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Basic Petri Net Concepts

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Summary. This article presents the basic place/transition net structure definition for building various types of Petri nets. The basic net structure fields include places, transitions, and arcs (place-transition, transition-place) which may be supplemented with other fields (e.g., capacity, weight, marking, etc.) as needed. The theorems included in this article are divided into the following categories: deadlocks, traps, and dual net theorems. Here, a dual net is taken as the result of inverting all arcs (place-transition arcs to transition-place arcs and vice-versa) in the original net.

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The articles [3], [1], [5], [6], [7], [4], and [2] provide the notation and terminology for this paper.

1. Basic Place/Transition Net Structure Definition

Let A, B be non empty sets and let r be a non empty relation between A and B. We see that the element of r is an element of $[:A, B:]$.

We consider place/transition net structures as systems

$\langle$ places, transitions, S-T arcs, T-S arcs $\rangle$,

where the places and the transitions constitute non empty sets, the S-T arcs constitute a non empty relation between the places and the transitions, and the T-S arcs constitute a non empty relation between the transitions and the places.

In the sequel P_1 is a place/transition net structure.

Let us consider P_1. A place of P_1 is an element of the places of P_1. A transition of P_1 is an element of the transitions of P_1. An S-T arc of P_1 is an element of the S-T arcs of P_1. A T-S arc of P_1 is an element of the T-S arcs of P_1.

Let us consider P_1 and let x be an S-T arc of P_1. Then $x_{\mathbf{1}}$ is a place of P_1. Then $x_{\mathbf{2}}$ is a transition of P_1.

Let us consider P_1 and let x be a T-S arc of P_1. Then $x_{\mathbf{1}}$ is a transition of P_1. Then $x_{\mathbf{2}}$ is a place of P_1.

In the sequel S_0 denotes a subset of the places of P_1.

Let us consider P_1, S_0. The functor ${}^{*}S_0$ yielding a subset of the transitions of P_1 is defined by:

(Def. 1) $\quad {}^{*}S_0 = \{t; t \text{ ranges over transitions of } P_1\colon \bigvee_{f:\text{T-S arc of } P_1} \bigvee_{s:\text{place of } P_1} (s \in S_0 \wedge f = \langle t, s \rangle)\}$.

The functor $\overline{S_0}$ yields a subset of the transitions of P_1 and is defined by:

(Def. 2) $\quad \overline{S_0} = \{t; t \text{ ranges over transitions of } P_1\colon \bigvee_{f:\text{S-T arc of } P_1} \bigvee_{s:\text{place of } P_1} (s \in S_0 \wedge f = \langle s, t \rangle)\}$.

One can prove the following propositions:

(1) $^*S_0 = \{f_{\mathbf{1}}; f$ ranges over T-S arcs of P_1: $f_{\mathbf{2}} \in S_0\}$.

(2) For every set x holds $x \in {}^*S_0$ iff there exists a T-S arc f of P_1 and there exists a place s of P_1 such that $s \in S_0$ and $f = \langle x, s\rangle$.

(3) $\overline{S_0} = \{f_{\mathbf{2}}; f$ ranges over S-T arcs of P_1: $f_{\mathbf{1}} \in S_0\}$.

(4) For every set x holds $x \in \overline{S_0}$ iff there exists an S-T arc f of P_1 and there exists a place s of P_1 such that $s \in S_0$ and $f = \langle s, x\rangle$.

In the sequel T_0 is a subset of the transitions of P_1.
Let us consider P_1, T_0. The functor $^*T_0$ yielding a subset of the places of P_1 is defined by:

(Def. 3) $^*T_0 = \{s; s$ ranges over places of P_1: $\bigvee_{f\,:\,\text{S-T arc of } P_1} \bigvee_{t\,:\,\text{transition of } P_1} (t \in T_0 \wedge f = \langle s, t\rangle)\}$.

The functor $\overline{T_0}$ yields a subset of the places of P_1 and is defined as follows:

(Def. 4) $\overline{T_0} = \{s; s$ ranges over places of P_1: $\bigvee_{f\,:\,\text{T-S arc of } P_1} \bigvee_{t\,:\,\text{transition of } P_1} (t \in T_0 \wedge f = \langle t, s\rangle)\}$.

Next we state several propositions:

(5) $^*T_0 = \{f_{\mathbf{1}}; f$ ranges over S-T arcs of P_1: $f_{\mathbf{2}} \in T_0\}$.

(6) Let x be a set. Then $x \in {}^*T_0$ if and only if there exists an S-T arc f of P_1 and there exists a transition t of P_1 such that $t \in T_0$ and $f = \langle x, t\rangle$.

(7) $\overline{T_0} = \{f_{\mathbf{2}}; f$ ranges over T-S arcs of P_1: $f_{\mathbf{1}} \in T_0\}$.

(8) Let x be a set. Then $x \in \overline{T_0}$ if and only if there exists a T-S arc f of P_1 and there exists a transition t of P_1 such that $t \in T_0$ and $f = \langle t, x\rangle$.

(9) $^*(\emptyset_{\text{the places of } P_1}) = \emptyset$.

(10) $\overline{\emptyset_{\text{the places of } P_1}} = \emptyset$.

(11) $^*(\emptyset_{\text{the transitions of } P_1}) = \emptyset$.

(12) $\overline{\emptyset_{\text{the transitions of } P_1}} = \emptyset$.

2. DEADLOCKS

Let us consider P_1 and let I_1 be a subset of the places of P_1. We say that I_1 is deadlock-like if and only if:

(Def. 5) $^*I_1$ is a subset of $\overline{I_1}$.

Let I_1 be a place/transition net structure. We say that I_1 has deadlocks if and only if:

(Def. 6) There exists a subset of the places of I_1 which is deadlock-like.

Let us observe that there exists a place/transition net structure which has deadlocks.

3. TRAPS

Let us consider P_1 and let I_1 be a subset of the places of P_1. We say that I_1 is trap-like if and only if:

(Def. 7) $\overline{I_1}$ is a subset of $^*I_1$.

Let I_1 be a place/transition net structure. We say that I_1 has traps if and only if:

(Def. 8) There exists a subset of the places of I_1 which is trap-like.

Let us mention that there exists a place/transition net structure which has traps.
Let A, B be non empty sets and let r be a non empty relation between A and B. Then $r^{\smile}$ is a non empty relation between B and A.

4. Duality Theorems for Place/Transition Nets

Let us consider P_1. The functor $P_1{}^\circ$ yielding a strict place/transition net structure is defined by:

(Def. 9) $P_1{}^\circ = \langle$the places of P_1, the transitions of P_1, (the T-S arcs of $P_1)^\smile$, (the S-T arcs of $P_1)^\smile\rangle$.

Next we state two propositions:

(13) $(P_1{}^\circ)^\circ =$ the place/transition net structure of P_1.

(14)(i) The places of $P_1 =$ the places of $P_1{}^\circ$,

(ii) the transitions of $P_1 =$ the transitions of $P_1{}^\circ$,

(iii) (the S-T arcs of $P_1)^\smile =$ the T-S arcs of $P_1{}^\circ$, and

(iv) (the T-S arcs of $P_1)^\smile =$ the S-T arcs of $P_1{}^\circ$.

Let us consider P_1 and let S_0 be a subset of the places of P_1. The functor $S_0{}^\circ$ yielding a subset of the places of $P_1{}^\circ$ is defined by:

(Def. 10) $S_0{}^\circ = S_0$.

Let us consider P_1 and let s be a place of P_1. The functor s° yielding a place of $P_1{}^\circ$ is defined by:

(Def. 11) $s^\circ = s$.

Let us consider P_1 and let S_0 be a subset of the places of $P_1{}^\circ$. The functor ${}^\circ S_0$ yields a subset of the places of P_1 and is defined as follows:

(Def. 12) ${}^\circ S_0 = S_0$.

Let us consider P_1 and let s be a place of $P_1{}^\circ$. The functor ${}^\circ s$ yielding a place of P_1 is defined by:

(Def. 13) ${}^\circ s = s$.

Let us consider P_1 and let T_0 be a subset of the transitions of P_1. The functor $T_0{}^\circ$ yields a subset of the transitions of $P_1{}^\circ$ and is defined by:

(Def. 14) $T_0{}^\circ = T_0$.

Let us consider P_1 and let t be a transition of P_1. The functor t° yielding a transition of $P_1{}^\circ$ is defined as follows:

(Def. 15) $t^\circ = t$.

Let us consider P_1 and let T_0 be a subset of the transitions of $P_1{}^\circ$. The functor ${}^\circ T_0$ yields a subset of the transitions of P_1 and is defined by:

(Def. 16) ${}^\circ T_0 = T_0$.

Let us consider P_1 and let t be a transition of $P_1{}^\circ$. The functor ${}^\circ t$ yielding a transition of P_1 is defined as follows:

(Def. 17) ${}^\circ t = t$.

In the sequel S is a subset of the places of P_1.
We now state several propositions:

(15) $\overline{S^\circ} = {}^*S$.

(16) ${}^*(S^\circ) = \overline{S}$.

(17) S is deadlock-like iff S° is trap-like.

(18) S is trap-like iff S° is deadlock-like.

(19) Let P_1 be a place/transition net structure, t be a transition of P_1, and S_0 be a subset of the places of P_1. Then $t \in \overline{S_0}$ if and only if ${}^*\{t\}$ meets S_0.

(20) Let P_1 be a place/transition net structure, t be a transition of P_1, and S_0 be a subset of the places of P_1. Then $t \in {}^*S_0$ if and only if $\overline{\{t\}}$ meets S_0.

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