JOURNAL OF FORMALIZED MATHEMATICS
Volume 2, Released 1990, Published 2003
Inst. of Computer Science, Univ. of Białystok

Schemes of Existence of Some Types of Functions

Jarosław Kotowicz
Warsaw University
Białystok

Summary. We prove some useful schemes of existence of real sequences, partial functions from a domain into a domain, partial functions from a set to a set and functions from a domain into a domain. At the beginning we prove some related auxiliary theorems related to the article [1].

MML Identifier: SCHEME1.

WWW: http://mizar.org/JFM/Vol2/scheme1.html

The articles [8], [5], [10], [9], [1], [11], [2], [12], [4], [3], [7], and [6] provide the notation and terminology for this paper.

We use the following convention: x, y are sets, n, m are natural numbers, and r is a real number.

Next we state four propositions:

(1) For every n there exists m such that $n = 2 \cdot m$ or $n = 2 \cdot m + 1$.

(2) For every n there exists m such that $n = 3 \cdot m$ or $n = 3 \cdot m + 1$ or $n = 3 \cdot m + 2$.

(3) For every n there exists m such that $n = 4 \cdot m$ or $n = 4 \cdot m + 1$ or $n = 4 \cdot m + 2$ or $n = 4 \cdot m + 3$.

(4) For every n there exists m such that $n = 5 \cdot m$ or $n = 5 \cdot m + 1$ or $n = 5 \cdot m + 2$ or $n = 5 \cdot m + 3$ or $n = 5 \cdot m + 4$.

In this article we present several logical schemes. The scheme *ExRealSubseq* deals with a sequence $\mathcal{A}$ of real numbers and a unary predicate $\mathcal{P}$, and states that:

There exists a sequence q of real numbers such that
(i) q is a subsequence of $\mathcal{A}$,
(ii) for every n holds $\mathcal{P}[q(n)]$, and
(iii) for every n such that for every r such that $r = \mathcal{A}(n)$ holds $\mathcal{P}[r]$ there exists m such that $\mathcal{A}(n) = q(m)$

provided the following condition is satisfied:

- For every n there exists m such that $n \leq m$ and $\mathcal{P}[\mathcal{A}(m)]$.

The scheme *ExRealSeq2* deals with two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding real numbers, and states that:

There exists a sequence s of real numbers such that for every n holds $s(2 \cdot n) = \mathcal{F}(n)$ and $s(2 \cdot n + 1) = \mathcal{G}(n)$

for all values of the parameter.

The scheme *ExRealSeq3* deals with three unary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding real numbers, and states that:

There exists a sequence s of real numbers such that for every n holds $s(3 \cdot n) = \mathcal{F}(n)$ and $s(3 \cdot n + 1) = \mathcal{G}(n)$ and $s(3 \cdot n + 2) = \mathcal{H}(n)$

for all values of the parameter.

The scheme *ExRealSeq4* deals with four unary functors $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and I yielding real numbers, and states that:

There exists a sequence s of real numbers such that for every n holds $s(4 \cdot n) = \mathcal{F}(n)$ and $s(4 \cdot n + 1) = \mathcal{G}(n)$ and $s(4 \cdot n + 2) = \mathcal{H}(n)$ and $s(4 \cdot n + 3) = I(n)$

for all values of the parameter.

The scheme *ExRealSeq5* deals with five unary functors $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, I, and $\mathcal{J}$ yielding real numbers, and states that:

There exists a sequence s of real numbers such that for every n holds $s(5 \cdot n) = \mathcal{F}(n)$ and $s(5 \cdot n + 1) = \mathcal{G}(n)$ and $s(5 \cdot n + 2) = \mathcal{H}(n)$ and $s(5 \cdot n + 3) = I(n)$ and $s(5 \cdot n + 4) = \mathcal{J}(n)$

for all values of the parameter.

The scheme *PartFuncExD2* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding elements of $\mathcal{B}$, and two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that

(i) for every element c of $\mathcal{A}$ holds $c \in \operatorname{dom} f$ iff $\mathcal{P}[c]$ or $\mathcal{Q}[c]$, and

(ii) for every element c of $\mathcal{A}$ such that $c \in \operatorname{dom} f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$

provided the parameters meet the following condition:

- For every element c of $\mathcal{A}$ such that $\mathcal{P}[c]$ holds not $\mathcal{Q}[c]$.

The scheme *PartFuncExD2'* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding elements of $\mathcal{B}$, and two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that

(i) for every element c of $\mathcal{A}$ holds $c \in \operatorname{dom} f$ iff $\mathcal{P}[c]$ or $\mathcal{Q}[c]$, and

(ii) for every element c of $\mathcal{A}$ such that $c \in \operatorname{dom} f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$

provided the following condition is satisfied:

- For every element c of $\mathcal{A}$ such that $\mathcal{P}[c]$ and $\mathcal{Q}[c]$ holds $\mathcal{F}(c) = \mathcal{G}(c)$.

The scheme *PartFuncExD2"* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding elements of $\mathcal{B}$, and a unary predicate $\mathcal{P}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that f is total and for every element c of $\mathcal{A}$ such that $c \in \operatorname{dom} f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if not $\mathcal{P}[c]$, then $f(c) = \mathcal{G}(c)$

for all values of the parameters.

The scheme *PartFuncExD3* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, three unary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding elements of $\mathcal{B}$, and three unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that

(i) for every element c of $\mathcal{A}$ holds $c \in \operatorname{dom} f$ iff $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$, and

(ii) for every element c of $\mathcal{A}$ such that $c \in \operatorname{dom} f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$ and if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$

provided the following condition is satisfied:

- For every element c of $\mathcal{A}$ holds if $\mathcal{P}[c]$, then not $\mathcal{Q}[c]$ and if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$ and if $\mathcal{Q}[c]$, then not $\mathcal{R}[c]$.

The scheme *PartFuncExD3'* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, three unary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding elements of $\mathcal{B}$, and three unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that

(i) for every element c of $\mathcal{A}$ holds $c \in \operatorname{dom} f$ iff $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$, and

(ii) for every element c of $\mathcal{A}$ such that $c \in \operatorname{dom} f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$ and if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$

provided the following condition is met:

- Let c be an element of $\mathcal{A}$. Then
  (i) if $\mathcal{P}[c]$ and $\mathcal{Q}[c]$, then $\mathcal{F}(c) = \mathcal{G}(c)$,
  (ii) if $\mathcal{P}[c]$ and $\mathcal{R}[c]$, then $\mathcal{F}(c) = \mathcal{H}(c)$, and
  (iii) if $\mathcal{Q}[c]$ and $\mathcal{R}[c]$, then $\mathcal{G}(c) = \mathcal{H}(c)$.

The scheme *PartFuncExD4* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, four unary functors $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and I yielding elements of $\mathcal{B}$, and four unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, $\mathcal{S}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that

(i) for every element c of $\mathcal{A}$ holds $c \in \operatorname{dom} f$ iff $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$ or $\mathcal{S}[c]$, and
(ii) for every element c of $\mathcal{A}$ such that $c \in \operatorname{dom} f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ and if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$ and if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$ and if $\mathcal{S}[c]$, then $f(c) = I(c)$

provided the following requirement is met:

- Let c be an element of $\mathcal{A}$. Then
  (i) if $\mathcal{P}[c]$, then not $\mathcal{Q}[c]$,
  (ii) if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$,
  (iii) if $\mathcal{P}[c]$, then not $\mathcal{S}[c]$,
  (iv) if $\mathcal{Q}[c]$, then not $\mathcal{R}[c]$,
  (v) if $\mathcal{Q}[c]$, then not $\mathcal{S}[c]$, and
  (vi) if $\mathcal{R}[c]$, then not $\mathcal{S}[c]$.

The scheme *PartFuncExS2* deals with sets $\mathcal{A}$, $\mathcal{B}$, two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding sets, and two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that
(i) for every x holds $x \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ but $\mathcal{P}[x]$ or $\mathcal{Q}[x]$, and
(ii) for every x such that $x \in \operatorname{dom} f$ holds if $\mathcal{P}[x]$, then $f(x) = \mathcal{F}(x)$ and if $\mathcal{Q}[x]$, then $f(x) = \mathcal{G}(x)$

provided the parameters meet the following requirements:

- For every x such that $x \in \mathcal{A}$ holds if $\mathcal{P}[x]$, then not $\mathcal{Q}[x]$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{P}[x]$ holds $\mathcal{F}(x) \in \mathcal{B}$, and
- For every x such that $x \in \mathcal{A}$ and $\mathcal{Q}[x]$ holds $\mathcal{G}(x) \in \mathcal{B}$.

The scheme *PartFuncExS3* deals with sets $\mathcal{A}$, $\mathcal{B}$, three unary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding sets, and three unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that
(i) for every x holds $x \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ but $\mathcal{P}[x]$ or $\mathcal{Q}[x]$ or $\mathcal{R}[x]$, and
(ii) for every x such that $x \in \operatorname{dom} f$ holds if $\mathcal{P}[x]$, then $f(x) = \mathcal{F}(x)$ and if $\mathcal{Q}[x]$, then $f(x) = \mathcal{G}(x)$ and if $\mathcal{R}[x]$, then $f(x) = \mathcal{H}(x)$

provided the parameters meet the following requirements:

- For every x such that $x \in \mathcal{A}$ holds if $\mathcal{P}[x]$, then not $\mathcal{Q}[x]$ and if $\mathcal{P}[x]$, then not $\mathcal{R}[x]$ and if $\mathcal{Q}[x]$, then not $\mathcal{R}[x]$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{P}[x]$ holds $\mathcal{F}(x) \in \mathcal{B}$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{Q}[x]$ holds $\mathcal{G}(x) \in \mathcal{B}$, and
- For every x such that $x \in \mathcal{A}$ and $\mathcal{R}[x]$ holds $\mathcal{H}(x) \in \mathcal{B}$.

The scheme *PartFuncExS4* deals with sets $\mathcal{A}$, $\mathcal{B}$, four unary functors $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and I yielding sets, and four unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, $\mathcal{S}$, and states that:

There exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that
(i) for every x holds $x \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ but $\mathcal{P}[x]$ or $\mathcal{Q}[x]$ or $\mathcal{R}[x]$ or $\mathcal{S}[x]$, and
(ii) for every x such that $x \in \operatorname{dom} f$ holds if $\mathcal{P}[x]$, then $f(x) = \mathcal{F}(x)$ and if $\mathcal{Q}[x]$, then $f(x) = \mathcal{G}(x)$ and if $\mathcal{R}[x]$, then $f(x) = \mathcal{H}(x)$ and if $\mathcal{S}[x]$, then $f(x) = I(x)$

provided the parameters satisfy the following conditions:

- Let given x such that $x \in \mathcal{A}$. Then
  (i) if $\mathcal{P}[x]$, then not $\mathcal{Q}[x]$,
  (ii) if $\mathcal{P}[x]$, then not $\mathcal{R}[x]$,
  (iii) if $\mathcal{P}[x]$, then not $\mathcal{S}[x]$,
  (iv) if $\mathcal{Q}[x]$, then not $\mathcal{R}[x]$,
  (v) if $\mathcal{Q}[x]$, then not $\mathcal{S}[x]$, and
  (vi) if $\mathcal{R}[x]$, then not $\mathcal{S}[x]$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{P}[x]$ holds $\mathcal{F}(x) \in \mathcal{B}$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{Q}[x]$ holds $\mathcal{G}(x) \in \mathcal{B}$,
- For every x such that $x \in \mathcal{A}$ and $\mathcal{R}[x]$ holds $\mathcal{H}(x) \in \mathcal{B}$, and
- For every x such that $x \in \mathcal{A}$ and $\mathcal{S}[x]$ holds $I(x) \in \mathcal{B}$.

The scheme *PartFuncExC D2* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, two binary functors $\mathcal{F}$ and $\mathcal{G}$ yielding elements of $\mathcal{C}$, and two binary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that

(i) for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\langle c, d\rangle \in \operatorname{dom} f$ iff $\mathcal{P}[c,d]$ or $\mathcal{Q}[c,d]$, and

(ii) for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ such that $\langle c, d\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{F}(c,d)$ and if $\mathcal{Q}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{G}(c,d)$

provided the following requirement is met:

- For every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ such that $\mathcal{P}[c,d]$ holds not $\mathcal{Q}[c,d]$.

The scheme *PartFuncExC D3* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, three binary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding elements of $\mathcal{C}$, and three binary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and states that:

There exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that

(i) for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\langle c, d\rangle \in \operatorname{dom} f$ iff $\mathcal{P}[c,d]$ or $\mathcal{Q}[c,d]$ or $\mathcal{R}[c,d]$, and

(ii) for every element c of $\mathcal{A}$ and for every element r of $\mathcal{B}$ such that $\langle c, r\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[c,r]$, then $f(\langle c, r\rangle) = \mathcal{F}(c,r)$ and if $\mathcal{Q}[c,r]$, then $f(\langle c, r\rangle) = \mathcal{G}(c,r)$ and if $\mathcal{R}[c,r]$, then $f(\langle c, r\rangle) = \mathcal{H}(c,r)$

provided the following requirement is met:

- Let c be an element of $\mathcal{A}$ and s be an element of $\mathcal{B}$. Then
  (i) if $\mathcal{P}[c,s]$, then not $\mathcal{Q}[c,s]$,
  (ii) if $\mathcal{P}[c,s]$, then not $\mathcal{R}[c,s]$, and
  (iii) if $\mathcal{Q}[c,s]$, then not $\mathcal{R}[c,s]$.

The scheme *PartFuncExC S2* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, two binary functors $\mathcal{F}$ and $\mathcal{G}$ yielding sets, and two binary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that

(i) for all x, y holds $\langle x, y\rangle \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ but $y \in \mathcal{B}$ but $\mathcal{P}[x,y]$ or $\mathcal{Q}[x,y]$, and

(ii) for all x, y such that $\langle x, y\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[x,y]$, then $f(\langle x, y\rangle) = \mathcal{F}(x,y)$ and if $\mathcal{Q}[x,y]$, then $f(\langle x, y\rangle) = \mathcal{G}(x,y)$

provided the following conditions are satisfied:

- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{P}[x,y]$, then not $\mathcal{Q}[x,y]$,
- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $\mathcal{P}[x,y]$ holds $\mathcal{F}(x,y) \in \mathcal{C}$, and
- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $\mathcal{Q}[x,y]$ holds $\mathcal{G}(x,y) \in \mathcal{C}$.

The scheme *PartFuncExC S3* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, three binary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding sets, and three binary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and states that:

There exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that

(i) for all x, y holds $\langle x, y\rangle \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ but $y \in \mathcal{B}$ but $\mathcal{P}[x,y]$ or $\mathcal{Q}[x,y]$ or $\mathcal{R}[x,y]$, and

(ii) for all x, y such that $\langle x, y\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[x,y]$, then $f(\langle x, y\rangle) = \mathcal{F}(x,y)$ and if $\mathcal{Q}[x,y]$, then $f(\langle x, y\rangle) = \mathcal{G}(x,y)$ and if $\mathcal{R}[x,y]$, then $f(\langle x, y\rangle) = \mathcal{H}(x,y)$

provided the parameters meet the following requirements:

- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{P}[x,y]$, then not $\mathcal{Q}[x,y]$ and if $\mathcal{P}[x,y]$, then not $\mathcal{R}[x,y]$ and if $\mathcal{Q}[x,y]$, then not $\mathcal{R}[x,y]$,
- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{P}[x,y]$, then $\mathcal{F}(x,y) \in \mathcal{C}$,
- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{Q}[x,y]$, then $\mathcal{G}(x,y) \in \mathcal{C}$, and
- For all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{R}[x,y]$, then $\mathcal{H}(x,y) \in \mathcal{C}$.

The scheme *ExFuncD3* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, three unary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding elements of $\mathcal{B}$, and three unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and states that:

There exists a function f from $\mathcal{A}$ into $\mathcal{B}$ such that for every element c of $\mathcal{A}$ holds

(i) if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$,
(ii) if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$, and
(iii) if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$

provided the parameters satisfy the following conditions:

- For every element c of $\mathcal{A}$ holds if $\mathcal{P}[c]$, then not $\mathcal{Q}[c]$ and if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$ and if $\mathcal{Q}[c]$, then not $\mathcal{R}[c]$, and
- For every element c of $\mathcal{A}$ holds $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$.

The scheme *ExFuncD4* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, four unary functors $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, and $\mathcal{I}$ yielding elements of $\mathcal{B}$, and four unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, $\mathcal{S}$, and states that:

There exists a function f from $\mathcal{A}$ into $\mathcal{B}$ such that for every element c of $\mathcal{A}$ holds

(i) if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$,
(ii) if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$,
(iii) if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$, and
(iv) if $\mathcal{S}[c]$, then $f(c) = \mathcal{I}(c)$

provided the parameters satisfy the following conditions:

- Let c be an element of $\mathcal{A}$. Then
  (i) if $\mathcal{P}[c]$, then not $\mathcal{Q}[c]$,
  (ii) if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$,
  (iii) if $\mathcal{P}[c]$, then not $\mathcal{S}[c]$,
  (iv) if $\mathcal{Q}[c]$, then not $\mathcal{R}[c]$,
  (v) if $\mathcal{Q}[c]$, then not $\mathcal{S}[c]$, and
  (vi) if $\mathcal{R}[c]$, then not $\mathcal{S}[c]$,
  and
- For every element c of $\mathcal{A}$ holds $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$ or $\mathcal{S}[c]$.

The scheme *FuncExC D2* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, two binary functors $\mathcal{F}$ and $\mathcal{G}$ yielding elements of $\mathcal{C}$, and a binary predicate $\mathcal{P}$, and states that:

There exists a function f from $[:\mathcal{A}, \mathcal{B}:]$ into $\mathcal{C}$ such that for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds
(i) if $\mathcal{P}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{F}(c,d)$, and
(ii) if not $\mathcal{P}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{G}(c,d)$

for all values of the parameters.

The scheme *FuncExC D3* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, three binary functors $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ yielding elements of $\mathcal{C}$, and three binary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and states that:

There exists a function f from $[:\mathcal{A}, \mathcal{B}:]$ into $\mathcal{C}$ such that
(i) for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\langle c, d\rangle \in \operatorname{dom} f$ iff $\mathcal{P}[c,d]$ or $\mathcal{Q}[c,d]$ or $\mathcal{R}[c,d]$, and
(ii) for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ such that $\langle c, d\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{F}(c,d)$ and if $\mathcal{Q}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{G}(c,d)$ and if $\mathcal{R}[c,d]$, then $f(\langle c, d\rangle) = \mathcal{H}(c,d)$

provided the parameters meet the following requirements:

- Let c be an element of $\mathcal{A}$ and d be an element of $\mathcal{B}$. Then
  (i) if $\mathcal{P}[c,d]$, then not $\mathcal{Q}[c,d]$,
  (ii) if $\mathcal{P}[c,d]$, then not $\mathcal{R}[c,d]$, and
  (iii) if $\mathcal{Q}[c,d]$, then not $\mathcal{R}[c,d]$,
  and
- For every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\mathcal{P}[c,d]$ or $\mathcal{Q}[c,d]$ or $\mathcal{R}[c,d]$.

REFERENCES

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/nat_1.html.

[2] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[3] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[4] Czesław Byliński. Partial functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/partfun1.html.

[5] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[6] Jarosław Kotowicz. Monotone real sequences. Subsequences. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/seqm_3.html.

[7] Jarosław Kotowicz. Real sequences and basic operations on them. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/seq_1.html.

[8] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[9] Andrzej Trybulec. Subsets of real numbers. *Journal of Formalized Mathematics*, Addenda, 2003. `http://mizar.org/JFM/Addenda/numbers.html`.

[10] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

[11] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

[12] Edmund Woronowicz. Relations defined on sets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relset_1.html`.

Received September 21, 1990

Published January 2, 2004