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Definitions and Basic Properties of Boolean and Union of Many Sorted Sets

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Summary. In the first part of this article I have proved theorems about boolean of many sorted sets which are corresponded to theorems about boolean of sets, whereas the second part of this article contains propositions about union of many sorted sets. Boolean as well as union of many sorted sets are defined as boolean and union on every sorts.

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The articles [9], [3], [11], [1], [2], [5], [4], [10], [7], [6], and [8] provide the notation and terminology for this paper.

1. Boolean of Many Sorted Sets

We adopt the following rules: x, y, I denote sets and A, B, X, Y denote many sorted sets indexed by I.

Let us consider I, A. The functor 2^A yielding a many sorted set indexed by I is defined by:

(Def. 1) For every set i such that $i \in I$ holds $2^A(i) = 2^{A(i)}$.

Let us consider I, A. Note that 2^A is non-empty.

Next we state a number of propositions:

(1) $X = 2^Y$ iff for every A holds $A \in X$ iff $A \subseteq Y$.

(2) $2^{\mathbf{0}_I} = I \longmapsto \{\emptyset\}$.

(3) $2^{I \longmapsto x} = I \longmapsto 2^x$.

(4) $2^{I \longmapsto \{x\}} = I \longmapsto \{\emptyset, \{x\}\}$.

(5) $\mathbf{0}_I \subseteq A$.

(6) If $A \subseteq B$, then $2^A \subseteq 2^B$.

(7) $2^A \cup 2^B \subseteq 2^{A \cup B}$.

(8) If $2^A \cup 2^B = 2^{A \cup B}$, then for every set i such that $i \in I$ holds $A(i)$ and $B(i)$ are $\subseteq$-comparable.

(9) $2^{A \cap B} = 2^A \cap 2^B$.

(10) $2^{A \setminus B} \subseteq (I \longmapsto \{\emptyset\}) \cup (2^A \setminus 2^B)$.

(11) $X \subseteq A \setminus B$ iff $X \subseteq A$ and X misses B.

(12) $2^{A \setminus B} \cup 2^{B \setminus A} \subseteq 2^{A \dot{-} B}$.

(13) $X \subseteq A \dot{-} B$ iff $X \subseteq A \cup B$ and X misses $A \cap B$.

(15)[1] If $X \subseteq A$ or $Y \subseteq A$, then $X \cap Y \subseteq A$.

(16) If $X \subseteq A$, then $X \setminus Y \subseteq A$.

(17) If $X \subseteq A$ and $Y \subseteq A$, then $X \dot{-} Y \subseteq A$.

(18) $[\![X, Y]\!] \subseteq 2^{2^{X \cup Y}}$.

(19) $X \subseteq A$ iff $X \in 2^A$.

(20) $\mathrm{MSFuncs}(A, B) \subseteq 2^{[\![A, B]\!]}$.

2. Union of Many Sorted Sets

Let us consider I, A. The functor $\bigcup A$ yielding a many sorted set indexed by I is defined by:

(Def. 2) For every set i such that $i \in I$ holds $(\bigcup A)(i) = \bigcup A(i)$.

Let us consider I. Note that $\bigcup(\mathbf{0}_I)$ is empty yielding.
Next we state a number of propositions:

(21) $A \in \bigcup X$ iff there exists Y such that $A \in Y$ and $Y \in X$.

(22) $\bigcup(\mathbf{0}_I) = \mathbf{0}_I$.

(23) $\bigcup(I \longmapsto x) = I \longmapsto \bigcup x$.

(24) $\bigcup(I \longmapsto \{x\}) = I \longmapsto x$.

(25) $\bigcup(I \longmapsto \{\{x\}, \{y\}\}) = I \longmapsto \{x, y\}$.

(26) If $X \in A$, then $X \subseteq \bigcup A$.

(27) If $A \subseteq B$, then $\bigcup A \subseteq \bigcup B$.

(28) $\bigcup(A \cup B) = \bigcup A \cup \bigcup B$.

(29) $\bigcup(A \cap B) \subseteq \bigcup A \cap \bigcup B$.

(30) $\bigcup(2^A) = A$.

(31) $A \subseteq 2^{\bigcup A}$.

(32) If $\bigcup Y \subseteq A$ and $X \in Y$, then $X \subseteq A$.

(33) Let Z be a many sorted set indexed by I and A be a non-empty many sorted set indexed by I. Suppose that for every many sorted set X indexed by I such that $X \in A$ holds $X \subseteq Z$. Then $\bigcup A \subseteq Z$.

(34) Let B be a many sorted set indexed by I and A be a non-empty many sorted set indexed by I. Suppose that for every many sorted set X indexed by I such that $X \in A$ holds $X \cap B = \mathbf{0}_I$. Then $\bigcup A \cap B = \mathbf{0}_I$.

(35) Let A, B be many sorted sets indexed by I. Suppose $A \cup B$ is non-empty. Suppose that for all many sorted sets X, Y indexed by I such that $X \neq Y$ and $X \in A \cup B$ and $Y \in A \cup B$ holds $X \cap Y = \mathbf{0}_I$. Then $\bigcup(A \cap B) = \bigcup A \cap \bigcup B$.

[1] The proposition (14) has been removed.

(36) Let A, X be many sorted sets indexed by I and B be a non-empty many sorted set indexed by I. Suppose $X \subseteq \bigcup(A \cup B)$ and for every many sorted set Y indexed by I such that $Y \in B$ holds $Y \cap X = \mathbf{0}_I$. Then $X \subseteq \bigcup A$.

(37) Let A be a locally-finite non-empty many sorted set indexed by I. Suppose that for all many sorted sets X, Y indexed by I such that $X \in A$ and $Y \in A$ holds $X \subseteq Y$ or $Y \subseteq X$. Then $\bigcup A \in A$.

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