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Some Properties of Cells and Arcs[1]

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The articles [28], [33], [2], [30], [15], [1], [34], [6], [4], [32], [7], [16], [5], [29], [27], [14], [3], [25], [26], [22], [11], [12], [9], [31], [18], [13], [21], [19], [20], [8], [24], [23], [10], and [17] provide the notation and terminology for this paper.

For simplicity, we use the following convention: E denotes a compact non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$, C denotes a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$, G denotes a Go-board, i, j, m, n denote natural numbers, and p denotes a point of $\mathcal{E}^2_{\rm T}$.

Let us note that every simple closed curve is non vertical and non horizontal.

Let T be a non empty topological space. One can check that there exists a union of components of T which is non empty.

We now state a number of propositions:

(1) Let T be a non empty topological space and A be a non empty union of components of T. If A is connected, then A is a component of T.

(2) For every finite sequence f holds f is empty iff $\mathrm{Rev}(f)$ is empty.

(3) Let D be a non empty set, f be a finite sequence of elements of D, and given i, j. If $1 \leq i$ and $i \leq \mathrm{len}\, f$ and $1 \leq j$ and $j \leq \mathrm{len}\, f$, then $\mathrm{mid}(f,i,j)$ is non empty.

(4) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2_{\rm T}$. If $1 \leq \mathrm{len}\, f$ and $p \in \widetilde{\mathcal{L}}(f)$, then $(\downharpoonleft f, p)(1) = f(1)$.

(5) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2_{\rm T}$. If f is a special sequence and $p \in \widetilde{\mathcal{L}}(f)$, then $(\downharpoonleft p, f)(\mathrm{len} \downharpoonleft p, f) = f(\mathrm{len}\, f)$.

(6) For every simple closed curve P holds $\mathrm{W_{max}}(P) \neq \mathrm{E_{max}}(P)$.

(7) Let D be a non empty set and f be a finite sequence of elements of D. If $1 \leq i$ and $i < \mathrm{len}\, f$, then $(\mathrm{mid}(f,i,\mathrm{len}\, f -' 1)) \frown \langle f_{\mathrm{len}\, f} \rangle = \mathrm{mid}(f,i,\mathrm{len}\, f)$.

(8) For all points p, q of $\mathcal{E}^2_{\rm T}$ such that $p \neq q$ and $\mathcal{L}(p,q)$ is vertical holds $\langle p,q \rangle$ is a special sequence.

(9) For all points p, q of $\mathcal{E}^2_{\rm T}$ such that $p \neq q$ and $\mathcal{L}(p,q)$ is horizontal holds $\langle p,q \rangle$ is a special sequence.

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(10) Let p, q be finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$ and v be a point of $\mathcal{E}^2_{\mathrm{T}}$. If p is in the area of q, then $p \circlearrowleft v$ is in the area of q.

(11) For every non trivial finite sequence p of elements of $\mathcal{E}^2_{\mathrm{T}}$ and for every point v of $\mathcal{E}^2_{\mathrm{T}}$ holds $p \circlearrowleft v$ is in the area of p.

(12) For every finite sequence f holds $\operatorname{Center} f \geq 1$.

(13) For every finite sequence f such that $\operatorname{len} f \geq 1$ holds $\operatorname{Center} f \leq \operatorname{len} f$.

(14) $\operatorname{Center} G \leq \operatorname{len} G$.

(15) For every finite sequence f such that $\operatorname{len} f \geq 2$ holds $\operatorname{Center} f > 1$.

(16) For every finite sequence f such that $\operatorname{len} f \geq 3$ holds $\operatorname{Center} f < \operatorname{len} f$.

(17) $\operatorname{Center} \operatorname{Gauge}(E,n) = 2^{n-'1} + 2$.

(18) $E \subseteq \operatorname{cell}(\operatorname{Gauge}(E,0),2,2)$.

(19) $\operatorname{cell}(\operatorname{Gauge}(E,0),2,2) \not\subseteq \operatorname{BDD} E$.

(20) $\operatorname{Gauge}(C,1) \circ (\operatorname{Center} \operatorname{Gauge}(C,1),1) = [\frac{\text{W-bound}(C)+\text{E-bound}(C)}{2}, \text{S-bound}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,1)))]$.

(21) $\operatorname{Gauge}(C,1) \circ (\operatorname{Center} \operatorname{Gauge}(C,1), \operatorname{len} \operatorname{Gauge}(C,1)) = [\frac{\text{W-bound}(C)+\text{E-bound}(C)}{2}, \text{N-bound}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,1)))]$.

(22) If $1 \leq j$ and $j < \operatorname{width} G$ and $1 \leq m$ and $m \leq \operatorname{len} G$ and $1 \leq n$ and $n \leq \operatorname{width} G$ and $p \in \operatorname{cell}(G, \operatorname{len} G, j)$ and $p_{\mathbf{1}} = (G \circ (m,n))_{\mathbf{1}}$, then $\operatorname{len} G = m$.

(23) Suppose $1 \leq i$ and $i \leq \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$ and $1 \leq m$ and $m \leq \operatorname{len} G$ and $1 \leq n$ and $n \leq \operatorname{width} G$ and $p \in \operatorname{cell}(G,i,j)$ and $p_{\mathbf{1}} = (G \circ (m,n))_{\mathbf{1}}$. Then $i = m$ or $i = m -' 1$.

(24) If $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq m$ and $m \leq \operatorname{len} G$ and $1 \leq n$ and $n \leq \operatorname{width} G$ and $p \in \operatorname{cell}(G,i,\operatorname{width} G)$ and $p_{\mathbf{2}} = (G \circ (m,n))_{\mathbf{2}}$, then $\operatorname{width} G = n$.

(25) Suppose $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j \leq \operatorname{width} G$ and $1 \leq m$ and $m \leq \operatorname{len} G$ and $1 \leq n$ and $n \leq \operatorname{width} G$ and $p \in \operatorname{cell}(G,i,j)$ and $p_{\mathbf{2}} = (G \circ (m,n))_{\mathbf{2}}$. Then $j = n$ or $j = n -' 1$.

(26) For every simple closed curve C and for every real number r such that $\text{W-bound}(C) \leq r$ and $r \leq \text{E-bound}(C)$ holds $\mathcal{L}([r, \text{S-bound}(C)], [r, \text{N-bound}(C)])$ meets $\operatorname{UpperArc}(C)$.

(27) For every simple closed curve C and for every real number r such that $\text{W-bound}(C) \leq r$ and $r \leq \text{E-bound}(C)$ holds $\mathcal{L}([r, \text{S-bound}(C)], [r, \text{N-bound}(C)])$ meets $\operatorname{LowerArc}(C)$.

(28) Let C be a simple closed curve and i be a natural number. If $1 < i$ and $i < \operatorname{len} \operatorname{Gauge}(C,n)$, then $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,1), \operatorname{Gauge}(C,n) \circ (i, \operatorname{len} \operatorname{Gauge}(C,n)))$ meets $\operatorname{UpperArc}(C)$.

(29) Let C be a simple closed curve and i be a natural number. If $1 < i$ and $i < \operatorname{len} \operatorname{Gauge}(C,n)$, then $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,1), \operatorname{Gauge}(C,n) \circ (i, \operatorname{len} \operatorname{Gauge}(C,n)))$ meets $\operatorname{LowerArc}(C)$.

(30) For every simple closed curve C holds $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (\operatorname{Center} \operatorname{Gauge}(C,n),1), \operatorname{Gauge}(C,n) \circ (\operatorname{Center} \operatorname{Gauge}(C,n), \operatorname{len} \operatorname{Gauge}(C,n)))$ meets $\operatorname{UpperArc}(C)$.

(31) For every simple closed curve C holds $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (\operatorname{Center} \operatorname{Gauge}(C,n),1), \operatorname{Gauge}(C,n) \circ (\operatorname{Center} \operatorname{Gauge}(C,n), \operatorname{len} \operatorname{Gauge}(C,n)))$ meets $\operatorname{LowerArc}(C)$.

(32) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i be a natural number. If $1 \leq i$ and $i \leq \operatorname{len} \operatorname{Gauge}(C,n)$, then $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,1), \operatorname{Gauge}(C,n) \circ (i, \operatorname{len} \operatorname{Gauge}(C,n)))$ meets $\operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n)))$.

(33) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i be a natural number. If $1 \leq i$ and $i \leq \operatorname{len} \operatorname{Gauge}(C,n)$, then $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i,1), \operatorname{Gauge}(C,n) \circ (i, \operatorname{len} \operatorname{Gauge}(C,n)))$ meets $\operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n)))$.

(34) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathcal{L}(\mathrm{Gauge}(C,n)\circ(\mathrm{Center\,Gauge}(C,n),1),\mathrm{Gauge}(C,n)\circ(\mathrm{Center\,Gauge}(C,n),\mathrm{len\,Gauge}(C,n)))$ meets $\mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$.

(35) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathcal{L}(\mathrm{Gauge}(C,n)\circ(\mathrm{Center\,Gauge}(C,n),1),\mathrm{Gauge}(C,n)\circ(\mathrm{Center\,Gauge}(C,n),\mathrm{len\,Gauge}(C,n)))$ meets $\mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$.

(36) If $j \leq \mathrm{width}\,G$, then $\mathrm{cell}(G,0,j)$ is not Bounded.

(37) If $i \leq \mathrm{width}\,G$, then $\mathrm{cell}(G,\mathrm{len}\,G,i)$ is not Bounded.

(38) If $j \leq \mathrm{width\,Gauge}(C,n)$, then $\mathrm{cell}(\mathrm{Gauge}(C,n),0,j) \subseteq \mathrm{UBD}\,C$.

(39) If $j \leq \mathrm{len\,Gauge}(E,n)$, then $\mathrm{cell}(\mathrm{Gauge}(E,n),\mathrm{len\,Gauge}(E,n),j) \subseteq \mathrm{UBD}\,E$.

(40) If $i \leq \mathrm{len\,Gauge}(C,n)$ and $j \leq \mathrm{width\,Gauge}(C,n)$ and $\mathrm{cell}(\mathrm{Gauge}(C,n),i,j) \subseteq \mathrm{BDD}\,C$, then $j > 1$.

(41) If $i \leq \mathrm{len\,Gauge}(C,n)$ and $j \leq \mathrm{width\,Gauge}(C,n)$ and $\mathrm{cell}(\mathrm{Gauge}(C,n),i,j) \subseteq \mathrm{BDD}\,C$, then $i > 1$.

(42) If $i \leq \mathrm{len\,Gauge}(C,n)$ and $j \leq \mathrm{width\,Gauge}(C,n)$ and $\mathrm{cell}(\mathrm{Gauge}(C,n),i,j) \subseteq \mathrm{BDD}\,C$, then $j+1 < \mathrm{width\,Gauge}(C,n)$.

(43) If $i \leq \mathrm{len\,Gauge}(C,n)$ and $j \leq \mathrm{width\,Gauge}(C,n)$ and $\mathrm{cell}(\mathrm{Gauge}(C,n),i,j) \subseteq \mathrm{BDD}\,C$, then $i+1 < \mathrm{len\,Gauge}(C,n)$.

(44) If there exist i, j such that $i \leq \mathrm{len\,Gauge}(C,n)$ and $j \leq \mathrm{width\,Gauge}(C,n)$ and $\mathrm{cell}(\mathrm{Gauge}(C,n),i,j) \subseteq \mathrm{BDD}\,C$, then $n \geq 1$.

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