JOURNAL OF FORMALIZED MATHEMATICS
Volume 5, Released 1993, Published 2003
Inst. of Computer Science, Univ. of Białystok

Two Programs for SCM. Part I - Preliminaries[1]

Grzegorz Bancerek
Polish Academy of Sciences
Institute of Mathematics
Warsaw

Piotr Rudnicki
University of Alberta
Department of Computing Science
Edmonton

Summary. In two articles (this one and [4]) we discuss correctness of two short programs for the **SCM** machine: one computes Fibonacci numbers and the other computes the *fusc* function of Dijkstra [9]. The limitations of current Mizar implementation rendered it impossible to present the correctness proofs for the programs in one article. This part is purely technical and contains a number of very specific lemmas about integer division, floor, exponentiation and logarithms. The formal definitions of the Fibonacci sequence and the *fusc* function may be of general interest.

MML Identifier: `PRE_FF`.

WWW: `http://mizar.org/JFM/Vol5/pre_ff.html`

The articles [7], [16], [2], [13], [14], [1], [12], [11], [10], [5], [6], [3], [8], and [15] provide the notation and terminology for this paper.

Let X_1, X_2 be non empty sets, let Y_1 be a non empty subset of X_1, let Y_2 be a non empty subset of X_2, and let x be an element of $[:Y_1, Y_2:]$. Then $x_{\mathbf{1}}$ is an element of Y_1. Then $x_{\mathbf{2}}$ is an element of Y_2.

In the sequel n denotes a natural number.

Let us consider n. The functor $\mathrm{Fib}(n)$ yielding a natural number is defined by the condition (Def. 1).

(Def. 1) There exists a function f_1 from $\mathbb{N}$ into $[:\mathbb{N}, \mathbb{N}:]$ such that $\mathrm{Fib}(n) = f_1(n)_{\mathbf{1}}$ and $f_1(0) = \langle 0, 1\rangle$ and for every natural number n holds $f_1(n+1) = \langle f_1(n)_{\mathbf{2}}, f_1(n)_{\mathbf{1}} + f_1(n)_{\mathbf{2}}\rangle$.

One can prove the following propositions:

(1) $\mathrm{Fib}(0) = 0$ and $\mathrm{Fib}(1) = 1$ and for every natural number n holds $\mathrm{Fib}(n+1+1) = \mathrm{Fib}(n) + \mathrm{Fib}(n+1)$.

(2) For every integer i holds $i \div 1 = i$.

(3) For all integers i, j such that $j > 0$ and $i \div j = 0$ holds $i < j$.

(4) For all integers i, j such that $0 \leq i$ and $i < j$ holds $i \div j = 0$.

(5) For all integers i, j, k such that $j > 0$ and $k > 0$ holds $i \div j \div k = i \div j \cdot k$.

(6) For every integer i holds $i \bmod 2 = 0$ or $i \bmod 2 = 1$.

(7) For every integer i such that i is a natural number holds $i \div 2$ is a natural number.

[1]This work was partially supported by NSERC Grant OGP9207 while the first author visited University of Alberta, May–June 1993.

(10)[1] For all real numbers a, b, c such that $a \leq b$ and $c > 1$ holds $c^a \leq c^b$.

(11) For all real numbers r, s such that $r \geq s$ holds $\lfloor r \rfloor \geq \lfloor s \rfloor$.

(12) For all real numbers a, b, c such that $a > 1$ and $b > 0$ and $c \geq b$ holds $\log_a c \geq \log_a b$.

(13) For every natural number n such that $n > 0$ holds $\lfloor \log_2(2 \cdot n) \rfloor + 1 \neq \lfloor \log_2(2 \cdot n + 1) \rfloor$.

(14) For every natural number n such that $n > 0$ holds $\lfloor \log_2(2 \cdot n) \rfloor + 1 \geq \lfloor \log_2(2 \cdot n + 1) \rfloor$.

(15) For every natural number n such that $n > 0$ holds $\lfloor \log_2(2 \cdot n) \rfloor = \lfloor \log_2(2 \cdot n + 1) \rfloor$.

(16) For every natural number n such that $n > 0$ holds $\lfloor \log_2 n \rfloor + 1 = \lfloor \log_2(2 \cdot n + 1) \rfloor$.

Let f be a function from $\mathbb{N}$ into $\mathbb{N}^*$ and let n be a natural number. Then $f(n)$ is a finite sequence of elements of $\mathbb{N}$.

Let n be a natural number. The functor $\mathrm{Fusc}(n)$ yielding a natural number is defined by:

(Def. 2)(i) $\mathrm{Fusc}(n) = 0$ if $n = 0$,

(ii) there exists a natural number l and there exists a function f_2 from $\mathbb{N}$ into $\mathbb{N}^*$ such that $l + 1 = n$ and $\mathrm{Fusc}(n) = f_2(l)_n$ and $f_2(0) = \langle 1 \rangle$ and for every natural number n holds for every natural number k such that $n + 2 = 2 \cdot k$ holds $f_2(n+1) = f_2(n) \frown \langle f_2(n)_k \rangle$ and for every natural number k such that $n + 2 = 2 \cdot k + 1$ holds $f_2(n+1) = f_2(n) \frown \langle f_2(n)_k + f_2(n)_{k+1} \rangle$, otherwise.

Next we state several propositions:

(17) $\mathrm{Fusc}(0) = 0$ and $\mathrm{Fusc}(1) = 1$ and for every natural number n holds $\mathrm{Fusc}(2 \cdot n) = \mathrm{Fusc}(n)$ and $\mathrm{Fusc}(2 \cdot n + 1) = \mathrm{Fusc}(n) + \mathrm{Fusc}(n + 1)$.

(18) For all natural numbers n_1, n'_1 such that $n_1 \neq 0$ and $n_1 = 2 \cdot n'_1$ holds $n'_1 < n_1$.

(19) For all natural numbers n_1, n'_1 such that $n_1 = 2 \cdot n'_1 + 1$ holds $n'_1 < n_1$.

(20) For all natural numbers A, B holds $B = A \cdot \mathrm{Fusc}(0) + B \cdot \mathrm{Fusc}(0 + 1)$.

(21) For all natural numbers n_1, n'_1, A, B, N such that $n_1 = 2 \cdot n'_1 + 1$ and $\mathrm{Fusc}(N) = A \cdot \mathrm{Fusc}(n_1) + B \cdot \mathrm{Fusc}(n_1 + 1)$ holds $\mathrm{Fusc}(N) = A \cdot \mathrm{Fusc}(n'_1) + (B + A) \cdot \mathrm{Fusc}(n'_1 + 1)$.

(22) For all natural numbers n_1, n'_1, A, B, N such that $n_1 = 2 \cdot n'_1$ and $\mathrm{Fusc}(N) = A \cdot \mathrm{Fusc}(n_1) + B \cdot \mathrm{Fusc}(n_1 + 1)$ holds $\mathrm{Fusc}(N) = (A + B) \cdot \mathrm{Fusc}(n'_1) + B \cdot \mathrm{Fusc}(n'_1 + 1)$.

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[1] The propositions (8) and (9) have been removed.

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Received October 8, 1993

Published January 2, 2004