JOURNAL OF FORMALIZED MATHEMATICS
Volume 14, Released 2002, Published 2003
Inst. of Computer Science, Univ. of Białystok

Some Remarks on Clockwise Oriented Sequences on Go-boards[1]

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Summary. The main goal of this paper is to present alternative characterizations of clockwise oriented sequences on Go-boards.

MML Identifier: JORDAN1I.

WWW: http://mizar.org/JFM/Vol14/jordan1i.html

The articles [22], [27], [12], [1], [3], [4], [2], [26], [13], [24], [21], [11], [20], [8], [9], [6], [25], [15], [10], [17], [23], [16], [5], [19], [18], [7], and [14] provide the notation and terminology for this paper.

1. PRELIMINARIES

In this paper i, j, k, n are natural numbers.

We now state several propositions:

(1) For all subsets A, B of $\mathcal{E}^n_{\rm T}$ such that A is Bounded or B is Bounded holds $A \cap B$ is Bounded.

(2) For all subsets A, B of $\mathcal{E}^n_{\rm T}$ such that A is not Bounded and B is Bounded holds $A \setminus B$ is not Bounded.

(3) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\rm T}$ holds $({\rm W}_{\rm min}(\widetilde{\mathcal{L}}({\rm Cage}(C,n)))) \hookleftarrow {\rm Cage}(C,n) > 1$.

(4) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\rm T}$ holds $({\rm E}_{\rm max}(\widetilde{\mathcal{L}}({\rm Cage}(C,n)))) \hookleftarrow {\rm Cage}(C,n) > 1$.

(5) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\rm T}$ holds $({\rm S}_{\rm max}(\widetilde{\mathcal{L}}({\rm Cage}(C,n)))) \hookleftarrow {\rm Cage}(C,n) > 1$.

2. ON BOUNDING POINTS OF CIRCULAR SEQUENCES

The following propositions are true:

(6) Let f be a non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\rm T}$. If $p \in {\rm rng}\, f$, then ${\rm leftcell}(f, p \hookleftarrow f) = {\rm leftcell}(f \circlearrowleft p, 1)$.

(7) Let f be a non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\rm T}$. If $p \in {\rm rng}\, f$, then ${\rm rightcell}(f, p \hookleftarrow f) = {\rm rightcell}(f \circlearrowleft p, 1)$.

[1]This work has been partially supported by CALCULEMUS grant HPRN-CT-2000-00102.

(8) For every compact connected non vertical non horizontal non empty subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{W}_{\min}(C) \in \mathrm{rightcell}(\mathrm{Cage}(C,n) \circlearrowleft \mathrm{W}_{\min}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))),1)$.

(9) For every compact connected non vertical non horizontal non empty subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{E}_{\max}(C) \in \mathrm{rightcell}(\mathrm{Cage}(C,n) \circlearrowleft \mathrm{E}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))),1)$.

(10) For every compact connected non vertical non horizontal non empty subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{S}_{\max}(C) \in \mathrm{rightcell}(\mathrm{Cage}(C,n) \circlearrowleft \mathrm{S}_{\max}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))),1)$.

3. On Clockwise Oriented Sequences

Next we state a number of propositions:

(11) Let f be a clockwise oriented non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\mathrm{T}}$. If $p_{\mathbf{1}} < \text{W-bound}(\widetilde{\mathcal{L}}(f))$, then $p \in \mathrm{LeftComp}(f)$.

(12) Let f be a clockwise oriented non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\mathrm{T}}$. If $p_{\mathbf{1}} > \text{E-bound}(\widetilde{\mathcal{L}}(f))$, then $p \in \mathrm{LeftComp}(f)$.

(13) Let f be a clockwise oriented non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\mathrm{T}}$. If $p_{\mathbf{2}} < \text{S-bound}(\widetilde{\mathcal{L}}(f))$, then $p \in \mathrm{LeftComp}(f)$.

(14) Let f be a clockwise oriented non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\mathrm{T}}$. If $p_{\mathbf{2}} > \text{N-bound}(\widetilde{\mathcal{L}}(f))$, then $p \in \mathrm{LeftComp}(f)$.

(15) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ and $\langle i,\, j\rangle \in$ the indices of G and $\langle i+1,\, j\rangle \in$ the indices of G and $f_k = G \circ (i+1, j)$ and $f_{k+1} = G \circ (i, j)$. Then $j < \mathrm{width}\, G$.

(16) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ and $\langle i,\, j\rangle \in$ the indices of G and $\langle i,\, j+1\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i, j+1)$. Then $i < \mathrm{len}\, G$.

(17) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ and $\langle i,\, j\rangle \in$ the indices of G and $\langle i+1,\, j\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i+1, j)$. Then $j > 1$.

(18) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ and $\langle i,\, j\rangle \in$ the indices of G and $\langle i,\, j+1\rangle \in$ the indices of G and $f_k = G \circ (i, j+1)$ and $f_{k+1} = G \circ (i, j)$. Then $i > 1$.

(19) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ and $\langle i,\, j\rangle \in$ the indices of G and $\langle i+1,\, j\rangle \in$ the indices of G and $f_k = G \circ (i+1, j)$ and $f_{k+1} = G \circ (i, j)$. Then $(f_k)_{\mathbf{2}} \neq \text{N-bound}(\widetilde{\mathcal{L}}(f))$.

(20) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ and $\langle i,\, j\rangle \in$ the indices of G and $\langle i,\, j+1\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i, j+1)$. Then $(f_k)_{\mathbf{1}} \neq \text{E-bound}(\widetilde{\mathcal{L}}(f))$.

(21) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ and $\langle i,\, j\rangle \in$ the indices of G and $\langle i+1,\, j\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i+1, j)$. Then $(f_k)_{\mathbf{2}} \neq \text{S-bound}(\widetilde{\mathcal{L}}(f))$.

(22) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and $\langle i, j\rangle \in$ the indices of G and $\langle i, j+1\rangle \in$ the indices of G and $f_k = G \circ (i, j+1)$ and $f_{k+1} = G \circ (i, j)$. Then $(f_k)_{\mathbf{1}} \neq \text{W-bound}(\widetilde{\mathcal{L}}(f))$.

(23) Let f be a clockwise oriented non constant standard special circular sequence, G be a Go-board, and k be a natural number. Suppose f is a sequence which elements belong to G and $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and $f_k = \mathrm{W_{min}}(\widetilde{\mathcal{L}}(f))$. Then there exist natural numbers i, j such that $\langle i, j\rangle \in$ the indices of G and $\langle i, j+1\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i, j+1)$.

(24) Let f be a clockwise oriented non constant standard special circular sequence, G be a Go-board, and k be a natural number. Suppose f is a sequence which elements belong to G and $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and $f_k = \mathrm{N_{min}}(\widetilde{\mathcal{L}}(f))$. Then there exist natural numbers i, j such that $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i+1, j)$.

(25) Let f be a clockwise oriented non constant standard special circular sequence, G be a Go-board, and k be a natural number. Suppose f is a sequence which elements belong to G and $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and $f_k = \mathrm{E_{max}}(\widetilde{\mathcal{L}}(f))$. Then there exist natural numbers i, j such that $\langle i, j+1\rangle \in$ the indices of G and $\langle i, j\rangle \in$ the indices of G and $f_k = G \circ (i, j+1)$ and $f_{k+1} = G \circ (i, j)$.

(26) Let f be a clockwise oriented non constant standard special circular sequence, G be a Go-board, and k be a natural number. Suppose f is a sequence which elements belong to G and $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and $f_k = \mathrm{S_{max}}(\widetilde{\mathcal{L}}(f))$. Then there exist natural numbers i, j such that $\langle i+1, j\rangle \in$ the indices of G and $\langle i, j\rangle \in$ the indices of G and $f_k = G \circ (i+1, j)$ and $f_{k+1} = G \circ (i, j)$.

(27) Let f be a non constant standard special circular sequence. Then f is clockwise oriented if and only if $(f \circlearrowleft \mathrm{W_{min}}(\widetilde{\mathcal{L}}(f)))_2 \in \mathrm{W_{most}}(\widetilde{\mathcal{L}}(f))$.

(28) Let f be a non constant standard special circular sequence. Then f is clockwise oriented if and only if $(f \circlearrowleft \mathrm{E_{max}}(\widetilde{\mathcal{L}}(f)))_2 \in \mathrm{E_{most}}(\widetilde{\mathcal{L}}(f))$.

(29) Let f be a non constant standard special circular sequence. Then f is clockwise oriented if and only if $(f \circlearrowleft \mathrm{S_{max}}(\widetilde{\mathcal{L}}(f)))_2 \in \mathrm{S_{most}}(\widetilde{\mathcal{L}}(f))$.

(30) Let C be a compact non vertical non horizontal non empty subset of $\mathcal{E}^2_{\mathrm{T}}$ satisfying conditions of simple closed curve and p be a point of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $p_{\mathbf{1}} = \frac{\text{W-bound}(C)+\text{E-bound}(C)}{2}$ and $i > 0$ and $1 \leq k$ and $k \leq \operatorname{width} \operatorname{Gauge}(C, i)$ and $\operatorname{Gauge}(C, i) \circ (\operatorname{Center} \operatorname{Gauge}(C, i), k) \in \operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C, i)))$ and $p_{\mathbf{2}} = \sup(\mathrm{proj2}^\circ(\mathcal{L}(\operatorname{Gauge}(C, 1) \circ (\operatorname{Center} \operatorname{Gauge}(C, 1), 1), \operatorname{Gauge}(C, i) \circ (\operatorname{Center} \operatorname{Gauge}(C, i), k)) \cap \operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C, i)))))$. Then there exists j such that $1 \leq j$ and $j \leq \operatorname{width} \operatorname{Gauge}(C, i)$ and $p = \operatorname{Gauge}(C, i) \circ (\operatorname{Center} \operatorname{Gauge}(C, i), j)$.

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Received March 1, 2002

Published January 2, 2004