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Improvement of Radix-2^k Signed-Digit Number for High Speed Circuit

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Summary. In this article, a new radix-2^k signed-digit number (Radix-2^k sub signed-digit number) is defined and its properties for hardware realization are discussed.

Until now, high speed calculation method with Radix-2^k signed-digit numbers is proposed, but this method used "Compares With 2" to calculate carry. "Compares with 2" is a very simple method, but it needs very complicated hardware especially when the value of k becomes large. In this article, we propose a subset of Radix-2^k signed-digit, named Radix-2^k sub signed-digit numbers. Radix-2^k sub signed-digit was designed so that the carry calculation use "bit compare" to hardware-realization simplifies more.

In the first section of this article, we defined the concept of Radix-2^k sub signed-digit numbers and proved some of their properties. In the second section, we defined the new carry calculation method in consideration of hardware-realization, and proved some of their properties. In the third section, we provide some functions for generating Radix-2^k sub signed-digit numbers from Radix-2^k signed-digit numbers. In the last section, we defined some functions for generation natural numbers from Radix-2^k sub signed-digit, and we clarified its correctness.

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WWW: `http://mizar.org/JFM/Vol15/radix_3.html`

The articles [8], [11], [9], [1], [4], [3], [10], [7], [2], [6], and [5] provide the notation and terminology for this paper.

1. Definition for Radix-2^k Sub Signed-Digit Number

We adopt the following convention: i, n, m, k, x are natural numbers and i_1, i_2 are integers.

We now state the proposition

(1) $(\mathrm{Radix}\,k)^n \cdot \mathrm{Radix}\,k = (\mathrm{Radix}\,k)^{n+1}$.

Let us consider k. The functor k −SD_Sub_S is defined as follows:

(Def. 1) k −SD_Sub_S $= \{e; e$ ranges over elements of $\mathbb{Z}$: $e \geq -\mathrm{Radix}(k -' 1) \wedge e \leq \mathrm{Radix}(k -' 1) - 1\}$.

Let us consider k. The functor k −SD_Sub is defined as follows:

(Def. 2) k −SD_Sub $= \{e; e$ ranges over elements of $\mathbb{Z}$: $e \geq -\mathrm{Radix}(k -' 1) - 1 \wedge e \leq \mathrm{Radix}(k -' 1)\}$.

We now state a number of propositions:

(2) If $i_1 \in k-\text{SD\_Sub}$, then $-\text{Radix}(k-'1)-1 \leq i_1$ and $i_1 \leq \text{Radix}(k-'1)$.

(3) For every natural number k holds $k-\text{SD\_Sub\_S} \subseteq k-\text{SD\_Sub}$.

(4) $k-\text{SD\_Sub\_S} \subseteq (k+1)-\text{SD\_Sub\_S}$.

(5) For every natural number k such that $2 \leq k$ holds $k-\text{SD\_Sub} \subseteq k-\text{SD}$.

(6) $0 \in 0-\text{SD\_Sub\_S}$.

(7) $0 \in k-\text{SD\_Sub\_S}$.

(8) $0 \in k-\text{SD\_Sub}$.

(9) For every set e such that $e \in k-\text{SD\_Sub}$ holds e is an integer.

(10) $k-\text{SD\_Sub} \subseteq \mathbb{Z}$.

(11) $k-\text{SD\_Sub\_S} \subseteq \mathbb{Z}$.

Let us consider k. Note that $k-\text{SD\_Sub\_S}$ is non empty and $k-\text{SD\_Sub}$ is non empty.
Let us consider k. Then $k-\text{SD\_Sub\_S}$ is a non empty subset of $\mathbb{Z}$.
Let us consider k. Then $k-\text{SD\_Sub}$ is a non empty subset of $\mathbb{Z}$.
In the sequel a is a n-tuple of $k-\text{SD}$ and a_1 is a n-tuple of $k-\text{SD\_Sub}$.
One can prove the following proposition

(12) If $i \in \text{Seg}\, n$, then $a_1(i)$ is an element of $k-\text{SD\_Sub}$.

2. Definition for New Carry Calculation Method

Let x be an integer and let k be a natural number. The functor $\text{SDSubAddCarry}(x,k)$ yields an integer and is defined by:

(Def. 3) $$\text{SDSubAddCarry}(x,k) = \begin{cases} 1, \text{ if } \text{Radix}(k-'1) \leq x, \\ -1, \text{ if } x < -\text{Radix}(k-'1), \\ 0, \text{ otherwise.} \end{cases}$$

Let x be an integer and let k be a natural number. The functor $\text{SDSubAddData}(x,k)$ yields an integer and is defined by:

(Def. 4) $\text{SDSubAddData}(x,k) = x - \text{Radix}\, k \cdot \text{SDSubAddCarry}(x,k)$.

Next we state several propositions:

(13) For every integer x and for every natural number k such that $2 \leq k$ holds $-1 \leq \text{SDSubAddCarry}(x,k)$ and $\text{SDSubAddCarry}(x,k) \leq 1$.

(14) If $2 \leq k$ and $i_1 \in k-\text{SD}$, then $\text{SDSubAddData}(i_1,k) \geq -\text{Radix}(k-'1)$ and $\text{SDSubAddData}(i_1,k) \leq \text{Radix}(k-'1)-1$.

(15) For every integer x and for every natural number k such that $2 \leq k$ holds $\text{SDSubAddCarry}(x,k) \in k-\text{SD\_Sub\_S}$.

(16) If $2 \leq k$ and $i_1 \in k-\text{SD}$ and $i_2 \in k-\text{SD}$, then $\text{SDSubAddData}(i_1,k) + \text{SDSubAddCarry}(i_2,k) \in k-\text{SD\_Sub}$.

(17) If $2 \leq k$, then $\text{SDSubAddCarry}(0,k) = 0$.

3. Definition for Translation from Radix-2^k Signed-Digit Number

Let i, k, n be natural numbers and let x be a n-tuple of $k-$SD_Sub. The functor DigA_SDSub(x,i) yielding an integer is defined as follows:

(Def. 5)(i) DigA_SDSub$(x,i) = x(i)$ if $i \in \mathrm{Seg}\,n$,

(ii) DigA_SDSub$(x,i) = 0$ if $i = 0$.

Let i, k, n be natural numbers and let x be a n-tuple of $k-$SD. The functor SD2SDSubDigit(x,i,k) yields an integer and is defined by:

(Def. 6) $\mathrm{SD2SDSubDigit}(x,i,k) = \begin{cases} \text{(i)} \quad \mathrm{SDSubAddData}(\mathrm{DigA}(x,i),k) + \mathrm{SDSubAddCarry}(\mathrm{DigA}(x,i -' 1),k), \text{ if } i \in \mathrm{Seg}\,n, \\ \text{(ii)} \quad \mathrm{SDSubAddCarry}(\mathrm{DigA}(x,i -' 1),k), \text{ if } i = n+1, \\ 0, \text{ otherwise.} \end{cases}$

The following proposition is true

(18) If $2 \leq k$ and $i \in \mathrm{Seg}(n+1)$, then SD2SDSubDigit(a,i,k) is an element of $k-$SD_Sub.

Let i, k, n be natural numbers and let x be a n-tuple of $k-$SD. Let us assume that $2 \leq k$ and $i \in \mathrm{Seg}(n+1)$. The functor SD2SDSubDigitS(x,i,k) yielding an element of $k-$SD_Sub is defined by:

(Def. 7) SD2SDSubDigitS$(x,i,k) =$ SD2SDSubDigit(x,i,k).

Let n, k be natural numbers and let x be a n-tuple of $k-$SD. The functor SD2SDSub$x$ yielding a $n+1$-tuple of $k-$SD_Sub is defined as follows:

(Def. 8) For every natural number i such that $i \in \mathrm{Seg}(n+1)$ holds DigA_SDSub(SD2SDSub$x$, i) $=$ SD2SDSubDigitS(x,i,k).

Next we state two propositions:

(19) If $i \in \mathrm{Seg}\,n$, then DigA_SDSub(a_1,i) is an element of $k-$SD_Sub.

(20) If $2 \leq k$ and $i_1 \in k-$SD and $i_2 \in k-$SD_Sub, then SDSubAddData$(i_1 + i_2, k) \in k-$SD_Sub_S.

4. Definiton for Translation from Radix-2^k Sub Signed-Digit Number to INT

Let i, k, n be natural numbers and let x be a n-tuple of $k-$SD_Sub. The functor DigB_SDSub(x,i) yielding an element of $\mathbb{Z}$ is defined by:

(Def. 9) DigB_SDSub$(x,i) =$ DigA_SDSub(x,i).

Let i, k, n be natural numbers and let x be a n-tuple of $k-$SD_Sub. The functor SDSub2INTDigit(x,i,k) yielding an element of $\mathbb{Z}$ is defined by:

(Def. 10) SDSub2INTDigit$(x,i,k) = (\mathrm{Radix}\,k)^{i -' 1} \cdot$ DigB_SDSub(x,i).

Let n, k be natural numbers and let x be a n-tuple of $k-$SD_Sub. The functor SDSub2INTx yields a n-tuple of $\mathbb{Z}$ and is defined as follows:

(Def. 11) For every natural number i such that $i \in \mathrm{Seg}\,n$ holds $(\mathrm{SDSub2INT}\,x)_i =$ SDSub2INTDigit(x,i,k).

Let n, k be natural numbers and let x be a n-tuple of $k-$SD_Sub. The functor SDSub2IntOut$x$ yielding an integer is defined as follows:

(Def. 12) SDSub2IntOut$x = \sum$ SDSub2INTx.

The following propositions are true:

(21) For every i such that $i \in \mathrm{Seg}\,n$ holds if $2 \leq k$, then DigA_SDSub(SD2SDSubDecSD$(m, n+1, k), i) =$ DigA_SDSub(SD2SDSubDecSD$(m \bmod (\mathrm{Radix}\,k)^n, n, k), i)$.

(22) For every n such that $n \geq 1$ and for all k, x such that $k \geq 2$ and x is represented by n, k holds $x =$ SDSub2IntOutSD2SDSubDecSD(x,n,k).

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