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On the Composition of Macro Instructions. Part II[1]

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Summary. We define the semantics of macro instructions (introduced in [19]) in terms of executions of $\mathbf{SCM}_{\mathrm{FSA}}$. In a similar way, we define the semantics of macro composition. Several attributes of macro instructions are introduced (paraclosed, parahalting, keeping 0) and their usage enables a systematic treatment of the composition of macro intructions. This article is continued in [1].

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The articles [14], [15], [3], [21], [22], [7], [8], [4], [2], [9], [10], [11], [16], [5], [13], [6], [20], [17], [18], [19], and [12] provide the notation and terminology for this paper.

1. Preliminaries

One can prove the following proposition

(3)[1] For all functions f, g and for every set A such that $A \cap \mathrm{dom} f \subseteq A \cap \mathrm{dom} g$ holds $(f {+\cdot} g {\restriction} A) {\restriction} A = g {\restriction} A$.

2. Properties of Start-At

For simplicity, we adopt the following convention: m, n denote natural numbers, x denotes a set, i denotes an instruction of $\mathbf{SCM}_{\mathrm{FSA}}$, I denotes a macro instruction, a denotes an integer location, f denotes a finite sequence location, l, l_1 denote instruction-locations of $\mathbf{SCM}_{\mathrm{FSA}}$, and s, s_1, s_2 denote states of $\mathbf{SCM}_{\mathrm{FSA}}$.

One can prove the following propositions:

(4) $\mathrm{Start\text{-}At}(\mathrm{insloc}(0)) \subseteq \mathrm{Initialized}(I)$.

(5) If $I {+\cdot} \mathrm{Start\text{-}At}(\mathrm{insloc}(n)) \subseteq s$, then $I \subseteq s$.

(6) $(I {+\cdot} \mathrm{Start\text{-}At}(\mathrm{insloc}(n))) {\restriction}$the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}} = I$.

(7) If $x \in \mathrm{dom} I$, then $I(x) = (I {+\cdot} \mathrm{Start\text{-}At}(\mathrm{insloc}(n)))(x)$.

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[1] The propositions (1) and (2) have been removed.

(8) If $\mathrm{Initialized}(I) \subseteq s$, then $I {+\cdot}\, \mathrm{Start\text{-}At}(\mathrm{insloc}(0)) \subseteq s$.

(9) $a \notin \mathrm{dom}\,\mathrm{Start\text{-}At}(l)$.

(10) $f \notin \mathrm{dom}\,\mathrm{Start\text{-}At}(l)$.

(11) $l_1 \notin \mathrm{dom}\,\mathrm{Start\text{-}At}(l)$.

(12) $a \notin \mathrm{dom}(I {+\cdot}\, \mathrm{Start\text{-}At}(l))$.

(13) $f \notin \mathrm{dom}(I {+\cdot}\, \mathrm{Start\text{-}At}(l))$.

(14) $s {+\cdot}\, I {+\cdot}\, \mathrm{Start\text{-}At}(\mathrm{insloc}(0)) = s {+\cdot}\, \mathrm{Start\text{-}At}(\mathrm{insloc}(0)) {+\cdot}\, I$.

3. PROPERTIES OF AMI STRUCTURES

In the sequel N is a non empty set with non empty elements.

Next we state two propositions:

(15) If $s = \mathrm{Following}(s)$, then for every n holds $(\mathrm{Computation}(s))(n) = s$.

(16) Let S be a halting IC-Ins-separated definite non empty non void AMI over N and s be a state of S. If s is halting, then $\mathrm{Result}(s) = (\mathrm{Computation}(s))(\mathrm{LifeSpan}(s))$.

Let us consider N, let S be an IC-Ins-separated definite non empty non void AMI over N, let s be a state of S, let l be an instruction-location of S, and let i be an instruction of S. Then $s {+\cdot}\, (l, i)$ is a state of S.

Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, let l_2 be an integer location, and let k be an integer. Then $s {+\cdot}\, (l_2, k)$ is a state of $\mathbf{SCM}_{\mathrm{FSA}}$.

We now state the proposition

(17) Let S be a steady-programmed IC-Ins-separated definite non empty non void AMI over N, s be a state of S, and given n. Then $s \restriction$the instruction locations of $S = (\mathrm{Computation}(s))(n) \restriction$the instruction locations of S.

4. EXECUTION OF MACRO INSTRUCTIONS

Let I be a macro instruction and let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. The functor $\mathrm{IExec}(I, s)$ yields a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and is defined by:

(Def. 1) $\mathrm{IExec}(I, s) = \mathrm{Result}(s {+\cdot}\, \mathrm{Initialized}(I)) {+\cdot}\, s \restriction$the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

Let I be a macro instruction. We say that I is paraclosed if and only if:

(Def. 2) For every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number n such that $I {+\cdot}\, \mathrm{Start\text{-}At}(\mathrm{insloc}(0)) \subseteq s$ holds $\mathbf{IC}_{(\mathrm{Computation}(s))(n)} \in \mathrm{dom}\, I$.

We say that I is parahalting if and only if:

(Def. 3) $I {+\cdot}\, \mathrm{Start\text{-}At}(\mathrm{insloc}(0))$ is halting.

We say that I is keeping 0 if and only if:

(Def. 4) For every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ such that $I {+\cdot}\, \mathrm{Start\text{-}At}(\mathrm{insloc}(0)) \subseteq s$ and for every natural number k holds $(\mathrm{Computation}(s))(k)(\mathrm{intloc}(0)) = s(\mathrm{intloc}(0))$.

Let us note that there exists a macro instruction which is parahalting.

We now state two propositions:

(18) For every parahalting macro instruction I such that $I {+\cdot}\, \mathrm{Start\text{-}At}(\mathrm{insloc}(0)) \subseteq s$ holds s is halting.

(19) For every parahalting macro instruction I such that $\mathrm{Initialized}(I) \subseteq s$ holds s is halting.

Let I be a parahalting macro instruction. One can verify that $\mathrm{Initialized}(I)$ is halting.
One can prove the following two propositions:

(20) $s_2 +\cdot (\mathbf{IC}_{(s_2)}, \mathrm{goto}\ (\mathbf{IC}_{(s_2)}))$ is not halting.

(21) Suppose that

(i) s_1 and s_2 are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$,

(ii) $I \subseteq s_1$,

(iii) $I \subseteq s_2$, and

(iv) for every m such that $m < n$ holds $\mathbf{IC}_{(\mathrm{Computation}(s_2))(m)} \in \mathrm{dom}\, I$.

Let given m. Suppose $m \leq n$. Then $(\mathrm{Computation}(s_1))(m)$ and $(\mathrm{Computation}(s_2))(m)$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

Let us observe that every macro instruction which is parahalting is also paraclosed and every macro instruction which is keeping 0 is also paraclosed.
One can prove the following three propositions:

(22) Let I be a parahalting macro instruction and a be a read-write integer location. If $a \notin \mathrm{UsedIntLoc}(I)$, then $(\mathrm{IExec}(I,s))(a) = s(a)$.

(23) For every parahalting macro instruction I such that $f \notin \mathrm{UsedInt}^{*}\mathrm{Loc}(I)$ holds $(\mathrm{IExec}(I,s))(f) = s(f)$.

(24) If $\mathbf{IC}_s = l$ and $s(l) = \mathrm{goto}\ l$, then s is not halting.

Let us observe that every macro instruction which is parahalting is also non empty.
We now state a number of propositions:

(25) For every parahalting macro instruction I holds $\mathrm{dom}\, I \neq \emptyset$.

(26) For every parahalting macro instruction I holds $\mathrm{insloc}(0) \in \mathrm{dom}\, I$.

(27) Let J be a parahalting macro instruction. Suppose $J +\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)) \subseteq s_1$. Let n be a natural number. Suppose $\mathrm{ProgramPart}(\mathrm{Relocated}(J,n)) \subseteq s_2$ and $\mathbf{IC}_{(s_2)} = \mathrm{insloc}(n)$ and $s_1 \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations}) = s_2 \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations})$. Let i be a natural number. Then $\mathbf{IC}_{(\mathrm{Computation}(s_1))(i)} + n = \mathbf{IC}_{(\mathrm{Computation}(s_2))(i)}$ and $\mathrm{IncAddr}(\mathrm{CurInstr}((\mathrm{Computation}(s_1))(i)), n) = \mathrm{CurInstr}((\mathrm{Computation}(s_2))(i))$ and $(\mathrm{Computation}(s_1))(i) \restriction (\text{Int-Locations} \cup$
$(\mathrm{Computation}(s_2))(i) \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations})$.

(28) Let I be a parahalting macro instruction. Suppose $I +\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)) \subseteq s_1$ and $I +\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)) \subseteq s_2$ and s_1 and s_2 are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$. Let k be a natural number. Then $(\mathrm{Computation}(s_1))(k)$ and $(\mathrm{Computation}(s_2))(k)$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$ and $\mathrm{CurInstr}((\mathrm{Computation}(s_1))(k)) = \mathrm{CurInstr}((\mathrm{Computation}(s_2))(k))$.

(29) Let I be a parahalting macro instruction. Suppose $I +\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)) \subseteq s_1$ and $I +\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)) \subseteq s_2$ and s_1 and s_2 are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$. Then $\mathrm{LifeSpan}(s_1) = \mathrm{LifeSpan}(s_2)$ and $\mathrm{Result}(s_1)$ and $\mathrm{Result}(s_2)$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

(30) For every parahalting macro instruction I holds $\mathbf{IC}_{\mathrm{IExec}(I,s)} = \mathbf{IC}_{\mathrm{Result}(s +\cdot \mathrm{Initialized}(I))}$.

(31) For every non empty macro instruction I holds $\mathrm{insloc}(0) \in \mathrm{dom}\, I$ and $\mathrm{insloc}(0) \in \mathrm{dom}\, \mathrm{Initialized}(I)$ and $\mathrm{insloc}(0) \in \mathrm{dom}(I +\cdot \mathrm{Start\text{-}At}(\mathrm{insloc}(0)))$.

(32) $x \in \mathrm{dom}\, \mathrm{Macro}(i)$ iff $x = \mathrm{insloc}(0)$ or $x = \mathrm{insloc}(1)$.

(33) $(\mathrm{Macro}(i))(\mathrm{insloc}(0)) = i$ and $(\mathrm{Macro}(i))(\mathrm{insloc}(1)) = \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$ and $(\mathrm{Initialized}(\mathrm{Macro}(i)))(\mathrm{insloc}(0)) = i$ and $(\mathrm{Initialized}(\mathrm{Macro}(i)))(\mathrm{insloc}(1)) = \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$ and $(\mathrm{Macro}(i) +\cdot \text{Start-At}(\mathrm{insloc}(0)))(\mathrm{insloc}(0)) = i$.

(34) If $\mathrm{Initialized}(I) \subseteq s$, then $\mathbf{IC}_s = \mathrm{insloc}(0)$.

One can verify that there exists a macro instruction which is keeping 0 and parahalting.
We now state the proposition

(35) For every keeping 0 parahalting macro instruction I holds $(\mathrm{IExec}(I,s))(\mathrm{intloc}(0)) = 1$.

5. THE COMPOSITION OF MACRO INSTRUCTIONS

Next we state several propositions:

(36) Let I be a paraclosed macro instruction and J be a macro instruction. Suppose $I +\cdot \text{Start-At}(\mathrm{insloc}(0)) \subseteq s$ and s is halting. Let given m. Suppose $m \leq \mathrm{LifeSpan}(s)$. Then $(\mathrm{Computation}(s))(m)$ and $(\mathrm{Computation}(s +\cdot (I;J)))(m)$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

(37) For every paraclosed macro instruction I such that $s +\cdot I$ is halting and $\mathrm{Directed}(I) \subseteq s$ and $\text{Start-At}(\mathrm{insloc}(0)) \subseteq s$ holds $\mathbf{IC}_{(\mathrm{Computation}(s))(\mathrm{LifeSpan}(s+\cdot I)+1)} = \mathrm{insloc}(\mathrm{card}\, I)$.

(38) Let I be a paraclosed macro instruction. If $s +\cdot I$ is halting and $\mathrm{Directed}(I) \subseteq s$ and $\text{Start-At}(\mathrm{insloc}(0)) \subseteq s$, then $(\mathrm{Computation}(s))(\mathrm{LifeSpan}(s +\cdot I)) \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations}) = (\mathrm{Computation}(s))(\mathrm{LifeSpan}(s +\cdot I) + 1) \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations})$.

(39) Let I be a parahalting macro instruction. Suppose $\mathrm{Initialized}(I) \subseteq s$. Let k be a natural number. If $k \leq \mathrm{LifeSpan}(s)$, then $\mathrm{CurInstr}((\mathrm{Computation}(s +\cdot \mathrm{Directed}(I)))(k)) \neq \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$.

(40) Let I be a paraclosed macro instruction. Suppose $s +\cdot (I +\cdot \text{Start-At}(\mathrm{insloc}(0)))$ is halting. Let J be a macro instruction and k be a natural number. Suppose $k \leq \mathrm{LifeSpan}(s +\cdot (I +\cdot \text{Start-At}(\mathrm{insloc}(0))))$. Then $(\mathrm{Computation}(s +\cdot (I +\cdot \text{Start-At}(\mathrm{insloc}(0)))))(k)$ and $(\mathrm{Computation}(s +\cdot ((I;J) +\cdot \text{Start-At}(\mathrm{insloc}(0)))))(k)$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

Let I, J be parahalting macro instructions. Observe that $I;J$ is parahalting.
The following two propositions are true:

(41) Let I be a keeping 0 macro instruction. Suppose $s +\cdot (I +\cdot \text{Start-At}(\mathrm{insloc}(0)))$ is not halting. Let J be a macro instruction and k be a natural number. Then $(\mathrm{Computation}(s +\cdot (I +\cdot \text{Start-At}(\mathrm{insloc}(0)))))(k)$ and $(\mathrm{Computation}(s +\cdot ((I;J) +\cdot \text{Start-At}(\mathrm{insloc}(0)))))(k)$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

(42) Let I be a keeping 0 macro instruction. Suppose $s +\cdot I$ is halting. Let J be a paraclosed macro instruction. Suppose $(I;J) +\cdot \text{Start-At}(\mathrm{insloc}(0)) \subseteq s$. Let k be a natural number. Then $(\mathrm{Computation}(\mathrm{Result}(s +\cdot I) +\cdot (J +\cdot \text{Start-At}(\mathrm{insloc}(0)))))(k) +\cdot \text{Start-At}(\mathbf{IC}_{(\mathrm{Computation}(\mathrm{Result}(s+\cdot I)+\cdot(J+\cdot \text{Start-At}(\mathrm{insloc}(0)))))(k)} + \mathrm{card}\, I)$ and $(\mathrm{Computation}(s +\cdot (I;J)))(\mathrm{LifeSpan}(s +\cdot I) + 1 + k)$ are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

Let I, J be keeping 0 macro instructions. Note that $I;J$ is keeping 0.
The following two propositions are true:

(43) Let I be a keeping 0 parahalting macro instruction and J be a parahalting macro instruction. Then $\mathrm{LifeSpan}(s +\cdot \mathrm{Initialized}(I;J)) = \mathrm{LifeSpan}(s +\cdot \mathrm{Initialized}(I)) + 1 + \mathrm{LifeSpan}(\mathrm{Result}(s +\cdot \mathrm{Initialized}(I)) +\cdot \mathrm{Initialized}(J))$.

(44) Let I be a keeping 0 parahalting macro instruction and J be a parahalting macro instruction. Then $\mathrm{IExec}(I;J,s) = \mathrm{IExec}(J, \mathrm{IExec}(I,s)) +\cdot \text{Start-At}(\mathbf{IC}_{\mathrm{IExec}(J,\mathrm{IExec}(I,s))} + \mathrm{card}\, I)$.

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