JOURNAL OF FORMALIZED MATHEMATICS
Volume 13, Released 2001, Published 2003
Inst. of Computer Science, Univ. of Białystok

On Cosets in Segre's Product of Partial Linear Spaces

Adam Naumowicz
University of Białystok

Summary. This paper is a continuation of [15]. We prove that the family of cosets in the Segre's product of partial linear spaces remains invariant under automorphisms.

MML Identifier: `PENCIL_2`.

WWW: `http://mizar.org/JFM/Vol13/pencil_2.html`

The articles [19], [10], [23], [2], [24], [16], [7], [9], [8], [5], [13], [1], [4], [3], [17], [20], [21], [6], [11], [25], [14], [18], [12], [22], and [15] provide the notation and terminology for this paper.

1. Preliminaries on Finite Sequences

Let D be a set, let p be a finite sequence of elements of D, and let i, j be natural numbers. The functor $\mathrm{Del}(p,i,j)$ yielding a finite sequence of elements of D is defined by:

(Def. 1) $\mathrm{Del}(p,i,j) = (p \restriction (i -' 1)) \frown (p_{\lfloor j})$.

Next we state several propositions:

(1) For every set D and for every finite sequence p of elements of D and for all natural numbers i, j holds $\mathrm{rng}\,\mathrm{Del}(p,i,j) \subseteq \mathrm{rng}\,p$.

(2) Let D be a set, p be a finite sequence of elements of D, and i, j be natural numbers. If $i \in \mathrm{dom}\,p$ and $j \in \mathrm{dom}\,p$, then $\mathrm{len}\,\mathrm{Del}(p,i,j) = ((\mathrm{len}\,p - j) + i) - 1$.

(3) Let D be a set, p be a finite sequence of elements of D, and i, j be natural numbers. If $i \in \mathrm{dom}\,p$ and $j \in \mathrm{dom}\,p$, then if $\mathrm{len}\,\mathrm{Del}(p,i,j) = 0$, then $i = 1$ and $j = \mathrm{len}\,p$.

(4) Let D be a set, p be a finite sequence of elements of D, and i, j, k be natural numbers. If $i \in \mathrm{dom}\,p$ and $1 \leq k$ and $k \leq i - 1$, then $(\mathrm{Del}(p,i,j))(k) = p(k)$.

(5) For all finite sequences p, q and for every natural number k such that $\mathrm{len}\,p + 1 \leq k$ holds $(p \frown q)(k) = q(k - \mathrm{len}\,p)$.

(6) Let D be a set, p be a finite sequence of elements of D, and i, j, k be natural numbers. Suppose $i \in \mathrm{dom}\,p$ and $j \in \mathrm{dom}\,p$ and $i \leq j$ and $i \leq k$ and $k \leq ((\mathrm{len}\,p - j) + i) - 1$. Then $(\mathrm{Del}(p,i,j))(k) = p((j -' i) + k + 1)$.

The scheme *FinSeqOneToOne* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, a finite sequence $\mathcal{D}$ of elements of $\mathcal{C}$, and a binary predicate $\mathcal{P}$, and states that:

There exists an one-to-one finite sequence g of elements of $\mathcal{C}$ such that $\mathcal{A} = g(1)$ and $\mathcal{B} = g(\mathrm{len}\,g)$ and $\mathrm{rng}\,g \subseteq \mathrm{rng}\,\mathcal{D}$ and for every natural number j such that $1 \leq j$ and $j < \mathrm{len}\,g$ holds $\mathcal{P}[g(j), g(j+1)]$

provided the parameters satisfy the following conditions:

- $\mathcal{A} = \mathcal{D}(1)$ and $\mathcal{B} = \mathcal{D}(\operatorname{len}\mathcal{D})$, and
- For every natural number i and for all sets d_1, d_2 such that $1 \leq i$ and $i < \operatorname{len}\mathcal{D}$ and $d_1 = \mathcal{D}(i)$ and $d_2 = \mathcal{D}(i+1)$ holds $\mathcal{P}[d_1, d_2]$.

2. Segre Cosets

We now state the proposition

(7) Let I be a non empty set, A be a 1-sorted yielding many sorted set indexed by I, L be a many sorted subset indexed by the support of A, i be an element of I, and S be a subset of $A(i)$. Then $L +\cdot (i, S)$ is a many sorted subset indexed by the support of A.

Let I be a non empty set and let A be a non-Trivial-yielding TopStruct-yielding many sorted set indexed by I. A subset of Segre_Product$A$ is called a Segre-Coset of A if it satisfies the condition (Def. 2).

(Def. 2) There exists a Segre-like non trivial-yielding many sorted subset L indexed by the support of A such that it $= \prod L$ and $L(\operatorname{index}(L)) = \Omega_{A(\operatorname{index}(L))}$.

One can prove the following proposition

(8) Let I be a non empty set, A be a non-Trivial-yielding TopStruct-yielding many sorted set indexed by I, and B_1, B_2 be Segre-Cosets of A. If $2 \subseteq \overline{\overline{B_1 \cap B_2}}$, then $B_1 = B_2$.

Let S be a topological structure and let X, Y be subsets of S. We say that X and Y are joinable if and only if the condition (Def. 3) is satisfied.

(Def. 3) There exists a finite sequence f of elements of $2^{\text{the carrier of } S}$ such that

(i) $X = f(1)$,

(ii) $Y = f(\operatorname{len} f)$,

(iii) for every subset W of S such that $W \in \operatorname{rng} f$ holds W is closed under lines and strong, and

(iv) for every natural number i such that $1 \leq i$ and $i < \operatorname{len} f$ holds $2 \subseteq \overline{\overline{f(i) \cap f(i+1)}}$.

The following propositions are true:

(9) Let S be a topological structure and X, Y be subsets of S. Suppose X and Y are joinable. Then there exists an one-to-one finite sequence f of elements of $2^{\text{the carrier of } S}$ such that

(i) $X = f(1)$,

(ii) $Y = f(\operatorname{len} f)$,

(iii) for every subset W of S such that $W \in \operatorname{rng} f$ holds W is closed under lines and strong, and

(iv) for every natural number i such that $1 \leq i$ and $i < \operatorname{len} f$ holds $2 \subseteq \overline{\overline{f(i) \cap f(i+1)}}$.

(10) Let S be a topological structure and X be a subset of S. If X is closed under lines and strong, then X and X are joinable.

(11) Let I be a non empty set, A be a PLS-yielding many sorted set indexed by I, and X, Y be subsets of Segre_Product$A$. Suppose that

(i) X is non trivial, closed under lines, and strong,

(ii) Y is non trivial, closed under lines, and strong, and

(iii) X and Y are joinable.

Let X_1, Y_1 be Segre-like non trivial-yielding many sorted subsets indexed by the support of A. Suppose $X = \prod X_1$ and $Y = \prod Y_1$. Then $\operatorname{index}(X_1) = \operatorname{index}(Y_1)$ and for every set i such that $i \neq \operatorname{index}(X_1)$ holds $X_1(i) = Y_1(i)$.

3. Collineations of Segre Product

Next we state the proposition

(12) Let S be a 1-sorted structure, T be a non empty 1-sorted structure, and f be a map from S into T. If f is bijective, then f^{-1} is bijective.

Let S, T be topological structures and let f be a map from S into T. We say that f is isomorphic if and only if:

(Def. 4) f is bijective and open and f^{-1} is bijective and open.

Let S be a non empty topological structure. Observe that there exists a map from S into S which is isomorphic.

Let S be a non empty topological structure. A collineation of S is an isomorphic map from S into S.

Let S be a non empty non void topological structure, let f be a collineation of S, and let l be a block of S. Then $f^{\circ}l$ is a block of S.

Let S be a non empty non void topological structure, let f be a collineation of S, and let l be a block of S. Then $f^{-1}(l)$ is a block of S.

Next we state a number of propositions:

(13) For every non empty topological structure S and for every collineation f of S holds f^{-1} is a collineation of S.

(14) Let S be a non empty topological structure, f be a collineation of S, and X be a subset of S. If X is non trivial, then $f^{\circ}X$ is non trivial.

(15) Let S be a non empty topological structure, f be a collineation of S, and X be a subset of S. If X is non trivial, then $f^{-1}(X)$ is non trivial.

(16) Let S be a non empty non void topological structure, f be a collineation of S, and X be a subset of S. If X is strong, then $f^{\circ}X$ is strong.

(17) Let S be a non empty non void topological structure, f be a collineation of S, and X be a subset of S. If X is strong, then $f^{-1}(X)$ is strong.

(18) Let S be a non empty non void topological structure, f be a collineation of S, and X be a subset of S. If X is closed under lines, then $f^{\circ}X$ is closed under lines.

(19) Let S be a non empty non void topological structure, f be a collineation of S, and X be a subset of S. If X is closed under lines, then $f^{-1}(X)$ is closed under lines.

(20) Let S be a non empty non void topological structure, f be a collineation of S, and X, Y be subsets of S. Suppose X is non trivial and Y is non trivial and X and Y are joinable. Then $f^{\circ}X$ and $f^{\circ}Y$ are joinable.

(21) Let S be a non empty non void topological structure, f be a collineation of S, and X, Y be subsets of S. Suppose X is non trivial and Y is non trivial and X and Y are joinable. Then $f^{-1}(X)$ and $f^{-1}(Y)$ are joinable.

(22) Let I be a non empty set and A be a PLS-yielding many sorted set indexed by I. Suppose that for every element i of I holds $A(i)$ is strongly connected. Let W be a subset of Segre_Product$A$. Suppose W is non trivial, strong, and closed under lines. Then $\bigcup\{Y;Y$ ranges over subsets of Segre_Product$A$: Y is non trivial, strong, and closed under lines $\wedge$ W and Y are joinable$\}$ is a Segre-Coset of A.

(23) Let I be a non empty set and A be a PLS-yielding many sorted set indexed by I. Suppose that for every element i of I holds $A(i)$ is strongly connected. Let B be a set. Then B is a Segre-Coset of A if and only if there exists a subset W of Segre_Product$A$ such that W is non trivial, strong, and closed under lines and $B = \bigcup\{Y;Y$ ranges over subsets of Segre_Product$A$: Y is non trivial, strong, and closed under lines $\wedge$ W and Y are joinable$\}$.

(24) Let I be a non empty set and A be a PLS-yielding many sorted set indexed by I. Suppose that for every element i of I holds $A(i)$ is strongly connected. Let B be a Segre-Coset of A and f be a collineation of Segre_Product$A$. Then $f^{\circ}B$ is a Segre-Coset of A.

(25) Let I be a non empty set and A be a PLS-yielding many sorted set indexed by I. Suppose that for every element i of I holds $A(i)$ is strongly connected. Let B be a Segre-Coset of A and f be a collineation of Segre_Product$A$. Then $f^{-1}(B)$ is a Segre-Coset of A.

REFERENCES

[1] Grzegorz Bancerek. Cardinal numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/card_1.html`.

[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/nat_1.html`.

[3] Grzegorz Bancerek. König's theorem. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/card_3.html`.

[4] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/finseq_1.html`.

[5] Józef Białas. Group and field definitions. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/realset1.html`.

[6] Ewa Burakowska. Subalgebras of many sorted algebra. Lattice of subalgebras. *Journal of Formalized Mathematics*, 6, 1994. `http://mizar.org/JFM/Vol6/msualg_2.html`.

[7] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_1.html`.

[8] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_2.html`.

[9] Czesław Byliński. Partial functions. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/partfun1.html`.

[10] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/zfmisc_1.html`.

[11] Agata Darmochwał. Families of subsets, subspaces and mappings in topological spaces. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/tops_2.html`.

[12] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\mathrm{T}}$. Arcs, line segments and special polygonal arcs. *Journal of Formalized Mathematics*, 3, 1991. `http://mizar.org/JFM/Vol3/topreal1.html`.

[13] Jarosław Kotowicz. Functions and finite sequences of real numbers. *Journal of Formalized Mathematics*, 5, 1993. `http://mizar.org/JFM/Vol5/rfinseq.html`.

[14] Beata Madras. Product of family of universal algebras. *Journal of Formalized Mathematics*, 5, 1993. `http://mizar.org/JFM/Vol5/pralg_1.html`.

[15] Adam Naumowicz. On Segre's product of partial line spaces. *Journal of Formalized Mathematics*, 12, 2000. `http://mizar.org/JFM/Vol12/pencil_1.html`.

[16] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Journal of Formalized Mathematics*, 5, 1993. `http://mizar.org/JFM/Vol5/binarith.html`.

[17] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/pre_topc.html`.

[18] Piotr Rudnicki and Andrzej Trybulec. Multivariate polynomials with arbitrary number of variables. *Journal of Formalized Mathematics*, 11, 1999. `http://mizar.org/JFM/Vol11/polynom1.html`.

[19] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[20] Andrzej Trybulec. Many-sorted sets. *Journal of Formalized Mathematics*, 5, 1993. `http://mizar.org/JFM/Vol5/pboole.html`.

[21] Andrzej Trybulec. Many sorted algebras. *Journal of Formalized Mathematics*, 6, 1994. `http://mizar.org/JFM/Vol6/msualg_1.html`.

[22] Wojciech A. Trybulec. Pigeon hole principle. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/finseq_4.html`.

[23] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

[24] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

[25] Mariusz Żynel and Adam Guzowski. T_0 topological spaces. *Journal of Formalized Mathematics*, 6, 1994. `http://mizar.org/JFM/Vol6/t_0topsp.html`.

Received August 14, 2001

Published January 2, 2004