JOURNAL OF FORMALIZED MATHEMATICS
Volume 2, Released 1990, Published 2003
Inst. of Computer Science, Univ. of Białystok

Linear Independence in Left Module over Domain[1]

Michał Muzalewski
Warsaw University
Białystok

Wojciech Skaba
Nicolaus Copernicus University
Toruń

Summary. Notion of a submodule generated by a set of vectors and linear independence of a set of vectors. A few theorems originated as a generalization of the theorems from the article [10].

MML Identifier: LMOD_5.

WWW: http://mizar.org/JFM/Vol2/lmod_5.html

The articles [7], [14], [4], [15], [2], [3], [8], [1], [9], [5], [6], [13], [12], and [11] provide the notation and terminology for this paper.

For simplicity, we use the following convention: x is a set, R is a ring, V is a left module over R, v, v_1, v_2 are vectors of V, and A, B are subsets of V.

Let R be a non empty double loop structure, let V be a non empty vector space structure over R, and let I_1 be a subset of V. We say that I_1 is linearly independent if and only if:

(Def. 1) For every linear combination l of I_1 such that $\sum l = 0_V$ holds the support of $l = \emptyset$.

We introduce I_1 is linearly dependent as an antonym of I_1 is linearly independent.

Next we state several propositions:

(2)[1] If $A \subseteq B$ and B is linearly independent, then A is linearly independent.

(3) If $0_R \neq \mathbf{1}_R$ and A is linearly independent, then $0_V \notin A$.

(4) $\emptyset_{\text{the carrier of } V}$ is linearly independent.

(5) If $0_R \neq \mathbf{1}_R$ and $\{v_1, v_2\}$ is linearly independent, then $v_1 \neq 0_V$ and $v_2 \neq 0_V$.

(6) If $0_R \neq \mathbf{1}_R$, then $\{v, 0_V\}$ is linearly dependent and $\{0_V, v\}$ is linearly dependent.

(7) Let R be an integral domain, V be a left module over R, L be a linear combination of V, and a be a scalar of R. If $a \neq 0_R$, then the support of $a \cdot L =$ the support of L.

(8) Let R be an integral domain, V be a left module over R, L be a linear combination of V, and a be a scalar of R. Then $\sum(a \cdot L) = a \cdot \sum L$.

For simplicity, we adopt the following convention: R is an integral domain, V is a left module over R, A, B are subsets of V, and l is a linear combination of A.

Let us consider R, let us consider V, and let us consider A. The functor $\mathrm{Lin}(A)$ yielding a strict subspace of V is defined by:

(Def. 2) The carrier of $\mathrm{Lin}(A) = \{\sum l\}$.

[1] Supported by RPBP.III-24.C6.

[1] The proposition (1) has been removed.

© Association of Mizar Users

We now state a number of propositions:

(9) $x \in \mathrm{Lin}(A)$ iff there exists l such that $x = \sum l$.

(10) If $x \in A$, then $x \in \mathrm{Lin}(A)$.

(11) $\mathrm{Lin}(\emptyset_{\text{the carrier of } V}) = \mathbf{0}_V$.

(12) If $\mathrm{Lin}(A) = \mathbf{0}_V$, then $A = \emptyset$ or $A = \{0_V\}$.

(13) For every strict subspace W of V such that $0_R \neq \mathbf{1}_R$ and $A =$ the carrier of W holds $\mathrm{Lin}(A) = W$.

(14) Let V be a strict left module over R and A be a subset of V. If $0_R \neq \mathbf{1}_R$ and $A =$ the carrier of V, then $\mathrm{Lin}(A) = V$.

(15) If $A \subseteq B$, then $\mathrm{Lin}(A)$ is a subspace of $\mathrm{Lin}(B)$.

(16) For every strict left module V over R and for all subsets A, B of V such that $\mathrm{Lin}(A) = V$ and $A \subseteq B$ holds $\mathrm{Lin}(B) = V$.

(17) $\mathrm{Lin}(A \cup B) = \mathrm{Lin}(A) + \mathrm{Lin}(B)$.

(18) $\mathrm{Lin}(A \cap B)$ is a subspace of $\mathrm{Lin}(A) \cap \mathrm{Lin}(B)$.

REFERENCES

[1] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/finseq_1.html.

[2] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[3] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[4] Agata Darmochwał. Finite sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/finset_1.html.

[5] Eugeniusz Kusak, Wojciech Leończuk, and Michał Muzalewski. Abelian groups, fields and vector spaces. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/vectsp_1.html.

[6] Michał Muzalewski. Construction of rings and left-, right-, and bi-modules over a ring. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/vectsp_2.html.

[7] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[8] Andrzej Trybulec. Function domains and Frænkel operator. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/fraenkel.html.

[9] Wojciech A. Trybulec. Vectors in real linear space. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/rlvect_1.html.

[10] Wojciech A. Trybulec. Basis of vector space. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/vectsp_7.html.

[11] Wojciech A. Trybulec. Linear combinations in vector space. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/vectsp_6.html.

[12] Wojciech A. Trybulec. Operations on subspaces in vector space. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/vectsp_5.html.

[13] Wojciech A. Trybulec. Subspaces and cosets of subspaces in vector space. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/vectsp_4.html.

[14] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[15] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_1.html.

Received October 22, 1990

Published January 2, 2004