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The `for` (going up) Macro Instruction

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Summary. We define a `for` type (going up) macro instruction in terms of the `while` macro. This gives an iterative macro with an explicit control variable. The `for` macro is used to define a macro for the selection sort acting on a finite sequence location of $\mathbf{SCM}_{\mathrm{FSA}}$. On the way, a macro for finding a minimum in a section of an array is defined.

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The articles [24], [35], [7], [26], [9], [8], [18], [25], [32], [6], [33], [36], [37], [12], [14], [13], [11], [19], [5], [17], [27], [23], [10], [15], [34], [20], [28], [31], [29], [30], [3], [22], [4], [2], [1], [16], and [21] provide the notation and terminology for this paper.

1. General Preliminaries

One can prove the following two propositions:

(1) Let X be a set, p be a permutation of X, and x, y be elements of X. Then $p +\cdot (x, p(y)) +\cdot (y, p(x))$ is a permutation of X.

(2) Let f be a function and x, y be sets. Suppose $x \in \operatorname{dom} f$ and $y \in \operatorname{dom} f$. Then there exists a permutation p of $\operatorname{dom} f$ such that $f +\cdot (x, f(y)) +\cdot (y, f(x)) = f \cdot p$.

Let A be a finite non empty real-membered set. Then $\inf A$ can be characterized by the condition:

(Def. 1) $\inf A \in A$ and for every real number k such that $k \in A$ holds $\inf A \leq k$.

We introduce $\min A$ as a synonym of $\inf A$.

Let X be a finite non empty natural-membered set. Observe that $\min X$ is integer.

Let F be a finite sequence of elements of $\mathbb{Z}$ and let m, n be natural numbers. Let us assume that $1 \leq m$ and $m \leq n$ and $n \leq \operatorname{len} F$. The functor $\min_m^n F$ yields a natural number and is defined as follows:

(Def. 3)[1] There exists a finite non empty subset X of $\mathbb{Z}$ such that $X = \operatorname{rng}\langle F(m), \ldots, F(n)\rangle$ and $(\min_m^n F) + 1 = (\min X) \leftarrow\!\!\!\!\!\!\;\rightharpoonup \langle F(m), \ldots, F(n)\rangle + m$.

We follow the rules: F, F_1 are finite sequences of elements of $\mathbb{Z}$ and k, m, n, m_1 are natural numbers.

We now state two propositions:

[1] The definition (Def. 2) has been removed.

(3) Suppose $1 \leq m$ and $m \leq n$ and $n \leq \operatorname{len} F$. Then $m_1 = \min_m^n F$ if and only if the following conditions are satisfied:

(i) $m \leq m_1$,

(ii) $m_1 \leq n$,

(iii) for every natural number i such that $m \leq i$ and $i \leq n$ holds $F(m_1) \leq F(i)$, and

(iv) for every natural number i such that $m \leq i$ and $i < m_1$ holds $F(m_1) < F(i)$.

(4) If $1 \leq m$ and $m \leq \operatorname{len} F$, then $\min_m^m F = m$.

Let F be a finite sequence of elements of $\mathbb{Z}$ and let m, n be natural numbers. We say that F is non decreasing on m, n if and only if:

(Def. 4) For all natural numbers i, j such that $m \leq i$ and $i \leq j$ and $j \leq n$ holds $F(i) \leq F(j)$.

Let F be a finite sequence of elements of $\mathbb{Z}$ and let n be a natural number. We say that F is split at n if and only if:

(Def. 5) For all natural numbers i, j such that $1 \leq i$ and $i \leq n$ and $n < j$ and $j \leq \operatorname{len} F$ holds $F(i) \leq F(j)$.

One can prove the following two propositions:

(5) Suppose $k+1 \leq \operatorname{len} F$ and $m_1 = \min_{(k+1)}^{(\operatorname{len} F)} F$ and F is split at k and F is non decreasing on $1, k$ and $F_1 = F +\cdot (k+1, F(m_1)) +\cdot (m_1, F(k+1))$. Then F_1 is non decreasing on $1, k+1$.

(6) If $k+1 \leq \operatorname{len} F$ and $m_1 = \min_{(k+1)}^{(\operatorname{len} F)} F$ and F is split at k and $F_1 = F +\cdot (k+1, F(m_1)) +\cdot (m_1, F(k+1))$, then F_1 is split at $k+1$.

2. $\mathbf{SCM}_{\mathrm{FSA}}$ PRELIMINARIES

For simplicity, we adopt the following rules: s is a state of $\mathbf{SCM}_{\mathrm{FSA}}$, a, c are read-write integer locations, a_1, b_1, c_1, d_1, x are integer locations, f is a finite sequence location, I, J are macro instructions, I_1 is a good macro instruction, and k is a natural number.

The following propositions are true:

(7) If I is closed on $\mathrm{Initialize}(s)$ and halting on $\mathrm{Initialize}(s)$ and I does not destroy a_1, then $(\mathrm{IExec}(I,s))(a_1) = (\mathrm{Initialize}(s))(a_1)$.

(8) If $s(\mathrm{intloc}(0)) = 1$, then $\mathrm{IExec}(\mathrm{Stop}_{\mathbf{SCM}_{\mathrm{FSA}}}, s) \restriction D = s \restriction D$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

(9) $\mathrm{Stop}_{\mathbf{SCM}_{\mathrm{FSA}}}$ does not refer a_1.

(10) If $a_1 \neq b_1$, then $c_1{:=}b_1$ does not refer a_1.

(11) $(\mathrm{Exec}(a{:=}f_{b_1}, s))(a) = s(f)_{|s(b_1)|}$.

(12) $(\mathrm{Exec}(f_{a_1}{:=}b_1, s))(f) = s(f) +\cdot (|s(a_1)|, s(b_1))$.

Let a be a read-write integer location, let b be an integer location, and let I, J be good macro instructions. Observe that **if** $a > b$ **then** I **else** J is good.

The following propositions are true:

(13) $\mathrm{UsedIntLoc}(\textbf{if } a_1 > b_1 \textbf{ then } I \textbf{ else } J) = \{a_1, b_1\} \cup \mathrm{UsedIntLoc}(I) \cup \mathrm{UsedIntLoc}(J)$.

(14) If I does not destroy a_1, then **while** $b_1 > 0$ **do** I does not destroy a_1.

(15) If $c_1 \neq a_1$ and I does not destroy c_1 and J does not destroy c_1, then **if** $a_1 > b_1$ **then** I **else** J does not destroy c_1.

3. THE for-up MACRO INSTRUCTION

Let a, b, c be integer locations, let I be a macro instruction, and let s be a state of $\mathbf{SCM}_{\rm FSA}$. The functor StepForUp(a,b,c,I,s) yields a function from $\mathbb{N}$ into $\prod$(the object kind of $\mathbf{SCM}_{\rm FSA}$) and is defined as follows:

(Def. 6) StepForUp$(a,b,c,I,s) = \mathit{StepWhile}{>}0(a_2, I;$ AddTo$(a,$ intloc$(0))$; SubFrom$(a_2,$ intloc$(0)), s+\cdot (a_2, (s(c)-s(b))+1)+\cdot (a, s(b)))$, where $a_2 = 1^{\rm st}$-RWNotIn$(\{a,b,c\} \cup$ UsedIntLoc$(I))$.

We now state several propositions:

(16) If $s($intloc$(0)) = 1$, then (StepForUp$(a,b_1,c_1,I,s))(0)($intloc$(0)) = 1$.

(17) (StepForUp$(a,b_1,c_1,I,s))(0)(a) = s(b_1)$.

(18) If $a \neq b_1$, then (StepForUp$(a,b_1,c_1,I,s))(0)(b_1) = s(b_1)$.

(19) If $a \neq c_1$, then (StepForUp$(a,b_1,c_1,I,s))(0)(c_1) = s(c_1)$.

(20) If $a \neq d_1$ and $d_1 \in$ UsedIntLoc(I), then (StepForUp$(a,b_1,c_1,I,s))(0)(d_1) = s(d_1)$.

(21) (StepForUp$(a,b_1,c_1,I,s))(0)(f) = s(f)$.

(22) Suppose $s($intloc$(0)) = 1$. Let a_2 be a read-write integer location. If $a_2 = 1^{\rm st}$-RWNotIn$(\{a,b_1,c_1\} \cup$ UsedIntLoc$(I))$, then IExec$((a_2{:=}c_1)$; SubFrom(a_2,b_1); AddTo$(a_2,$ intloc$(0))$; $(a{:=}b_1), s){\restriction}D = (s+\cdot (a_2,(s(c_1)-s(b_1))+1)+\cdot (a,s(b_1))){\restriction}D$, where $D =$ Int-Locations $\cup$ FinSeq-Locations.

Let a, b, c be integer locations, let I be a macro instruction, and let s be a state of $\mathbf{SCM}_{\rm FSA}$. We say that ProperForUpBody a, b, c, I, s if and only if:

(Def. 7) For every natural number i such that $i < (s(c)-s(b))+1$ holds I is closed on (StepForUp$(a,b,c,I,s))(i)$ and halting on (StepForUp$(a,b,c,I,s))(i)$.

We now state several propositions:

(23) For every parahalting macro instruction I holds ProperForUpBody a_1, b_1, c_1, I, s.

(24) If (StepForUp$(a,b_1,c_1,I_1,s))(k)($intloc$(0)) = 1$ and I_1 is closed on (StepForUp$(a,b_1,c_1,I_1,s))(k)$ and halting on (StepForUp$(a,b_1,c_1,I_1,s))(k)$, then (StepForUp$(a,b_1,c_1,I_1,s))(k+1)($intloc$(0)) = 1$.

(25) Suppose $s($intloc$(0)) = 1$ and ProperForUpBody a, b_1, c_1, I_1, s. Let given k. Suppose $k \leq (s(c_1)-s(b_1))+1$. Then

(i) (StepForUp$(a,b_1,c_1,I_1,s))(k)($intloc$(0)) = 1$,

(ii) if I_1 does not destroy a, then (StepForUp$(a,b_1,c_1,I_1,s))(k)(a) = k+s(b_1)$ and (StepForUp$(a,b_1,c_1,I_1,s))(k)(a) \leq s(c_1)+1$, and

(iii) (StepForUp$(a,b_1,c_1,I_1,s))(k)(1^{\rm st}$-RWNotIn$(\{a,b_1,c_1\} \cup$ UsedIntLoc$(I_1))) + k = (s(c_1)-s(b_1))+1$.

(26) Suppose $s($intloc$(0)) = 1$ and ProperForUpBody a, b_1, c_1, I_1, s. Let given k. Then (StepForUp$(a,b_1,c_1,I_1,s))(k)(1^{\rm st}$-RWNotIn$(\{a,b_1,c_1\} \cup$ UsedIntLoc$(I_1))) > 0$ if and only if $k < (s(c_1)-s(b_1))+1$.

(27) Suppose $s($intloc$(0)) = 1$ and ProperForUpBody a, b_1, c_1, I_1, s and $k < (s(c_1)-s(b_1))+1$. Then (StepForUp$(a,b_1,c_1,I_1,s))(k+1){\restriction}(\{a,b_1,c_1\} \cup$ UsedIntLoc$(I_1) \cup F_2) =$ IExec$(I_1;$ AddTo$(a,$ intloc$(0))$, (StepForUp$(a,b_1,c_1,I_1,s))(k)){\restriction}(\{a,b_1,c_1\} \cup$ UsedIntLoc$(I_1) \cup F_2)$, where $F_2 =$ FinSeq-Locations.

Let a, b, c be integer locations and let I be a macro instruction. The functor for-up(a,b,c,I) yields a macro instruction and is defined by:

(Def. 8) for-up(a,b,c,I) $=$ $(a_2{:=}c)$; SubFrom(a_2,b); AddTo$(a_2,\mathrm{intloc}(0))$; $(a{:=}b)$; (**while** $a_2 >$ 0 **do** (I; AddTo$(a,\mathrm{intloc}(0))$; SubFrom$(a_2,\mathrm{intloc}(0))$))), where $a_2 = 1^{\mathrm{st}}$-RWNotIn$(\{a,b,c\} \cup$ UsedIntLoc$(I))$.

The following proposition is true

(28) $\{a_1,b_1,c_1\} \cup \mathrm{UsedIntLoc}(I) \subseteq \mathrm{UsedIntLoc}(\text{for-up}(a_1,b_1,c_1,I))$.

Let a be a read-write integer location, let b, c be integer locations, and let I be a good macro instruction. One can check that for-up(a,b,c,I) is good.

We now state four propositions:

(29) If $a \neq a_1$ and $a_1 \neq 1^{\mathrm{st}}$-RWNotIn$(\{a,b_1,c_1\} \cup \mathrm{UsedIntLoc}(I))$ and I does not destroy a_1, then for-up(a,b_1,c_1,I) does not destroy a_1.

(30) Suppose $s(\mathrm{intloc}(0)) = 1$ and $s(b_1) > s(c_1)$. Then for every x such that $x \neq a$ and $x \in \{b_1,c_1\} \cup \mathrm{UsedIntLoc}(I)$ holds $(\mathrm{IExec}(\text{for-up}(a,b_1,c_1,I),s))(x) = s(x)$ and for every f holds $(\mathrm{IExec}(\text{for-up}(a,b_1,c_1,I),s))(f) = s(f)$.

(31) Suppose $s(\mathrm{intloc}(0)) = 1$ but $k = (s(c_1) - s(b_1)) + 1$ but ProperForUpBody a, b_1, c_1, I_1, s or I_1 is parahalting. Then $\mathrm{IExec}(\text{for-up}(a,b_1,c_1,I_1),s){\restriction}D = (\mathrm{StepForUp}(a,b_1,c_1,I_1,s))(k){\restriction}D$, where $D = \text{Int-Locations} \cup \text{FinSeq-Locations}$.

(32) Suppose $s(\mathrm{intloc}(0)) = 1$ but ProperForUpBody a, b_1, c_1, I_1, s or I_1 is parahalting. Then for-up(a,b_1,c_1,I_1) is closed on s and for-up(a,b_1,c_1,I_1) is halting on s.

4. Finding Minimum in a Section of an Array

Let s_1, f_1, m_2 be integer locations and let f be a finite sequence location. The functor FinSeqMin(f,s_1,f_1,m_2) yielding a macro instruction is defined by:

(Def. 9) FinSeqMin$(f,s_1,f_1,m_2) = (m_2{:=}s_1)$; for-up$(c_2,s_1,f_1,(a_3{:=}f_{c_2})$; $(a_4{:=}f_{m_2})$; (**if** $a_4 > a_3$ **then** Macro$(m_2{:=}c_2)$ **else** (Sto
where $c_2 = 3^{\mathrm{rd}}$-RWNotIn$(\{s_1,f_1,m_2\})$, $a_3 = 1^{\mathrm{st}}$-RWNotIn$(\{s_1,f_1,m_2\})$, and $a_4 = 2^{\mathrm{nd}}$-RWNotIn$(\{s_1,f_1,m_2\})$.

Let s_1, f_1 be integer locations, let m_2 be a read-write integer location, and let f be a finite sequence location. Observe that FinSeqMin(f,s_1,f_1,m_2) is good.

We now state several propositions:

(33) If $c \neq a_1$, then FinSeqMin(f,a_1,b_1,c) does not destroy a_1.

(34) $\{a_1,b_1,c\} \subseteq \mathrm{UsedIntLoc}(\mathrm{FinSeqMin}(f,a_1,b_1,c))$.

(35) If $s(\mathrm{intloc}(0)) = 1$, then FinSeqMin$(f,a_1,b_1,c)$ is closed on s and FinSeqMin(f,a_1,b_1,c) is halting on s.

(36) If $a_1 \neq c$ and $b_1 \neq c$ and $s(\mathrm{intloc}(0)) = 1$, then $(\mathrm{IExec}(\mathrm{FinSeqMin}(f,a_1,b_1,c),s))(f) = s(f)$ and $(\mathrm{IExec}(\mathrm{FinSeqMin}(f,a_1,b_1,c),s))(a_1) = s(a_1)$ and $(\mathrm{IExec}(\mathrm{FinSeqMin}(f,a_1,b_1,c),s))(b_1) = s(b_1)$.

(37) If $1 \leq s(a_1)$ and $s(a_1) \leq s(b_1)$ and $s(b_1) \leq \mathrm{len}\, s(f)$ and $a_1 \neq c$ and $b_1 \neq c$ and $s(\mathrm{intloc}(0)) = 1$, then $(\mathrm{IExec}(\mathrm{FinSeqMin}(f,a_1,b_1,c),s))(c) = \min_{|s(a_1)|}^{|s(b_1)|} s(f)$.

5. A SWAP MACRO INSTRUCTION

Let f be a finite sequence location and let a, b be integer locations. The functor $\mathrm{swap}(f,a,b)$ yields a macro instruction and is defined by:

(Def. 10) $\mathrm{swap}(f,a,b) = (a_3{:=}f_a);\ (a_4{:=}f_b);\ (f_a{:=}a_4);\ (f_b{:=}a_3)$, where $a_3 = 1^{\mathrm{st}}\text{-RWNotIn}(\{a,b\})$ and $a_4 = 2^{\mathrm{nd}}\text{-RWNotIn}(\{a,b\})$.

Let f be a finite sequence location and let a, b be integer locations. Observe that $\mathrm{swap}(f,a,b)$ is good and parahalting.

One can prove the following propositions:

(38) If $c_1 \neq 1^{\mathrm{st}}\text{-RWNotIn}(\{a_1,b_1\})$ and $c_1 \neq 2^{\mathrm{nd}}\text{-RWNotIn}(\{a_1,b_1\})$, then $\mathrm{swap}(f,a_1,b_1)$ does not destroy c_1.

(39) If $1 \leq s(a_1)$ and $s(a_1) \leq \mathrm{len}\, s(f)$ and $1 \leq s(b_1)$ and $s(b_1) \leq \mathrm{len}\, s(f)$ and $s(\mathrm{intloc}(0)) = 1$, then $(\mathrm{IExec}(\mathrm{swap}(f,a_1,b_1),s))(f) = s(f) +\cdot (s(a_1), s(f)(s(b_1))) +\cdot (s(b_1), s(f)(s(a_1)))$.

(40) Suppose $1 \leq s(a_1)$ and $s(a_1) \leq \mathrm{len}\, s(f)$ and $1 \leq s(b_1)$ and $s(b_1) \leq \mathrm{len}\, s(f)$ and $s(\mathrm{intloc}(0)) = 1$. Then $(\mathrm{IExec}(\mathrm{swap}(f,a_1,b_1),s))(f)(s(a_1)) = s(f)(s(b_1))$ and $(\mathrm{IExec}(\mathrm{swap}(f,a_1,b_1),s))(f)(s(b_1)) = s(f)(s(a_1))$.

(41) $\{a_1,b_1\} \subseteq \mathrm{UsedIntLoc}(\mathrm{swap}(f,a_1,b_1))$.

(42) $\mathrm{UsedInt}^*\,\mathrm{Loc}(\mathrm{swap}(f,a_1,b_1)) = \{f\}$.

6. SELECTION SORT

Let f be a finite sequence location. The functor $\text{Selection-sort}\, f$ yielding a macro instruction is defined as follows:

(Def. 11) $\text{Selection-sort}\, f = (f_1{:=}\mathrm{len} f);\ \text{for-up}(c_2, \mathrm{intloc}(0), f_1, \mathrm{FinSeqMin}(f,c_2,f_1,m_2);\ \mathrm{swap}(f,c_2,m_2))$, where $f_1 = 1^{\mathrm{st}}\text{-NotUsed}(\mathrm{swap}(f,c_2,m_2))$, $c_2 = 1^{\mathrm{st}}\text{-RWNotIn}(\emptyset_{\text{Int-Locations}})$, and $m_2 = 2^{\mathrm{nd}}\text{-RWNotIn}(\emptyset_{\text{Int-Locations}})$.

We now state the proposition

(43) Let S be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $S = \mathrm{IExec}(\text{Selection-sort}\, f, s)$. Then $S(f)$ is non decreasing on 1, $\mathrm{len}\, S(f)$ and there exists a permutation p of $\mathrm{dom}\, s(f)$ such that $S(f) = s(f) \cdot p$.

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