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# Fanoian, Pappian and Desarguesian Affine Spaces

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**Summary.** We introduce basic types of affine spaces such as Desarguesian, Fanoian, Pappian, and translation affine and ordered affine sapces and we prove that suitably choosen analytically defined affine structures satisfy the required properties.

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The articles [10], [9], [2], [3], [6], [7], [4], [5], [8], and [1] provide the notation and terminology for this paper.

Let $O_1$ be an ordered affine space. Observe that $\Lambda(O_1)$ is affine space-like and non trivial.

Let $O_1$ be an ordered affine plane. Observe that $\Lambda(O_1)$ is 2-dimensional.

We now state several propositions:

(2)[1] Let $O_1$ be an ordered affine space and $x$ be a set. Then
(i) $x$ is an element of $O_1$ iff $x$ is an element of $\Lambda(O_1)$, and
(ii) $x$ is a subset of $O_1$ iff $x$ is a subset of $\Lambda(O_1)$.

(3) Let $O_1$ be an ordered affine space, $a$, $b$, $c$ be elements of the carrier of $O_1$, and $a'$, $b'$, $c'$ be elements of $\Lambda(O_1)$. If $a = a'$ and $b = b'$ and $c = c'$, then $\mathbf{L}(a,b,c)$ iff $\mathbf{L}(a',b',c')$.

(4) Let $V$ be a real linear space and $x$ be a set. Then $x$ is an element of OASpace $V$ if and only if $x$ is a vector of $V$.

(5) Let $V$ be a real linear space and $O_1$ be an ordered affine space. Suppose $O_1 = \mathrm{OASpace}\,V$. Let $a$, $b$, $c$, $d$ be elements of $O_1$ and $u$, $v$, $w$, $y$ be vectors of $V$. If $a = u$ and $b = v$ and $c = w$ and $d = y$, then $a,b \parallel c,d$ iff $u,v \parallel w,y$.

(6) Let $V$ be a real linear space and $O_1$ be an ordered affine space. Suppose $O_1 = \mathrm{OASpace}\,V$. Then there exist vectors $u$, $v$ of $V$ such that for all real numbers $a$, $b$ if $a \cdot u + b \cdot v = 0_V$, then $a = 0$ and $b = 0$.

Let $A_1$ be an affine space. We introduce $A_1$ satisfies **PAP'** as a synonym of $A_1$ is Pappian.

Let $A_1$ be an affine space. We introduce $A_1$ satisfies **DES'** as a synonym of $A_1$ is Desarguesian.

Let $A_1$ be an affine space. We introduce $A_1$ satisfies **TDES'** as a synonym of $A_1$ is Moufangian.

Let $A_1$ be an affine space. We introduce $A_1$ satisfies **des'** as a synonym of $A_1$ is translational.

Let $A_1$ be an affine space. Let us observe that $A_1$ is Fanoian if and only if:

(Def. 5)[2] For all elements $a$, $b$, $c$, $d$ of $A_1$ such that $a,b \upuparrows c,d$ and $a,c \upuparrows b,d$ and $a,d \upuparrows b,c$ holds $a,b \upuparrows a,c$.

[1] The proposition (1) has been removed.
[2] The definitions (Def. 1)–(Def. 4) have been removed.

We introduce $A_1$ satisfies Fano Axiom as a synonym of $A_1$ is Fanoian.

Let $I_1$ be an ordered affine space. We say that $I_1$ is Pappian if and only if:

(Def. 11)[3] $\Lambda(I_1)$ satisfies **PAP'**.

We say that $I_1$ is Desarguesian if and only if:

(Def. 12) $\Lambda(I_1)$ satisfies **DES'**.

We say that $I_1$ is Moufangian if and only if:

(Def. 13) $\Lambda(I_1)$ satisfies **TDES'**.

We say that $I_1$ is translation if and only if:

(Def. 14) $\Lambda(I_1)$ satisfies **des'**.

Let $O_1$ be an ordered affine space. We say that $O_1$ satisfies Desargues Axiom if and only if the condition (Def. 15) is satisfied.

(Def. 15) Let $o$, $a$, $b$, $c$, $a_1$, $b_1$, $c_1$ be elements of $O_1$. Suppose $o,a \upuparrows o,a_1$ and $o,b \upuparrows o,b_1$ and $o,c \upuparrows o,c_1$ and not $\mathbf{L}(o,a,b)$ and not $\mathbf{L}(o,a,c)$ and $a,b \upuparrows a_1,b_1$ and $a,c \upuparrows a_1,c_1$. Then $b,c \upuparrows b_1,c_1$.

We introduce $O_1$ satisfies **DES** as a synonym of $O_1$ satisfies Desargues Axiom.

Let $O_1$ be an ordered affine space. We say that $O_1$ satisfies $\mathbf{DES}_1$ if and only if the condition (Def. 16) is satisfied.

(Def. 16) Let $o$, $a$, $b$, $c$, $a_1$, $b_1$, $c_1$ be elements of $O_1$. Suppose $a,o \upuparrows o,a_1$ and $b,o \upuparrows o,b_1$ and $c,o \upuparrows o,c_1$ and not $\mathbf{L}(o,a,b)$ and not $\mathbf{L}(o,a,c)$ and $a,b \upuparrows b_1,a_1$ and $a,c \upuparrows c_1,a_1$. Then $b,c \upuparrows c_1,b_1$.

We introduce $O_1$ satisfies $\mathbf{DES}_1$ as a synonym of $O_1$ satisfies $\mathbf{DES}_1$.

We now state a number of propositions:

(11)[4] For every ordered affine space $O_1$ such that $O_1$ satisfies $\mathbf{DES}_1$ holds $O_1$ satisfies **DES**.

(12) Let $O_1$ be an ordered affine space and $o$, $a$, $b$, $a'$, $b'$ be elements of the carrier of $O_1$. If not $\mathbf{L}(o,a,b)$ and $a,o \upuparrows o,a'$ and $\mathbf{L}(o,b,b')$ and $a,b \parallel a',b'$, then $b,o \upuparrows o,b'$ and $a,b \upuparrows b',a'$.

(13) Let $O_1$ be an ordered affine space and $o$, $a$, $b$, $a'$, $b'$ be elements of the carrier of $O_1$. If not $\mathbf{L}(o,a,b)$ and $o,a \upuparrows o,a'$ and $\mathbf{L}(o,b,b')$ and $a,b \parallel a',b'$, then $o,b \upuparrows o,b'$ and $a,b \upuparrows a',b'$.

(14) For every ordered affine space $O_2$ such that $O_2$ satisfies $\mathbf{DES}_1$ holds $\Lambda(O_2)$ satisfies **DES'**.

(15) Let $V$ be a real linear space, $o$, $u$, $v$, $u_1$, $v_1$ be vectors of $V$, and $r$ be a real number. Suppose $o - u = r \cdot (u_1 - o)$ and $r \neq 0$ and $o,v \parallel o,v_1$ and $o,u \nparallel o,v$ and $u,v \parallel u_1,v_1$. Then $v_1 = u_1 + (-r)^{-1} \cdot (v-u)$ and $v_1 = o + (-r)^{-1} \cdot (v-o)$ and $v - u = (-r) \cdot (v_1 - u_1)$.

(17)[5] Let $V$ be a real linear space and $O_1$ be an ordered affine space. If $O_1 = \mathrm{OASpace}\,V$, then $O_1$ satisfies $\mathbf{DES}_1$.

(18) Let $V$ be a real linear space and $O_1$ be an ordered affine space. Suppose $O_1 = \mathrm{OASpace}\,V$. Then $O_1$ satisfies $\mathbf{DES}_1$ and $O_1$ satisfies **DES**.

(19) Let $V$ be a real linear space and $O_1$ be an ordered affine space. If $O_1 = \mathrm{OASpace}\,V$, then $\Lambda(O_1)$ satisfies **PAP'**.

(20) Let $V$ be a real linear space and $O_1$ be an ordered affine space. If $O_1 = \mathrm{OASpace}\,V$, then $\Lambda(O_1)$ satisfies **DES'**.

(21) For every affine space $A_1$ such that $A_1$ satisfies **DES'** holds $A_1$ satisfies **TDES'**.

[3] The definitions (Def. 6)–(Def. 10) have been removed.
[4] The propositions (7)–(10) have been removed.
[5] The proposition (16) has been removed.

(22) Let $V$ be a real linear space and $O_1$ be an ordered affine space. If $O_1 = \mathrm{OASpace}\, V$, then $\Lambda(O_1)$ satisfies **TDES'**.

(23) Let $V$ be a real linear space and $O_1$ be an ordered affine space. If $O_1 = \mathrm{OASpace}\, V$, then $\Lambda(O_1)$ satisfies **des'**.

(24) For every ordered affine space $O_1$ holds $\Lambda(O_1)$ satisfies Fano Axiom.

Let us note that there exists an ordered affine space which is Pappian, Desarguesian, Moufangian, and translation.

Let us note that there exists an affine plane which is strict, Fanoian, Pappian, Desarguesian, Moufangian, and translational.

Let us observe that there exists an affine space which is strict, Fanoian, Pappian, Desarguesian, Moufangian, and translational.

Let $O_1$ be an ordered affine space. One can check that $\Lambda(O_1)$ is Fanoian.

Let $O_1$ be a Pappian ordered affine space. One can verify that $\Lambda(O_1)$ is Pappian.

Let $O_1$ be a Desarguesian ordered affine space. Note that $\Lambda(O_1)$ is Desarguesian.

Let $O_1$ be a Moufangian ordered affine space. One can verify that $\Lambda(O_1)$ is Moufangian.

Let $O_1$ be a translation ordered affine space. Observe that $\Lambda(O_1)$ is translational.

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