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Fano-Desargues Parallelity Spaces[1]

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Summary. This article is the second part of Parallelity Space. It contains definition of a Fano-Desargues space, axioms of a Fano-Desargues parallelity space, definition of the relations: collinearity, parallelogram and directed congruence and some basic facts concerned with them.

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The articles [1], [7], [5], [4], [6], [2], and [3] provide the notation and terminology for this paper.

In this paper F is a field.

We now state the proposition

(1) Aff_{F^3} is a parallelity space.

We use the following convention: a, b, c, d, p, q, r are elements of Aff_{F^3}, e, f, g, h are elements of $[\!:$ the carrier of F, the carrier of F, the carrier of $F\,:\!]$, and K, L are elements of F.

Next we state several propositions:

(2) $a,b \parallel c,d$ if and only if there exist e, f, g, h such that $\langle a,b,c,d\rangle = \langle e,f,g,h\rangle$ but there exists K such that $K\cdot(e_{\mathbf{1}}-f_{\mathbf{1}}) = g_{\mathbf{1}}-h_{\mathbf{1}}$ and $K\cdot(e_{\mathbf{2}}-f_{\mathbf{2}}) = g_{\mathbf{2}}-h_{\mathbf{2}}$ and $K\cdot(e_{\mathbf{3}}-f_{\mathbf{3}}) = g_{\mathbf{3}}-h_{\mathbf{3}}$ or $e_{\mathbf{1}}-f_{\mathbf{1}} = 0_F$ and $e_{\mathbf{2}}-f_{\mathbf{2}} = 0_F$ and $e_{\mathbf{3}}-f_{\mathbf{3}} = 0_F$.

(3) If $a,b \nparallel a,c$ and $\langle a,b,a,c\rangle = \langle e,f,e,g\rangle$, then $e \neq f$ and $e \neq g$ and $f \neq g$.

(4) Suppose $a,b \nparallel a,c$ and $\langle a,b,a,c\rangle = \langle e,f,e,g\rangle$ and $K\cdot(e_{\mathbf{1}}-f_{\mathbf{1}}) = L\cdot(e_{\mathbf{1}}-g_{\mathbf{1}})$ and $K\cdot(e_{\mathbf{2}}-f_{\mathbf{2}}) = L\cdot(e_{\mathbf{2}}-g_{\mathbf{2}})$ and $K\cdot(e_{\mathbf{3}}-f_{\mathbf{3}}) = L\cdot(e_{\mathbf{3}}-g_{\mathbf{3}})$. Then $K = 0_F$ and $L = 0_F$.

(5) If $a,b \nparallel a,c$ and $a,b \parallel c,d$ and $a,c \parallel b,d$ and $\langle a,b,c,d\rangle = \langle e,f,g,h\rangle$, then $h_{\mathbf{1}} = (f_{\mathbf{1}}+g_{\mathbf{1}})-e_{\mathbf{1}}$ and $h_{\mathbf{2}} = (f_{\mathbf{2}}+g_{\mathbf{2}})-e_{\mathbf{2}}$ and $h_{\mathbf{3}} = (f_{\mathbf{3}}+g_{\mathbf{3}})-e_{\mathbf{3}}$.

(6) There exist a, b, c such that $a,b \nparallel a,c$.

(7) If $\mathbf{1}_F+\mathbf{1}_F \neq 0_F$ and $b,c \parallel a,d$ and $a,b \parallel c,d$ and $a,c \parallel b,d$, then $a,b \parallel a,c$.

(8) If $a,p \nparallel a,b$ and $a,p \nparallel a,c$ and $a,p \parallel b,q$ and $a,p \parallel c,r$ and $a,b \parallel p,q$ and $a,c \parallel p,r$, then $b,c \parallel q,r$.

Let I_1 be a parallelity space. We say that I_1 is Fano-Desarques space-like if and only if the conditions (Def. 1) are satisfied.

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(Def. 1)(i) There exist elements a, b, c of I_1 such that $a,b \nparallel a,c$,

(ii) for all elements a, b, c, d of I_1 such that $b,c \parallel a,d$ and $a,b \parallel c,d$ and $a,c \parallel b,d$ holds $a,b \parallel a,c$, and

(iii) for all elements a, b, c, p, q, r of I_1 such that $a,p \nparallel a,b$ and $a,p \nparallel a,c$ and $a,p \parallel b,q$ and $a,p \parallel c,r$ and $a,b \parallel p,q$ and $a,c \parallel p,r$ holds $b,c \parallel q,r$.

Let us note that there exists a parallelity space which is strict and Fano-Desarques space-like.

A Fano-Desarques space is a Fano-Desarques space-like parallelity space.

We use the following convention: F_1 denotes a Fano-Desarques space and a, b, c, d, p, q, r, s, o, x, y denote elements of F_1.

The following proposition is true

(13)[1] If $p \neq q$, then there exists r such that $p,q \nparallel p,r$.

Let us consider F_1, a, b, c. We say that a, b and c are collinear if and only if:

(Def. 2) $a,b \parallel a,c$.

One can prove the following propositions:

(15)[2] Suppose a, b and c are collinear. Then

(i) a, c and b are collinear,

(ii) c, b and a are collinear,

(iii) b, a and c are collinear,

(iv) b, c and a are collinear, and

(v) c, a and b are collinear.

(17)[3] If a, b and c are not collinear and $a,b \parallel p,q$ and $a,c \parallel p,r$ and $p \neq q$ and $p \neq r$, then p, q and r are not collinear.

(18) If $a = b$ or $b = c$ or $c = a$, then a, b and c are collinear.

(19) Suppose $a \neq b$ and a, b and p are collinear and a, b and q are collinear and a, b and r are collinear. Then p, q and r are collinear.

(20) If $p \neq q$, then there exists r such that p, q and r are not collinear.

(21) If a, b and c are collinear and a, b and d are collinear, then $a,b \parallel c,d$.

(22) If a, b and c are not collinear and $a,b \parallel c,d$, then a, b and d are not collinear.

(23) Suppose a, b and c are not collinear and $a,b \parallel c,d$ and $c \neq d$. Then a, b and x are not collinear or c, d and x are not collinear.

(24) If o, a and b are not collinear, then o, a and x are not collinear or o, b and x are not collinear or $o = x$.

(25) Suppose $o \neq a$ and $o \neq b$ and o, a and b are collinear and o, a and p are collinear and o, b and q are collinear. Then $a,b \parallel p,q$.

(26) Suppose that

(i) $a,b \nparallel c,d$,

(ii) a, b and p are collinear,

(iii) a, b and q are collinear,

(iv) c, d and p are collinear, and

(v) c, d and q are collinear.

Then $p = q$.

[1] The propositions (9)–(12) have been removed.
[2] The proposition (14) has been removed.
[3] The proposition (16) has been removed.

(27) If $a \neq b$ and a, b and c are collinear and $a,b \parallel c,d$, then $a,c \parallel b,d$.

(28) If $a \neq b$ and a, b and c are collinear and $a,b \parallel c,d$, then $c,b \parallel c,d$.

(29) Suppose that

(i) o, a and c are not collinear,

(ii) o, a and b are collinear,

(iii) o, c and p are collinear,

(iv) o, c and q are collinear,

(v) $a,c \parallel b,p$, and

(vi) $a,c \parallel b,q$.

Then $p = q$.

(30) If $a \neq b$ and a, b and c are collinear and a, b and d are collinear, then a, c and d are collinear.

(31) If a, b and c are collinear and a, c and d are collinear and $a \neq c$, then b, c and d are collinear.

Let us consider F_1, a, b, c, d. We say that a, b, c, d form a parallelogram if and only if:

(Def. 3) a, b and c are not collinear and $a,b \parallel c,d$ and $a,c \parallel b,d$.

Next we state a number of propositions:

(34)[4] If a, b, c, d form a parallelogram, then $a \neq b$ and $b \neq c$ and $c \neq a$ and $a \neq d$ and $b \neq d$ and $c \neq d$.

(35) Suppose a, b, c, d form a parallelogram. Then

(i) a, b and c are not collinear,

(ii) b, a and d are not collinear,

(iii) c, d and a are not collinear, and

(iv) d, c and b are not collinear.

(36) Suppose a, b, c, d form a parallelogram. Then a, b and c are not collinear and b, a and d are not collinear and c, d and a are not collinear and d, c and b are not collinear and a, c and b are not collinear and b, a and c are not collinear and b, c and a are not collinear and c, a and b are not collinear and c, b and a are not collinear and b, d and a are not collinear and a, b and d are not collinear and a, d and b are not collinear and d, a and b are not collinear and d, b and a are not collinear and c, a and d are not collinear and a, c and d are not collinear and a, d and c are not collinear and d, a and c are not collinear and d, c and a are not collinear and d, b and c are not collinear and b, c and d are not collinear and b, d and c are not collinear and c, b and d are not collinear and c, d and b are not collinear.

(37) If a, b, c, d form a parallelogram, then a, b and x are not collinear or c, d and x are not collinear.

(38) If a, b, c, d form a parallelogram, then a, c, b, d form a parallelogram.

(39) If a, b, c, d form a parallelogram, then c, d, a, b form a parallelogram.

(40) If a, b, c, d form a parallelogram, then b, a, d, c form a parallelogram.

(41) Suppose a, b, c, d form a parallelogram. Then a, c, b, d form a parallelogram and c, d, a, b form a parallelogram and b, a, d, c form a parallelogram and c, a, d, b form a parallelogram and d, b, c, a form a parallelogram and b, d, a, c form a parallelogram and d, c, b, a form a parallelogram.

[4] The propositions (32) and (33) have been removed.

(42) If a, b and c are not collinear, then there exists d such that a, b, c, d form a parallelogram.

(43) If a, b, c, p form a parallelogram and a, b, c, q form a parallelogram, then $p = q$.

(44) If a, b, c, d form a parallelogram, then $a,d \nparallel b,c$.

(45) If a, b, c, d form a parallelogram, then a, b, d, c do not form a parallelogram.

(46) If $a \neq b$, then there exists c such that a, b and c are collinear and $c \neq a$ and $c \neq b$.

(47) If a, p, b, q form a parallelogram and a, p, c, r form a parallelogram, then $b,c \parallel q,r$.

(48) Suppose b, q and c are not collinear and a, p, b, q form a parallelogram and a, p, c, r form a parallelogram. Then b, q, c, r form a parallelogram.

(49) Suppose that
(i) a, b and c are collinear,
(ii) $b \neq c$,
(iii) a, p, b, q form a parallelogram, and
(iv) a, p, c, r form a parallelogram.
Then b, q, c, r form a parallelogram.

(50) Suppose that
(i) a, p, b, q form a parallelogram,
(ii) a, p, c, r form a parallelogram, and
(iii) b, q, d, s form a parallelogram.
Then $c,d \parallel r,s$.

(51) If $a \neq b$, then there exist c, d such that a, b, c, d form a parallelogram.

(52) If $a \neq d$, then there exist b, c such that a, b, c, d form a parallelogram.

Let us consider F_1, a, b, r, s. We say that a, b congr r, s if and only if:

(Def. 4) $a = b$ and $r = s$ or there exist p, q such that p, q, a, b form a parallelogram and p, q, r, s form a parallelogram.

Next we state a number of propositions:

(55)[5] If a, a congr b, c, then $b = c$.

(56) If a, b congr c, c, then $a = b$.

(57) If a, b congr b, a, then $a = b$.

(58) If a, b congr c, d, then $a,b \parallel c,d$.

(59) If a, b congr c, d, then $a,c \parallel b,d$.

(60) If a, b congr c, d and a, b and c are not collinear, then a, b, c, d form a parallelogram.

(61) If a, b, c, d form a parallelogram, then a, b congr c, d.

(62) Suppose a, b congr c, d and a, b and c are collinear and r, s, a, b form a parallelogram. Then r, s, c, d form a parallelogram.

(63) If a, b congr c, x and a, b congr c, y, then $x = y$.

(64) There exists d such that a, b congr c, d.

[5] The propositions (53) and (54) have been removed.

(66)[6] a, b congr a, b.

(67) If r, s congr a, b and r, s congr c, d, then a, b congr c, d.

(68) If a, b congr c, d, then c, d congr a, b.

(69) If a, b congr c, d, then b, a congr d, c.

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[6] The proposition (65) has been removed.