JOURNAL OF FORMALIZED MATHEMATICS
Volume 11, Released 1999, Published 2003
Inst. of Computer Science, Univ. of Białystok

Gauges

Czesław Byliński
University of Białystok

MML Identifier: JORDAN8.

WWW: http://mizar.org/JFM/Vol11/jordan8.html

The articles [19], [5], [10], [1], [16], [18], [21], [4], [2], [3], [20], [12], [11], [17], [7], [8], [9], [13], [14], [6], and [15] provide the notation and terminology for this paper.

For simplicity, we use the following convention: i, i_1, i_2, j, j_1, j_2, k, m, n are natural numbers, D is a set, f is a finite sequence of elements of D, and G is a matrix over D.

One can prove the following propositions:

(1) If $\mathrm{len}\, f \geq 2$, then $f{\restriction}2 = \langle f_1, f_2 \rangle$.

(2) If $k+1 \leq \mathrm{len}\, f$, then $f{\restriction}(k+1) = (f{\restriction}k) \frown \langle f_{k+1} \rangle$.

(3) ε_D is a sequence which elements belong to G.

(5)[1] Let D be a non empty set, f be a finite sequence of elements of D, and G be a matrix over D. Suppose f is a sequence which elements belong to G. Then $f_{\lfloor m}$ is a sequence which elements belong to G.

(6) Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ and f is a sequence which elements belong to G. Then there exist natural numbers i_1, j_1, i_2, j_2 such that

(i) $\langle i_1, j_1 \rangle \in$ the indices of G,

(ii) $f_k = G \circ (i_1, j_1)$,

(iii) $\langle i_2, j_2 \rangle \in$ the indices of G,

(iv) $f_{k+1} = G \circ (i_2, j_2)$, and

(v) $i_1 = i_2$ and $j_1 + 1 = j_2$ or $i_1 + 1 = i_2$ and $j_1 = j_2$ or $i_1 = i_2 + 1$ and $j_1 = j_2$ or $i_1 = i_2$ and $j_1 = j_2 + 1$.

In the sequel G is a Go-board and p is a point of $\mathcal{E}^2_{\mathrm{T}}$.

Next we state several propositions:

(7) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose f is a sequence which elements belong to G. Then f is standard and special.

(8) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $\mathrm{len}\, f \geq 2$ and f is a sequence which elements belong to G. Then f is non constant.

[1] The proposition (4) has been removed.

(9) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that

(i) f is a sequence which elements belong to G,

(ii) there exist i, j such that $\langle i, j\rangle \in$ the indices of G and $p = G \circ (i, j)$, and

(iii) for all i_1, j_1, i_2, j_2 such that $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $f_{\mathrm{len}\, f} = G \circ (i_1, j_1)$ and $p = G \circ (i_2, j_2)$ holds $|i_2 - i_1| + |j_2 - j_1| = 1$.

Then $f \frown \langle p\rangle$ is a sequence which elements belong to G.

(10) If $i+k < \mathrm{len}\, G$ and $1 \leq j$ and $j < \mathrm{width}\, G$ and $\mathrm{cell}(G, i, j)$ meets $\mathrm{cell}(G, i+k, j)$, then $k \leq 1$.

(11) For every non empty compact subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds C is vertical iff $\mathrm{E\text{-}bound}(C) \leq \mathrm{W\text{-}bound}(C)$.

(12) For every non empty compact subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds C is horizontal iff $\mathrm{N\text{-}bound}(C) \leq \mathrm{S\text{-}bound}(C)$.

Let C be a subset of $\mathcal{E}^2_{\mathrm{T}}$ and let n be a natural number. The functor $\mathrm{Gauge}(C,n)$ yields a matrix over $\mathcal{E}^2_{\mathrm{T}}$ and is defined by the conditions (Def. 1).

(Def. 1)(i) $\mathrm{len}\, \mathrm{Gauge}(C,n) = 2^n + 3$,

(ii) $\mathrm{len}\, \mathrm{Gauge}(C,n) = \mathrm{width}\, \mathrm{Gauge}(C,n)$, and

(iii) for all i, j such that $\langle i, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ holds $\mathrm{Gauge}(C,n) \circ (i, j) = [\mathrm{W\text{-}bound}(C) + \frac{\mathrm{E\text{-}bound}(C) - \mathrm{W\text{-}bound}(C)}{2^n} \cdot (i-2), \mathrm{S\text{-}bound}(C) + \frac{\mathrm{N\text{-}bound}(C) - \mathrm{S\text{-}bound}(C)}{2^n} \cdot (j-2)]$.

Let C be a non empty subset of $\mathcal{E}^2_{\mathrm{T}}$ and let n be a natural number. Note that $\mathrm{Gauge}(C,n)$ is non empty yielding, line **X**-constant, and column **Y**-constant.

Let C be a compact non vertical non horizontal non empty subset of $\mathcal{E}^2_{\mathrm{T}}$ and let us consider n. Note that $\mathrm{Gauge}(C,n)$ is line **Y**-increasing and column **X**-increasing.

In the sequel T is a non empty subset of $\mathcal{E}^2_{\mathrm{T}}$.

Next we state several propositions:

(13) $\mathrm{len}\, \mathrm{Gauge}(T,n) \geq 4$.

(14) If $1 \leq j$ and $j \leq \mathrm{len}\, \mathrm{Gauge}(T,n)$, then $(\mathrm{Gauge}(T,n) \circ (2, j))_{\mathbf{1}} = \mathrm{W\text{-}bound}(T)$.

(15) If $1 \leq j$ and $j \leq \mathrm{len}\, \mathrm{Gauge}(T,n)$, then $(\mathrm{Gauge}(T,n) \circ (\mathrm{len}\, \mathrm{Gauge}(T,n) -' 1, j))_{\mathbf{1}} = \mathrm{E\text{-}bound}(T)$.

(16) If $1 \leq i$ and $i \leq \mathrm{len}\, \mathrm{Gauge}(T,n)$, then $(\mathrm{Gauge}(T,n) \circ (i, 2))_{\mathbf{2}} = \mathrm{S\text{-}bound}(T)$.

(17) If $1 \leq i$ and $i \leq \mathrm{len}\, \mathrm{Gauge}(T,n)$, then $(\mathrm{Gauge}(T,n) \circ (i, \mathrm{len}\, \mathrm{Gauge}(T,n) -' 1))_{\mathbf{2}} = \mathrm{N\text{-}bound}(T)$.

In the sequel C denotes a compact non vertical non horizontal non empty subset of $\mathcal{E}^2_{\mathrm{T}}$.

The following four propositions are true:

(18) If $i \leq \mathrm{len}\, \mathrm{Gauge}(C,n)$, then $\mathrm{cell}(\mathrm{Gauge}(C,n), i, \mathrm{len}\, \mathrm{Gauge}(C,n))$ misses C.

(19) If $j \leq \mathrm{len}\, \mathrm{Gauge}(C,n)$, then $\mathrm{cell}(\mathrm{Gauge}(C,n), \mathrm{len}\, \mathrm{Gauge}(C,n), j)$ misses C.

(20) If $i \leq \mathrm{len}\, \mathrm{Gauge}(C,n)$, then $\mathrm{cell}(\mathrm{Gauge}(C,n), i, 0)$ misses C.

(21) If $j \leq \mathrm{len}\, \mathrm{Gauge}(C,n)$, then $\mathrm{cell}(\mathrm{Gauge}(C,n), 0, j)$ misses C.

REFERENCES

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/nat_1.html.

[2] Grzegorz Bancerek. Countable sets and Hessenberg's theorem. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/card_4.html.

[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/finseq_1.html.

[4] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[5] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[6] Czesław Byliński and Piotr Rudnicki. Bounding boxes for compact sets in $\mathcal{E}^2$. *Journal of Formalized Mathematics*, 9, 1997. http://mizar.org/JFM/Vol9/pscomp_1.html.

[7] Agata Darmochwał. Compact spaces. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/compts_1.html.

[8] Agata Darmochwał. The Euclidean space. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/euclid.html.

[9] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\rm T}$. Arcs, line segments and special polygonal arcs. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/topreal1.html.

[10] Krzysztof Hryniewiecki. Basic properties of real numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/real_1.html.

[11] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/matrix_1.html.

[12] Jarosław Kotowicz. Functions and finite sequences of real numbers. *Journal of Formalized Mathematics*, 5, 1993. http://mizar.org/JFM/Vol5/rfinseq.html.

[13] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-Board — part I. *Journal of Formalized Mathematics*, 4, 1992. http://mizar.org/JFM/Vol4/goboard1.html.

[14] Yatsuka Nakamura and Czesław Byliński. Extremal properties of vertices on special polygons, part I. *Journal of Formalized Mathematics*, 6, 1994. http://mizar.org/JFM/Vol6/sppol_1.html.

[15] Yatsuka Nakamura and Andrzej Trybulec. Decomposing a Go-Board into cells. *Journal of Formalized Mathematics*, 7, 1995. http://mizar.org/JFM/Vol7/goboard5.html.

[16] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Journal of Formalized Mathematics*, 5, 1993. http://mizar.org/JFM/Vol5/binarith.html.

[17] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/pre_topc.html.

[18] Jan Popiołek. Some properties of functions modul and signum. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/absvalue.html.

[19] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[20] Wojciech A. Trybulec. Pigeon hole principle. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/finseq_4.html.

[21] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_1.html.

Received January 22, 1999

Published January 2, 2004