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N-Tuples and Cartesian Products for $n = 7$[1]

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Summary. This article defines ordered n-tuples, projections and Cartesian products for $n = 7$. We prove many theorems concerning the basic properties of the n-tuples and Cartesian products that may be utilized in several further, more challenging applications. A few of these theorems are a strightforward consequence of the regularity axiom. The article originated as an upgrade of the article [5].

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The articles [4], [1], [6], [5], [2], and [3] provide the notation and terminology for this paper.

For simplicity, we adopt the following rules: x, x_1, x_2, x_3, x_4, x_5, x_6, x_7 are sets, y, y_1, y_2, y_3, y_4, y_5, y_6, y_7 are sets, X, X_1, X_2, X_3, X_4, X_5, X_6, X_7 are sets, Y, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}, Y_{11} are sets, Z is a set, x_8 is an element of X_1, x_9 is an element of X_2, x_{10} is an element of X_3, x_{11} is an element of X_4, x_{12} is an element of X_5, and x_{13} is an element of X_6.

Next we state two propositions:

(1) Suppose $X \neq \emptyset$. Then there exists Y such that

(i) $Y \in X$, and

(ii) for all Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10} such that $Y_1 \in Y_2$ and $Y_2 \in Y_3$ and $Y_3 \in Y_4$ and $Y_4 \in Y_5$ and $Y_5 \in Y_6$ and $Y_6 \in Y_7$ and $Y_7 \in Y_8$ and $Y_8 \in Y_9$ and $Y_9 \in Y_{10}$ and $Y_{10} \in Y$ holds Y_1 misses X.

(2) Suppose $X \neq \emptyset$. Then there exists Y such that

(i) $Y \in X$, and

(ii) for all Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}, Y_{11} such that $Y_1 \in Y_2$ and $Y_2 \in Y_3$ and $Y_3 \in Y_4$ and $Y_4 \in Y_5$ and $Y_5 \in Y_6$ and $Y_6 \in Y_7$ and $Y_7 \in Y_8$ and $Y_8 \in Y_9$ and $Y_9 \in Y_{10}$ and $Y_{10} \in Y_{11}$ and $Y_{11} \in Y$ holds Y_1 misses X.

Let us consider $x_1, x_2, x_3, x_4, x_5, x_6, x_7$. The functor $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7\rangle$ is defined as follows:

(Def. 1) $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7\rangle = \langle\langle x_1,x_2,x_3,x_4,x_5,x_6\rangle, x_7\rangle$.

We now state several propositions:

(3) $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7\rangle = \langle\langle\langle\langle\langle\langle x_1, x_2\rangle, x_3\rangle, x_4\rangle, x_5\rangle, x_6\rangle, x_7\rangle$.

(5)[1] $\langle x_1,x_2,x_3,x_4,x_5,x_6,x_7\rangle = \langle\langle x_1,x_2,x_3,x_4,x_5\rangle, x_6, x_7\rangle$.

[1] Supported by RPBP.III-24.C6.
[1] The proposition (4) has been removed.

(6) $\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle = \langle \langle x_1, x_2, x_3, x_4 \rangle, x_5, x_6, x_7 \rangle$.

(7) $\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle = \langle \langle x_1, x_2, x_3 \rangle, x_4, x_5, x_6, x_7 \rangle$.

(8) $\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle = \langle \langle x_1, x_2 \rangle, x_3, x_4, x_5, x_6, x_7 \rangle$.

(9) If $\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle = \langle y_1, y_2, y_3, y_4, y_5, y_6, y_7 \rangle$, then $x_1 = y_1$ and $x_2 = y_2$ and $x_3 = y_3$ and $x_4 = y_4$ and $x_5 = y_5$ and $x_6 = y_6$ and $x_7 = y_7$.

(10) If $X \neq \emptyset$, then there exists x such that $x \in X$ and it is not true that there exist x_1, x_2, x_3, x_4, x_5, x_6, x_7 such that $x_1 \in X$ or $x_2 \in X$ but $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$.

Let us consider X_1, X_2, X_3, X_4, X_5, X_6, X_7. The functor $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$ yields a set and is defined by:

(Def. 2) $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :] = [: [: X_1, X_2, X_3, X_4, X_5\ X_6 :], X_7 :]$.

The following propositions are true:

(11) $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :] = [: [: [: [: [: [: X_1, X_2 :], X_3 :], X_4 :], X_5 :], X_6 :], X_7 :]$.

(13)[2] $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :] = [: [: X_1, X_2, X_3, X_4, X_5 :], X_6, X_7 :]$.

(14) $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :] = [: [: X_1, X_2, X_3, X_4 :], X_5, X_6, X_7 :]$.

(15) $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :] = [: [: X_1, X_2, X_3 :], X_4, X_5, X_6, X_7 :]$.

(16) $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :] = [: [: X_1, X_2 :], X_3, X_4, X_5, X_6\ X_7 :]$.

(17) $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ iff $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :] \neq \emptyset$.

(18) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$. Suppose $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :] = [: Y_1, Y_2, Y_3, Y_4, Y_5\ Y_6\ Y_7 :]$. Then $X_1 = Y_1$ and $X_2 = Y_2$ and $X_3 = Y_3$ and $X_4 = Y_4$ and $X_5 = Y_5$ and $X_6 = Y_6$ and $X_7 = Y_7$.

(19) If $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :] \neq \emptyset$ and $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :] = [: Y_1, Y_2, Y_3, Y_4, Y_5\ Y_6\ Y_7 :]$, then $X_1 = Y_1$ and $X_2 = Y_2$ and $X_3 = Y_3$ and $X_4 = Y_4$ and $X_5 = Y_5$ and $X_6 = Y_6$ and $X_7 = Y_7$.

(20) If $[: X, X, X, X, X\ X\ X :] = [: Y, Y, Y, Y, Y\ Y\ Y :]$, then $X = Y$.

In the sequel x_{14} denotes an element of X_7.

One can prove the following proposition

(21) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$. Then there exist x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} such that $x = \langle x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} \rangle$.

Let us consider X_1, X_2, X_3, X_4, X_5, X_6, X_7. Let us assume that $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$. The functor $x_{\mathbf{1}}$ yielding an element of X_1 is defined as follows:

(Def. 3) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$, then $x_{\mathbf{1}} = x_1$.

The functor $x_{\mathbf{2}}$ yields an element of X_2 and is defined by:

(Def. 4) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$, then $x_{\mathbf{2}} = x_2$.

The functor $x_{\mathbf{3}}$ yields an element of X_3 and is defined as follows:

(Def. 5) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$, then $x_{\mathbf{3}} = x_3$.

The functor $x_{\mathbf{4}}$ yields an element of X_4 and is defined by:

[2] The proposition (12) has been removed.

(Def. 6) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$, then $x_{\mathbf{4}} = x_4$.

The functor $x_{\mathbf{5}}$ yields an element of X_5 and is defined by:

(Def. 7) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$, then $x_{\mathbf{5}} = x_5$.

The functor $x_{\mathbf{6}}$ yielding an element of X_6 is defined as follows:

(Def. 8) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$, then $x_{\mathbf{6}} = x_6$.

The functor $x_{\mathbf{7}}$ yields an element of X_7 and is defined by:

(Def. 9) If $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$, then $x_{\mathbf{7}} = x_7$.

We now state several propositions:

(22) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$ and given $x_1, x_2, x_3, x_4, x_5, x_6, x_7$. Suppose $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$. Then $x_{\mathbf{1}} = x_1$ and $x_{\mathbf{2}} = x_2$ and $x_{\mathbf{3}} = x_3$ and $x_{\mathbf{4}} = x_4$ and $x_{\mathbf{5}} = x_5$ and $x_{\mathbf{6}} = x_6$ and $x_{\mathbf{7}} = x_7$.

(23) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$. Then $x = \langle x_{\mathbf{1}}, x_{\mathbf{2}}, x_{\mathbf{3}}, x_{\mathbf{4}}, x_{\mathbf{5}}, x_{\mathbf{6}}, x_{\mathbf{7}} \rangle$.

(24) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$. Then $x_{\mathbf{1}} = ((((((x \textbf{ qua } \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}}$ and $x_{\mathbf{2}} = ((((((x \textbf{ qua } \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{3}} = (((((x \textbf{ qua } \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{4}} = ((((x \textbf{ qua } \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{5}} = (((x \textbf{ qua } \mathrm{set})_{\mathbf{1}})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{6}} = ((x \textbf{ qua } \mathrm{set})_{\mathbf{1}})_{\mathbf{2}}$ and $x_{\mathbf{7}} = (x \textbf{ qua } \mathrm{set})_{\mathbf{2}}$.

(25) Suppose $X_1 \subseteq [: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$ or $X_1 \subseteq [: X_2, X_3, X_4, X_5, X_6\ X_7\ X_1 :]$ or $X_1 \subseteq [: X_3, X_4, X_5, X_6, X_7\ X_1\ X_2 :]$ or $X_1 \subseteq [: X_4, X_5, X_6, X_7, X_1\ X_2\ X_3 :]$ or $X_1 \subseteq [: X_5, X_6, X_7, X_1, X_2\ X_3\ X_4 :]$ or $X_1 \subseteq [: X_6, X_7, X_1, X_2, X_3\ X_4\ X_5 :]$ or $X_1 \subseteq [: X_7, X_1, X_2, X_3, X_4\ X_5\ X_6 :]$. Then $X_1 = \emptyset$.

(26) Suppose $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$ meets $[: Y_1, Y_2, Y_3, Y_4, Y_5\ Y_6\ Y_7 :]$. Then X_1 meets Y_1 and X_2 meets Y_2 and X_3 meets Y_3 and X_4 meets Y_4 and X_5 meets Y_5 and X_6 meets Y_6 and X_7 meets Y_7.

(27) $[: \{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}\ \{x_6\}\ \{x_7\} :] = \{\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle\}$.

For simplicity, we adopt the following rules: A_1 denotes a subset of X_1, A_2 denotes a subset of X_2, A_3 denotes a subset of X_3, A_4 denotes a subset of X_4, A_5 denotes a subset of X_5, A_6 denotes a subset of X_6, A_7 denotes a subset of X_7, and x denotes an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$.

One can prove the following propositions:

(28) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$. Let given $x_1, x_2, x_3, x_4, x_5, x_6, x_7$. Suppose $x = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$. Then $x_{\mathbf{1}} = x_1$ and $x_{\mathbf{2}} = x_2$ and $x_{\mathbf{3}} = x_3$ and $x_{\mathbf{4}} = x_4$ and $x_{\mathbf{5}} = x_5$ and $x_{\mathbf{6}} = x_6$ and $x_{\mathbf{7}} = x_7$.

(29) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and for all $x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}$ such that $x = \langle x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} \rangle$ holds $y_1 = x_8$. Then $y_1 = x_{\mathbf{1}}$.

(30) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and for all $x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}$ such that $x = \langle x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} \rangle$ holds $y_2 = x_9$. Then $y_2 = x_{\mathbf{2}}$.

(31) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and for all $x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}$ such that $x = \langle x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} \rangle$ holds $y_3 = x_{10}$. Then $y_3 = x_{\mathbf{3}}$.

(32) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and for all $x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}$ such that $x = \langle x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} \rangle$ holds $y_4 = x_{11}$. Then $y_4 = x_{\mathbf{4}}$.

(33) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and for all x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} such that $x = \langle x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} \rangle$ holds $y_5 = x_{12}$. Then $y_5 = x_{\mathbf{5}}$.

(34) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and for all x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} such that $x = \langle x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} \rangle$ holds $y_6 = x_{13}$. Then $y_6 = x_{\mathbf{6}}$.

(35) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and for all x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} such that $x = \langle x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14} \rangle$ holds $y_7 = x_{14}$. Then $y_7 = x_{\mathbf{7}}$.

(36) Suppose $y \in [: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$. Then there exist x_1, x_2, x_3, x_4, x_5, x_6, x_7 such that $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$ and $x_4 \in X_4$ and $x_5 \in X_5$ and $x_6 \in X_6$ and $x_7 \in X_7$ and $y = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$.

(37) $\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle \in [: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$ iff $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$ and $x_4 \in X_4$ and $x_5 \in X_5$ and $x_6 \in X_6$ and $x_7 \in X_7$.

(38) Suppose that for every y holds $y \in Z$ iff there exist x_1, x_2, x_3, x_4, x_5, x_6, x_7 such that $x_1 \in X_1$ and $x_2 \in X_2$ and $x_3 \in X_3$ and $x_4 \in X_4$ and $x_5 \in X_5$ and $x_6 \in X_6$ and $x_7 \in X_7$ and $y = \langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$. Then $Z = [: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$.

(39) Suppose $X_1 \neq \emptyset$ and $X_2 \neq \emptyset$ and $X_3 \neq \emptyset$ and $X_4 \neq \emptyset$ and $X_5 \neq \emptyset$ and $X_6 \neq \emptyset$ and $X_7 \neq \emptyset$ and $Y_1 \neq \emptyset$ and $Y_2 \neq \emptyset$ and $Y_3 \neq \emptyset$ and $Y_4 \neq \emptyset$ and $Y_5 \neq \emptyset$ and $Y_6 \neq \emptyset$ and $Y_7 \neq \emptyset$. Let x be an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$ and y be an element of $[: Y_1, Y_2, Y_3, Y_4, Y_5\ Y_6\ Y_7 :]$. Suppose $x = y$. Then $x_{\mathbf{1}} = y_{\mathbf{1}}$ and $x_{\mathbf{2}} = y_{\mathbf{2}}$ and $x_{\mathbf{3}} = y_{\mathbf{3}}$ and $x_{\mathbf{4}} = y_{\mathbf{4}}$ and $x_{\mathbf{5}} = y_{\mathbf{5}}$ and $x_{\mathbf{6}} = y_{\mathbf{6}}$ and $x_{\mathbf{7}} = y_{\mathbf{7}}$.

(40) Let x be an element of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$. Suppose $x \in [: A_1, A_2, A_3, A_4, A_5\ A_6\ A_7 :]$. Then $x_{\mathbf{1}} \in A_1$ and $x_{\mathbf{2}} \in A_2$ and $x_{\mathbf{3}} \in A_3$ and $x_{\mathbf{4}} \in A_4$ and $x_{\mathbf{5}} \in A_5$ and $x_{\mathbf{6}} \in A_6$ and $x_{\mathbf{7}} \in A_7$.

(41) If $X_1 \subseteq Y_1$ and $X_2 \subseteq Y_2$ and $X_3 \subseteq Y_3$ and $X_4 \subseteq Y_4$ and $X_5 \subseteq Y_5$ and $X_6 \subseteq Y_6$ and $X_7 \subseteq Y_7$, then $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :] \subseteq [: Y_1, Y_2, Y_3, Y_4, Y_5\ Y_6\ Y_7 :]$.

(42) $[: A_1, A_2, A_3, A_4, A_5\ A_6\ A_7 :]$ is a subset of $[: X_1, X_2, X_3, X_4, X_5\ X_6\ X_7 :]$.

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