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Some Elementary Notions of the Theory of Petri Nets

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Summary. Some fundamental notions of the theory of Petri nets are described in Mizar formalism. A Petri net is defined as a triple of the form $\langle$places, transitions, flow$\rangle$ with places and transitions being disjoint sets and flow being a relation included in places $\times$ transitions.

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The articles [2], [1], [3], and [4] provide the notation and terminology for this paper.

We introduce nets which are systems

$\langle$ places, transitions, a flow relation $\rangle$,

where the places and the transitions constitute sets and the flow relation is a binary relation.

In the sequel x, y are sets and N is a net.

Let N be a net. We say that N is Petri if and only if the conditions (Def. 1) are satisfied.

(Def. 1)(i) The places of N misses the transitions of N, and

(ii) the flow relation of $N \subseteq [$:the places of N, the transitions of N:$] \cup [$:the transitions of N, the places of N:$]$.

We introduce N is a Petri net as a synonym of N is Petri.

Let N be a net. The functor Elements(N) is defined by:

(Def. 2) Elements$(N) =$ (the places of $N) \cup$ (the transitions of N).

We now state several propositions:

(4)[1] The places of $N \subseteq$ Elements(N).

(5) The transitions of $N \subseteq$ Elements(N).

(6) $x \in$ Elements(N) iff $x \in$ the places of N or $x \in$ the transitions of N.

(7) Suppose Elements$(N) \neq \emptyset$. Suppose x is an element of Elements(N). Then x is an element of the places of N and an element of the transitions of N.

(8) If x is an element of the places of N and the places of $N \neq \emptyset$, then x is an element of Elements(N).

(9) If x is an element of the transitions of N and the transitions of $N \neq \emptyset$, then x is an element of Elements(N).

[1] The propositions (1)–(3) have been removed.

Let us observe that $\langle \emptyset, \emptyset, \emptyset \rangle$ is Petri.
Let us mention that there exists a net which is strict and Petri.
A Petri net is a Petri net.
The following propositions are true:

(11)[2] For every Petri net N holds $x \notin$ the places of N or $x \notin$ the transitions of N.

(12) Let N be a Petri net. Suppose $\langle x, y \rangle \in$ the flow relation of N and $x \in$ the transitions of N. Then $y \in$ the places of N.

(13) Let N be a Petri net. Suppose $\langle x, y \rangle \in$ the flow relation of N and $y \in$ the transitions of N. Then $x \in$ the places of N.

(14) Let N be a Petri net. Suppose $\langle x, y \rangle \in$ the flow relation of N and $x \in$ the places of N. Then $y \in$ the transitions of N.

(15) Let N be a Petri net. Suppose $\langle x, y \rangle \in$ the flow relation of N and $y \in$ the places of N. Then $x \in$ the transitions of N.

Let N be a Petri net and let us consider x, y. We say that x is a pre-element of y in N if and only if:

(Def. 5)[3] $\langle y, x \rangle \in$ the flow relation of N and $x \in$ the transitions of N.

We say that x is a post-element of y in N if and only if:

(Def. 6) $\langle x, y \rangle \in$ the flow relation of N and $x \in$ the transitions of N.

Let N be a net and let x be an element of Elements(N). The functor Pre(N,x) is defined as follows:

(Def. 7) $y \in$ Pre(N,x) iff $y \in$ Elements(N) and $\langle y, x \rangle \in$ the flow relation of N.

The functor Post(N,x) is defined by:

(Def. 8) $y \in$ Post(N,x) iff $y \in$ Elements(N) and $\langle x, y \rangle \in$ the flow relation of N.

Next we state several propositions:

(16) For every Petri net N and for every element x of Elements(N) holds Pre(N,x) $\subseteq$ Elements(N).

(17) For every Petri net N and for every element x of Elements(N) holds Pre(N,x) $\subseteq$ Elements(N).

(18) For every Petri net N and for every element x of Elements(N) holds Post(N,x) $\subseteq$ Elements(N).

(19) For every Petri net N and for every element x of Elements(N) holds Post(N,x) $\subseteq$ Elements(N).

(20) Let N be a Petri net and y be an element of Elements(N). Suppose $y \in$ the transitions of N. Then $x \in$ Pre(N,y) if and only if y is a pre-element of x in N.

(21) Let N be a Petri net and y be an element of Elements(N). Suppose $y \in$ the transitions of N. Then $x \in$ Post(N,y) if and only if y is a post-element of x in N.

Let N be a Petri net and let x be an element of Elements(N). Let us assume that Elements(N) $\neq \emptyset$. The functor enter(N,x) is defined by:

[2] The proposition (10) has been removed.
[3] The definitions (Def. 3) and (Def. 4) have been removed.

(Def. 9) If $x \in$ the places of N, then $\mathrm{enter}(N,x) = \{x\}$ and if $x \in$ the transitions of N, then $\mathrm{enter}(N,x) = \mathrm{Pre}(N,x)$.

One can prove the following three propositions:

(22) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{enter}(N,x) = \{x\}$ or $\mathrm{enter}(N,x) = \mathrm{Pre}(N,x)$.

(23) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{enter}(N,x) \subseteq \mathrm{Elements}(N)$.

(24) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{enter}(N,x) \subseteq \mathrm{Elements}(N)$.

Let N be a Petri net and let x be an element of $\mathrm{Elements}(N)$. Let us assume that $\mathrm{Elements}(N) \neq \emptyset$. The functor $\mathrm{exit}(N,x)$ yields a set and is defined by:

(Def. 10) If $x \in$ the places of N, then $\mathrm{exit}(N,x) = \{x\}$ and if $x \in$ the transitions of N, then $\mathrm{exit}(N,x) = \mathrm{Post}(N,x)$.

We now state three propositions:

(25) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{exit}(N,x) = \{x\}$ or $\mathrm{exit}(N,x) = \mathrm{Post}(N,x)$.

(26) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{exit}(N,x) \subseteq \mathrm{Elements}(N)$.

(27) For every Petri net N and for every element x of $\mathrm{Elements}(N)$ such that $\mathrm{Elements}(N) \neq \emptyset$ holds $\mathrm{exit}(N,x) \subseteq \mathrm{Elements}(N)$.

Let N be a Petri net and let x be an element of $\mathrm{Elements}(N)$. The functor $\mathrm{field}(N,x)$ is defined as follows:

(Def. 11) $\mathrm{field}(N,x) = \mathrm{enter}(N,x) \cup \mathrm{exit}(N,x)$.

Let N be a net and let x be an element of the transitions of N. The functor $\mathrm{Prec}(N,x)$ is defined by:

(Def. 12) $y \in \mathrm{Prec}(N,x)$ iff $y \in$ the places of N and $\langle y, x \rangle \in$ the flow relation of N.

The functor $\mathrm{Postc}(N,x)$ is defined by:

(Def. 13) $y \in \mathrm{Postc}(N,x)$ iff $y \in$ the places of N and $\langle x, y \rangle \in$ the flow relation of N.

Let N be a Petri net and let X be a set. The functor $\mathrm{Entr}(N,X)$ is defined by:

(Def. 14) $x \in \mathrm{Entr}(N,X)$ iff $x \subseteq \mathrm{Elements}(N)$ and there exists an element y of $\mathrm{Elements}(N)$ such that $y \in X$ and $x = \mathrm{enter}(N,y)$.

The functor $\mathrm{Ext}(N,X)$ is defined as follows:

(Def. 15) $x \in \mathrm{Ext}(N,X)$ iff $x \subseteq \mathrm{Elements}(N)$ and there exists an element y of $\mathrm{Elements}(N)$ such that $y \in X$ and $x = \mathrm{exit}(N,y)$.

The following propositions are true:

(28) Let N be a Petri net, x be an element of $\mathrm{Elements}(N)$, and X be a set. If $\mathrm{Elements}(N) \neq \emptyset$ and $X \subseteq \mathrm{Elements}(N)$ and $x \in X$, then $\mathrm{enter}(N,x) \in \mathrm{Entr}(N,X)$.

(29) Let N be a Petri net, x be an element of $\mathrm{Elements}(N)$, and X be a set. If $\mathrm{Elements}(N) \neq \emptyset$ and $X \subseteq \mathrm{Elements}(N)$ and $x \in X$, then $\mathrm{exit}(N,x) \in \mathrm{Ext}(N,X)$.

Let N be a Petri net and let X be a set. The functor $\mathrm{Input}(N,X)$ is defined as follows:

(Def. 16) $\mathrm{Input}(N,X) = \bigcup \mathrm{Entr}(N,X)$.

The functor $\mathrm{Output}(N,X)$ is defined by:

(Def. 17) $\mathrm{Output}(N,X) = \bigcup \mathrm{Ext}(N,X)$.

We now state four propositions:

(30) Let N be a Petri net, given x, and X be a set. Suppose $\mathrm{Elements}(N) \neq \emptyset$ and $X \subseteq \mathrm{Elements}(N)$. Then $x \in \mathrm{Input}(N,X)$ if and only if there exists an element y of $\mathrm{Elements}(N)$ such that $y \in X$ and $x \in \mathrm{enter}(N,y)$.

(31) Let N be a Petri net, given x, and X be a set. Suppose $\mathrm{Elements}(N) \neq \emptyset$ and $X \subseteq \mathrm{Elements}(N)$. Then $x \in \mathrm{Output}(N,X)$ if and only if there exists an element y of $\mathrm{Elements}(N)$ such that $y \in X$ and $x \in \mathrm{exit}(N,y)$.

(32) Let N be a Petri net, X be a subset of $\mathrm{Elements}(N)$, and x be an element of $\mathrm{Elements}(N)$. Suppose $\mathrm{Elements}(N) \neq \emptyset$. Then $x \in \mathrm{Input}(N,X)$ if and only if one of the following conditions is satisfied:

(i) $x \in X$ and $x \in$ the places of N, or

(ii) there exists an element y of $\mathrm{Elements}(N)$ such that $y \in X$ and $y \in$ the transitions of N and y is a pre-element of x in N.

(33) Let N be a Petri net, X be a subset of $\mathrm{Elements}(N)$, and x be an element of $\mathrm{Elements}(N)$. Suppose $\mathrm{Elements}(N) \neq \emptyset$. Then $x \in \mathrm{Output}(N,X)$ if and only if one of the following conditions is satisfied:

(i) $x \in X$ and $x \in$ the places of N, or

(ii) there exists an element y of $\mathrm{Elements}(N)$ such that $y \in X$ and $y \in$ the transitions of N and y is a post-element of x in N.

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