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An Extension of SCM

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The articles [16], [8], [24], [2], [1], [20], [17], [13], [19], [25], [6], [7], [18], [15], [21], [4], [10], [11], [22], [3], [14], [9], [23], [12], and [5] provide the notation and terminology for this paper.

In this paper x denotes a set and k denotes a natural number.

The subset $\text{Data-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$ of $\mathbb{Z}$ is defined by:

(Def. 1) $\text{Data-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} = \text{Data-Loc}_{\mathrm{SCM}}$.

The subset $\text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$ of $\mathbb{Z}$ is defined as follows:

(Def. 2) $\text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} = \mathbb{Z} \setminus \mathbb{N}$.

The subset $\text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$ of $\mathbb{Z}$ is defined as follows:

(Def. 3) $\text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} = \text{Instr-Loc}_{\mathrm{SCM}}$.

One can check the following observations:

* $\text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$ is non empty,
* $\text{Data-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$ is non empty, and
* $\text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$ is non empty.

For simplicity, we adopt the following convention: J, K denote elements of $\mathbb{Z}_{13}$, a denotes an element of $\text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$, b, c, c_1 denote elements of $\text{Data-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$, and f, f_1 denote elements of $\text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

The subset $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$ of $[: \mathbb{Z}_{13}, (\bigcup\{\mathbb{Z}, \mathbb{Z}^*\} \cup \mathbb{Z})^* :]$ is defined as follows:

(Def. 4) $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}} = \mathrm{Instr}_{\mathrm{SCM}} \cup \{\langle J, \langle c, f, b\rangle\rangle : J \in \{9, 10\}\} \cup \{\langle K, \langle c_1, f_1\rangle\rangle : K \in \{11, 12\}\}$.

We now state the proposition

(2)[1] $\mathrm{Instr}_{\mathrm{SCM}} \subseteq \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

Let us observe that $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$ is non empty.

Let I be an element of $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$. The functor $\mathrm{InsCode}(I)$ yields a natural number and is defined by:

(Def. 5) $\mathrm{InsCode}(I) = I_{\mathbf{1}}$.

[1] The proposition (1) has been removed.

We now state two propositions:

(3) For every element I of $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$ such that $\mathrm{InsCode}(I) \leq 8$ holds $I \in \mathrm{Instr}_{\mathrm{SCM}}$.

(4) $\langle 0, \emptyset \rangle \in \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

The function $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}$ from $\mathbb{Z}$ into $\{\mathbb{Z}, \mathbb{Z}^*\} \cup \{\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}, \text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}\}$ is defined by:

(Def. 6) $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}} = (\mathbb{Z} \longmapsto \mathbb{Z}^*) {+\cdot} \mathrm{OK}_{\mathrm{SCM}} {+\cdot} (\mathrm{Instr}_{\mathrm{SCM}} \dot{\longmapsto} \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}) \cdot (\mathrm{OK}_{\mathrm{SCM}} \restriction \text{Instr-Loc}_{\mathrm{SCM}})$.

The following propositions are true:

(6)[2] If $x \in \{9, 10\}$, then $\langle x, \langle c, f, b \rangle \rangle \in \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(7) If $x \in \{11, 12\}$, then $\langle x, \langle c, f \rangle \rangle \in \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(8) $\mathbb{Z} = \{0\} \cup \text{Data-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} \cup \text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} \cup \text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(9) $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(0) = \text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(10) $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(b) = \mathbb{Z}$.

(11) $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(a) = \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(12) $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(f) = \mathbb{Z}^*$.

(13) $\text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} \neq \mathbb{Z}$ and $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}} \neq \mathbb{Z}$ and $\text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} \neq \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$ and $\text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} \neq \mathbb{Z}^*$ and $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}} \neq \mathbb{Z}^*$.

(14) For every integer i such that $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(i) = \text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$ holds $i = 0$.

(15) For every integer i such that $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(i) = \mathbb{Z}$ holds $i \in \text{Data-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(16) For every integer i such that $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(i) = \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$ holds $i \in \text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(17) For every integer i such that $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(i) = \mathbb{Z}^*$ holds $i \in \text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

An $\mathbf{SCM}_{\mathrm{FSA}}$-state is an element of $\prod(\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}})$.
The following two propositions are true:

(18) For every $\mathbf{SCM}_{\mathrm{FSA}}$-state s and for every element I of $\mathrm{Instr}_{\mathrm{SCM}}$ holds $s \restriction \mathbb{N} {+\cdot} (\text{Instr-Loc}_{\mathrm{SCM}} \longmapsto I)$ is a **SCM**-state.

(19) For every $\mathbf{SCM}_{\mathrm{FSA}}$-state s and for every **SCM**-state s' holds $s {+\cdot} s' {+\cdot} s \restriction \text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$ is an $\mathbf{SCM}_{\mathrm{FSA}}$-state.

Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state and let u be an element of $\text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$. The functor $\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s, u)$ yields an $\mathbf{SCM}_{\mathrm{FSA}}$-state and is defined by:

(Def. 7) $\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s, u) = s {+\cdot} (0 \dot{\longmapsto} u)$.

Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state, let t be an element of $\text{Data-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$, and let u be an integer. The functor $\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s, t, u)$ yielding an $\mathbf{SCM}_{\mathrm{FSA}}$-state is defined by:

(Def. 8) $\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s, t, u) = s {+\cdot} (t \dot{\longmapsto} u)$.

Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state, let t be an element of $\text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$, and let u be a finite sequence of elements of $\mathbb{Z}$. The functor $\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s, t, u)$ yields an $\mathbf{SCM}_{\mathrm{FSA}}$-state and is defined as follows:

(Def. 9) $\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s, t, u) = s {+\cdot} (t \dot{\longmapsto} u)$.

[2] The proposition (5) has been removed.

Let s be an $\mathbf{SCM}_{\rm FSA}$-state and let a be an element of Data-Loc$_{\rm SCM_{FSA}}$. Then $s(a)$ is an integer.

Let s be an $\mathbf{SCM}_{\rm FSA}$-state and let a be an element of Data*-Loc$_{\rm SCM_{FSA}}$. Then $s(a)$ is a finite sequence of elements of $\mathbb{Z}$.

Let x be an element of Instr$_{\rm SCM_{FSA}}$. Let us assume that there exist c, f, b, J such that $x = \langle J, \langle c, f, b\rangle\rangle$. The functor x int-addr$_1$ yielding an element of Data-Loc$_{\rm SCM_{FSA}}$ is defined as follows:

(Def. 10) There exist c, f, b such that $\langle c, f, b\rangle = x_{\mathbf{2}}$ and x int-addr$_1 = c$.

The functor x int-addr$_2$ yields an element of Data-Loc$_{\rm SCM_{FSA}}$ and is defined by:

(Def. 11) There exist c, f, b such that $\langle c, f, b\rangle = x_{\mathbf{2}}$ and x int-addr$_2 = b$.

The functor x coll-addr$_1$ yields an element of Data*-Loc$_{\rm SCM_{FSA}}$ and is defined as follows:

(Def. 12) There exist c, f, b such that $\langle c, f, b\rangle = x_{\mathbf{2}}$ and x coll-addr$_1 = f$.

Let x be an element of Instr$_{\rm SCM_{FSA}}$. Let us assume that there exist c, f, J such that $x = \langle J, \langle c, f\rangle\rangle$. The functor x int-addr$_3$ yields an element of Data-Loc$_{\rm SCM_{FSA}}$ and is defined by:

(Def. 13) There exist c, f such that $\langle c, f\rangle = x_{\mathbf{2}}$ and x int-addr$_3 = c$.

The functor x coll-addr$_2$ yields an element of Data*-Loc$_{\rm SCM_{FSA}}$ and is defined as follows:

(Def. 14) There exist c, f such that $\langle c, f\rangle = x_{\mathbf{2}}$ and x coll-addr$_2 = f$.

Let l be an element of Instr-Loc$_{\rm SCM_{FSA}}$. The functor Next(l) yielding an element of Instr-Loc$_{\rm SCM_{FSA}}$ is defined as follows:

(Def. 15) There exists an element L of Instr-Loc$_{\rm SCM}$ such that $L = l$ and $\mathrm{Next}(l) = \mathrm{Next}(L)$.

Let s be an $\mathbf{SCM}_{\rm FSA}$-state. The functor $\mathbf{IC}_s$ yields an element of Instr-Loc$_{\rm SCM_{FSA}}$ and is defined by:

(Def. 16) $\mathbf{IC}_s = s(0)$.

Let x be an element of Instr$_{\rm SCM_{FSA}}$ and let s be an $\mathbf{SCM}_{\rm FSA}$-state. The functor Exec-Res$_{\rm SCM_{FSA}}(x, s)$ yields an $\mathbf{SCM}_{\rm FSA}$-state and is defined as follows:

(Def. 17)(i) There exists an element x' of Instr$_{\rm SCM}$ and there exists a $\mathbf{SCM}$-state s' such that $x = x'$ and $s' = s{\restriction}\mathbb{N}{+\cdot}(\text{Instr-Loc}_{\rm SCM} \longmapsto x')$ and $\text{Exec-Res}_{\rm SCM_{FSA}}(x, s) = s{+\cdot}\text{Exec-Res}_{\rm SCM}(x', s'){+\cdot}s{\restriction}\text{Instr-Loc}_{\rm SCM_{FSA}}$ if $\mathrm{InsCode}(x) \leq 8$,

(ii) there exists an integer i and there exists k such that $k = |s(x \text{ int-addr}_2)|$ and $i = s(x \text{ coll-addr}_1)_k$ and $\text{Exec-Res}_{\rm SCM_{FSA}}(x, s) = \mathrm{Chg}_{\rm SCM_{FSA}}(\mathrm{Chg}_{\rm SCM_{FSA}}(s, x \text{ int-addr}_1, i), \mathrm{Next}(\mathbf{IC}_s))$ if $\mathrm{InsCode}(x) = 9$,

(iii) there exists a finite sequence f of elements of $\mathbb{Z}$ and there exists k such that $k = |s(x \text{ int-addr}_2)|$ and $f = s(x \text{ coll-addr}_1) +\cdot (k, s(x \text{ int-addr}_1))$ and $\text{Exec-Res}_{\rm SCM_{FSA}}(x, s) = \mathrm{Chg}_{\rm SCM_{FSA}}(\mathrm{Chg}_{\rm SCM_{FSA}}(s, x \text{ coll-addr}_1, f), \mathrm{Next}(\mathbf{IC}_s))$ if $\mathrm{InsCode}(x) = 10$,

(iv) $\text{Exec-Res}_{\rm SCM_{FSA}}(x, s) = \mathrm{Chg}_{\rm SCM_{FSA}}(\mathrm{Chg}_{\rm SCM_{FSA}}(s, x \text{ int-addr}_3, \mathrm{len}\, s(x \text{ coll-addr}_2)), \mathrm{Next}(\mathbf{IC}_s))$ if $\mathrm{InsCode}(x) = 11$,

(v) there exists a finite sequence f of elements of $\mathbb{Z}$ and there exists k such that $k = |s(x \text{ int-addr}_3)|$ and $f = k \mapsto 0$ and $\text{Exec-Res}_{\rm SCM_{FSA}}(x, s) = \mathrm{Chg}_{\rm SCM_{FSA}}(\mathrm{Chg}_{\rm SCM_{FSA}}(s, x \text{ coll-addr}_2, f), \mathrm{Next}(\mathbf{IC}_s))$ if $\mathrm{InsCode}(x) = 12$,

(vi) $\text{Exec-Res}_{\rm SCM_{FSA}}(x, s) = s$, otherwise.

The function Exec$_{\rm SCM_{FSA}}$ from Instr$_{\rm SCM_{FSA}}$ into $(\prod(\mathrm{OK}_{\rm SCM_{FSA}}))^{\Pi(\mathrm{OK}_{\rm SCM_{FSA}})}$ is defined by:

(Def. 18) For every element x of Instr$_{\rm SCM_{FSA}}$ and for every $\mathbf{SCM}_{\rm FSA}$-state y holds (Exec$_{\rm SCM_{FSA}}(x)$ **qua** element of $(\prod(\mathrm{OK}_{\rm SCM_{FSA}}))^{\Pi(\mathrm{OK}_{\rm SCM_{FSA}})})(y) = \text{Exec-Res}_{\rm SCM_{FSA}}(x, y)$.

The following propositions are true:

(20) For every $\mathbf{SCM}_{\rm FSA}$-state s and for every element u of Instr-Loc$_{\rm SCM_{FSA}}$ holds $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,u))(0) = u$.

(21) For every $\mathbf{SCM}_{\rm FSA}$-state s and for every element u of Instr-Loc$_{\rm SCM_{FSA}}$ and for every element m_1 of Data-Loc$_{\rm SCM_{FSA}}$ holds $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,u))(m_1) = s(m_1)$.

(22) For every $\mathbf{SCM}_{\rm FSA}$-state s and for every element u of Instr-Loc$_{\rm SCM_{FSA}}$ and for every element p of Data*-Loc$_{\rm SCM_{FSA}}$ holds $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,u))(p) = s(p)$.

(23) For every $\mathbf{SCM}_{\rm FSA}$-state s and for all elements u, v of Instr-Loc$_{\rm SCM_{FSA}}$ holds $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,u))(v) = s(v)$.

(24) For every $\mathbf{SCM}_{\rm FSA}$-state s and for every element t of Data-Loc$_{\rm SCM_{FSA}}$ and for every integer u holds $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,t,u))(0) = s(0)$.

(25) For every $\mathbf{SCM}_{\rm FSA}$-state s and for every element t of Data-Loc$_{\rm SCM_{FSA}}$ and for every integer u holds $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,t,u))(t) = u$.

(26) Let s be an $\mathbf{SCM}_{\rm FSA}$-state, t be an element of Data-Loc$_{\rm SCM_{FSA}}$, u be an integer, and m_1 be an element of Data-Loc$_{\rm SCM_{FSA}}$. If $m_1 \neq t$, then $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,t,u))(m_1) = s(m_1)$.

(27) Let s be an $\mathbf{SCM}_{\rm FSA}$-state, t be an element of Data-Loc$_{\rm SCM_{FSA}}$, u be an integer, and f be an element of Data*-Loc$_{\rm SCM_{FSA}}$. Then $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,t,u))(f) = s(f)$.

(28) Let s be an $\mathbf{SCM}_{\rm FSA}$-state, t be an element of Data-Loc$_{\rm SCM_{FSA}}$, u be an integer, and v be an element of Instr-Loc$_{\rm SCM_{FSA}}$. Then $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,t,u))(v) = s(v)$.

(29) Let s be an $\mathbf{SCM}_{\rm FSA}$-state, t be an element of Data*-Loc$_{\rm SCM_{FSA}}$, and u be a finite sequence of elements of $\mathbb{Z}$. Then $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,t,u))(t) = u$.

(30) Let s be an $\mathbf{SCM}_{\rm FSA}$-state, t be an element of Data*-Loc$_{\rm SCM_{FSA}}$, u be a finite sequence of elements of $\mathbb{Z}$, and m_1 be an element of Data*-Loc$_{\rm SCM_{FSA}}$. If $m_1 \neq t$, then $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,t,u))(m_1) = s(m_1)$.

(31) Let s be an $\mathbf{SCM}_{\rm FSA}$-state, t be an element of Data*-Loc$_{\rm SCM_{FSA}}$, u be a finite sequence of elements of $\mathbb{Z}$, and a be an element of Data-Loc$_{\rm SCM_{FSA}}$. Then $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,t,u))(a) = s(a)$.

(32) Let s be an $\mathbf{SCM}_{\rm FSA}$-state, t be an element of Data*-Loc$_{\rm SCM_{FSA}}$, u be a finite sequence of elements of $\mathbb{Z}$, and v be an element of Instr-Loc$_{\rm SCM_{FSA}}$. Then $(\mathrm{Chg}_{\rm SCM_{FSA}}(s,t,u))(v) = s(v)$.

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