedge\ A \subseteq F\} \neq \emptyset$. Then $\mathrm{MaxADSet}(A) \subseteq \bigcap\{F; F$ ranges over subsets of Y: F is closed $\wedge\ A \subseteq F\}$.

4. Anti-Discrete and Maximal Anti-discrete Subsets of Topological Spaces

Let X be a non empty topological space and let A be a subset of X. Let us observe that A is anti-discrete if and only if:

(Def. 12) For every point x of X such that $x \in A$ holds $A \subseteq \overline{\{x\}}$.

Let X be a non empty topological space and let A be a subset of X. Let us observe that A is anti-discrete if and only if:

(Def. 13) For every point x of X such that $x \in A$ holds $\overline{A} = \overline{\{x\}}$.

Let X be a non empty topological space and let A be a subset of X. Let us observe that A is anti-discrete if and only if:

(Def. 14) For all points x, y of X such that $x \in A$ and $y \in A$ holds $\overline{\{x\}} = \overline{\{y\}}$.

In the sequel X is a non empty topological space.

Next we state four propositions:

(44) For every point x of X and for every subset D of X such that D is anti-discrete and $\overline{\{x\}} \subseteq D$ holds $D = \overline{\{x\}}$.

(45) Let A be a subset of X. Then A is anti-discrete and closed if and only if for every point x of X such that $x \in A$ holds $A = \overline{\{x\}}$.

(46) For every subset A of X such that A is anti-discrete and A is not open holds A is boundary.

(47) For every point x of X such that $\overline{\{x\}} = \{x\}$ holds $\{x\}$ is maximal anti-discrete.

In the sequel x, y are points of X.

We now state several propositions:

(48) $\mathrm{MaxADSet}(x) \subseteq \bigcap\{G; G$ ranges over subsets of X: G is open $\wedge\ x \in G\}$.

(49) $\mathrm{MaxADSet}(x) \subseteq \bigcap\{F; F$ ranges over subsets of X: F is closed $\wedge\ x \in F\}$.

(50) $\mathrm{MaxADSet}(x) \subseteq \overline{\{x\}}$.

(51) $\mathrm{MaxADSet}(x) = \mathrm{MaxADSet}(y)$ iff $\overline{\{x\}} = \overline{\{y\}}$.

(52) $\mathrm{MaxADSet}(x)$ misses $\mathrm{MaxADSet}(y)$ iff $\overline{\{x\}} \neq \overline{\{y\}}$.

Let X be a non empty topological space and let x be a point of X. Then $\mathrm{MaxADSet}(x)$ is a non empty subset of X and it can be characterized by the condition:

(Def. 15) $\mathrm{MaxADSet}(x) = \overline{\{x\}} \cap \bigcap\{G; G$ ranges over subsets of X: G is open $\wedge\, x \in G\}$.

Next we state four propositions:

(53) Let x, y be points of X. Then $\overline{\{x\}} \subseteq \overline{\{y\}}$ if and only if $\bigcap\{G; G$ ranges over subsets of X: G is open $\wedge\, y \in G\} \subseteq \bigcap\{G; G$ ranges over subsets of X: G is open $\wedge\, x \in G\}$.

(54) For all points x, y of X holds $\overline{\{x\}} \subseteq \overline{\{y\}}$ iff $\mathrm{MaxADSet}(y) \subseteq \bigcap\{G; G$ ranges over subsets of X: G is open $\wedge\, x \in G\}$.

(55) Let x, y be points of X. Then $\mathrm{MaxADSet}(x)$ misses $\mathrm{MaxADSet}(y)$ if and only if one of the following conditions is satisfied:

(i) there exists a subset V of X such that V is open and $\mathrm{MaxADSet}(x) \subseteq V$ and V misses $\mathrm{MaxADSet}(y)$, or

(ii) there exists a subset W of X such that W is open and W misses $\mathrm{MaxADSet}(x)$ and $\mathrm{MaxADSet}(y) \subseteq W$.

(56) Let x, y be points of X. Then $\mathrm{MaxADSet}(x)$ misses $\mathrm{MaxADSet}(y)$ if and only if one of the following conditions is satisfied:

(i) there exists a subset E of X such that E is closed and $\mathrm{MaxADSet}(x) \subseteq E$ and E misses $\mathrm{MaxADSet}(y)$, or

(ii) there exists a subset F of X such that F is closed and F misses $\mathrm{MaxADSet}(x)$ and $\mathrm{MaxADSet}(y) \subseteq F$.

In the sequel A, B are subsets of X and P, Q are subsets of X.
Next we state a number of propositions:

(57) $\mathrm{MaxADSet}(A) \subseteq \bigcap\{G; G$ ranges over subsets of X: G is open $\wedge\, A \subseteq G\}$.

(58) If P is open, then $\mathrm{MaxADSet}(P) = P$.

(59) $\mathrm{MaxADSet}(\mathrm{Int}A) = \mathrm{Int}A$.

(60) $\mathrm{MaxADSet}(A) \subseteq \bigcap\{F; F$ ranges over subsets of X: F is closed $\wedge\, A \subseteq F\}$.

(61) $\mathrm{MaxADSet}(A) \subseteq \overline{A}$.

(62) If P is closed, then $\mathrm{MaxADSet}(P) = P$.

(63) $\mathrm{MaxADSet}(\overline{A}) = \overline{A}$.

(64) $\overline{\mathrm{MaxADSet}(A)} = \overline{A}$.

(65) If $\mathrm{MaxADSet}(A) = \mathrm{MaxADSet}(B)$, then $\overline{A} = \overline{B}$.

(66) If P is closed or Q is closed, then $\mathrm{MaxADSet}(P \cap Q) = \mathrm{MaxADSet}(P) \cap \mathrm{MaxADSet}(Q)$.

(67) If P is open or Q is open, then $\mathrm{MaxADSet}(P \cap Q) = \mathrm{MaxADSet}(P) \cap \mathrm{MaxADSet}(Q)$.

5. Maximal Anti-Discrete Subspaces

In the sequel Y is a non empty topological structure.

The following propositions are true:

(68) Let Y_0 be a subspace of Y and A be a subset of Y. Suppose $A =$ the carrier of Y_0. If Y_0 is anti-discrete, then A is anti-discrete.

(69) Let Y_0 be a subspace of Y. Suppose Y_0 is topological space-like. Let A be a subset of Y. Suppose $A =$ the carrier of Y_0. If A is anti-discrete, then Y_0 is anti-discrete.

In the sequel X denotes a non empty topological space and Y_0 denotes a non empty subspace of X.

We now state four propositions:

(70) If for every open subspace X_0 of X holds Y_0 misses X_0 or Y_0 is a subspace of X_0, then Y_0 is anti-discrete.

(71) If for every closed subspace X_0 of X holds Y_0 misses X_0 or Y_0 is a subspace of X_0, then Y_0 is anti-discrete.

(72) Let Y_0 be an anti-discrete subspace of X and X_0 be an open subspace of X. Then Y_0 misses X_0 or Y_0 is a subspace of X_0.

(73) Let Y_0 be an anti-discrete subspace of X and X_0 be a closed subspace of X. Then Y_0 misses X_0 or Y_0 is a subspace of X_0.

Let Y be a non empty topological structure and let I_1 be a subspace of Y. We say that I_1 is maximal anti-discrete if and only if the conditions (Def. 16) are satisfied.

(Def. 16)(i) I_1 is anti-discrete, and

(ii) for every subspace Y_0 of Y such that Y_0 is anti-discrete holds if the carrier of $I_1 \subseteq$ the carrier of Y_0, then the carrier of $I_1 =$ the carrier of Y_0.

Let Y be a non empty topological structure. Note that every subspace of Y which is maximal anti-discrete is also anti-discrete and every subspace of Y which is non anti-discrete is also non maximal anti-discrete.

Next we state the proposition

(74) Let Y_0 be a non empty subspace of X and A be a subset of X. Suppose $A =$ the carrier of Y_0. Then Y_0 is maximal anti-discrete if and only if A is maximal anti-discrete.

Let X be a non empty topological space. One can check the following observations:

* every non empty subspace of X which is open and anti-discrete is also maximal anti-discrete,
* every non empty subspace of X which is open and non maximal anti-discrete is also non anti-discrete,
* every non empty subspace of X which is anti-discrete and non maximal anti-discrete is also non open,
* every non empty subspace of X which is closed and anti-discrete is also maximal anti-discrete,
* every non empty subspace of X which is closed and non maximal anti-discrete is also non anti-discrete, and
* every non empty subspace of X which is anti-discrete and non maximal anti-discrete is also non closed.

Let Y be a topological structure and let x be a point of Y. The functor $\mathrm{MaxADSspace}(x)$ yielding a strict subspace of Y is defined by:

(Def. 17) The carrier of $\mathrm{MaxADSspace}(x) = \mathrm{MaxADSet}(x)$.

Let Y be a non empty topological structure and let x be a point of Y. One can check that $\mathrm{MaxADSspace}(x)$ is non empty.

Next we state three propositions:

(75) For every point x of Y holds $\mathrm{Sspace}(x)$ is a subspace of $\mathrm{MaxADSspace}(x)$.

(76) Let x, y be points of Y. Then y is a point of $\mathrm{MaxADSspace}(x)$ if and only if the topological structure of $\mathrm{MaxADSspace}(y) =$ the topological structure of $\mathrm{MaxADSspace}(x)$.

(77) Let x, y be points of Y. Then

(i) the carrier of $\mathrm{MaxADSspace}(x)$ misses the carrier of $\mathrm{MaxADSspace}(y)$, or

(ii) the topological structure of $\mathrm{MaxADSspace}(x) =$ the topological structure of $\mathrm{MaxADSspace}(y)$.

Let X be a non empty topological space. Observe that there exists a subspace of X which is maximal anti-discrete and strict.

Let X be a non empty topological space and let x be a point of X. One can check that $\mathrm{MaxADSspace}(x)$ is maximal anti-discrete.

One can prove the following three propositions:

(78) Let X_0 be a closed non empty subspace of X and x be a point of X. If x is a point of X_0, then $\mathrm{MaxADSspace}(x)$ is a subspace of X_0.

(79) Let X_0 be an open non empty subspace of X and x be a point of X. If x is a point of X_0, then $\mathrm{MaxADSspace}(x)$ is a subspace of X_0.

(80) For every point x of X such that $\overline{\{x\}} = \{x\}$ holds $\mathrm{Sspace}(x)$ is maximal anti-discrete.

Let Y be a topological structure and let A be a subset of Y. The functor $\mathrm{Sspace}(A)$ yields a strict subspace of Y and is defined as follows:

(Def. 18) The carrier of $\mathrm{Sspace}(A) = A$.

Let Y be a non empty topological structure and let A be a non empty subset of Y. One can verify that $\mathrm{Sspace}(A)$ is non empty.

The following propositions are true:

(81) Every non empty subset A of Y is a subset of $\mathrm{Sspace}(A)$.

(82) Let Y_0 be a subspace of Y and A be a non empty subset of Y. If A is a subset of Y_0, then $\mathrm{Sspace}(A)$ is a subspace of Y_0.

Let Y be a non trivial non empty topological structure. One can verify that there exists a subspace of Y which is non proper and strict.

Let Y be a non trivial non empty topological structure and let A be a non trivial non empty subset of Y. One can verify that $\mathrm{Sspace}(A)$ is non trivial.

Let Y be a non empty topological structure and let A be a non proper non empty subset of Y. Note that $\mathrm{Sspace}(A)$ is non proper.

Let Y be a non empty topological structure and let A be a subset of Y. The functor $\mathrm{MaxADSspace}(A)$ yielding a strict subspace of Y is defined by:

(Def. 19) The carrier of $\mathrm{MaxADSspace}(A) = \mathrm{MaxADSet}(A)$.

Let Y be a non empty topological structure and let A be a non empty subset of Y. One can verify that $\mathrm{MaxADSspace}(A)$ is non empty.

The following propositions are true:

(83) Every non empty subset A of Y is a subset of MaxADSspace(A).

(84) For every non empty subset A of Y holds Sspace(A) is a subspace of MaxADSspace(A).

(85) For every point x of Y holds the topological structure of MaxADSspace(x) = the topological structure of MaxADSspace$(\{x\})$.

(86) For all non empty subsets A, B of Y such that $A \subseteq B$ holds MaxADSspace(A) is a subspace of MaxADSspace(B).

(87) For every non empty subset A of Y holds the topological structure of MaxADSspace(A) = the topological structure of MaxADSspace(MaxADSet(A)).

(88) For all non empty subsets A, B of Y such that A is a subset of MaxADSspace(B) holds MaxADSspace(A) is a subspace of MaxADSspace(B).

(89) Let A, B be non empty subsets of Y. Then B is a subset of MaxADSspace(A) and A is a subset of MaxADSspace(B) if and only if the topological structure of MaxADSspace(A) = the topological structure of MaxADSspace(B).

Let Y be a non trivial non empty topological structure and let A be a non trivial non empty subset of Y. One can verify that MaxADSspace(A) is non trivial.

Let Y be a non empty topological structure and let A be a non proper non empty subset of Y. Observe that MaxADSspace(A) is non proper.

One can prove the following propositions:

(90) Let X_0 be an open subspace of X and A be a non empty subset of X. If A is a subset of X_0, then MaxADSspace(A) is a subspace of X_0.

(91) Let X_0 be a closed subspace of X and A be a non empty subset of X. If A is a subset of X_0, then MaxADSspace(A) is a subspace of X_0.

REFERENCES

[1] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/zfmisc_1.html`.

[2] Zbigniew Karno. The lattice of domains of an extremally disconnected space. *Journal of Formalized Mathematics*, 4, 1992. `http://mizar.org/JFM/Vol4/tdlat_3.html`.

[3] Zbigniew Karno. Separated and weakly separated subspaces of topological spaces. *Journal of Formalized Mathematics*, 4, 1992. `http://mizar.org/JFM/Vol4/tsep_1.html`.

[4] Zbigniew Karno. Maximal discrete subspaces of almost discrete topological spaces. *Journal of Formalized Mathematics*, 5, 1993. `http://mizar.org/JFM/Vol5/tex_2.html`.

[5] Kazimierz Kuratowski. *Topology*, volume I. PWN - Polish Scientific Publishers, Academic Press, Warsaw, New York and London, 1966.

[6] Beata Padlewska. Families of sets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/setfam_1.html`.

[7] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/pre_topc.html`.

[8] M. H. Stone. Application of Boolean algebras to topology. *Math. Sb.*, 1:765–771, 1936.

[9] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[10] Andrzej Trybulec. A Borsuk theorem on homotopy types. *Journal of Formalized Mathematics*, 3, 1991. `http://mizar.org/JFM/Vol3/borsuk_1.html`.

[11] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

[12] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/tops_1.html`.

Received July 26, 1994

Published January 2, 2004