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The Ordering of Points on a Curve. Part IV[1]

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The articles [20], [23], [24], [3], [4], [1], [21], [22], [13], [14], [19], [7], [18], [6], [11], [15], [2], [8], [9], [5], [10], [17], [12], and [16] provide the notation and terminology for this paper.

1. Preliminaries

For simplicity, we adopt the following convention: n is an element of $\mathbb{N}$, V is a subset of $\mathcal{E}^n_{\rm T}$, s, s_1, s_2, t, t_1, t_2 are points of $\mathcal{E}^n_{\rm T}$, C is a simple closed curve, P is a subset of $\mathcal{E}^2_{\rm T}$, and a, p, p_1, p_2, q, q_1, q_2 are points of $\mathcal{E}^2_{\rm T}$.

We now state several propositions:

(1) For all real numbers a, b holds $(a-b)^{\mathbf{2}} = (b-a)^{\mathbf{2}}$.

(2) Let S, T be non empty topological spaces, f be a map from S into T, and A be a subset of T. If f is a homeomorphism and A is connected, then $f^{-1}(A)$ is connected.

(3) Let S, T be non empty topological structures, f be a map from S into T, and A be a subset of T. If f is a homeomorphism and A is compact, then $f^{-1}(A)$ is compact.

(4) $\mathrm{proj2}^\circ\,\mathrm{NorthHalfline}\,a$ is lower bounded.

(5) $\mathrm{proj2}^\circ\,\mathrm{SouthHalfline}\,a$ is upper bounded.

(6) $\mathrm{proj1}^\circ\,\mathrm{WestHalfline}\,a$ is upper bounded.

(7) $\mathrm{proj1}^\circ\,\mathrm{EastHalfline}\,a$ is lower bounded.

Let us consider a. One can verify the following observations:

* $\mathrm{proj2}^\circ\,\mathrm{NorthHalfline}\,a$ is non empty,
* $\mathrm{proj2}^\circ\,\mathrm{SouthHalfline}\,a$ is non empty,
* $\mathrm{proj1}^\circ\,\mathrm{WestHalfline}\,a$ is non empty, and
* $\mathrm{proj1}^\circ\,\mathrm{EastHalfline}\,a$ is non empty.

The following four propositions are true:

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(8) $\inf(\mathrm{proj2}^\circ \mathrm{NorthHalfline}\, a) = a_{\mathbf{2}}$.

(9) $\sup(\mathrm{proj2}^\circ \mathrm{SouthHalfline}\, a) = a_{\mathbf{2}}$.

(10) $\sup(\mathrm{proj1}^\circ \mathrm{WestHalfline}\, a) = a_{\mathbf{1}}$.

(11) $\inf(\mathrm{proj1}^\circ \mathrm{EastHalfline}\, a) = a_{\mathbf{1}}$.

Let us consider a. One can check the following observations:

* NorthHalfline a is closed,
* SouthHalfline a is closed,
* EastHalfline a is closed, and
* WestHalfline a is closed.

The following propositions are true:

(12) If $a \in \mathrm{BDD}\, P$, then $\mathrm{NorthHalfline}\, a \not\subseteq \mathrm{UBD}\, P$.

(13) If $a \in \mathrm{BDD}\, P$, then $\mathrm{SouthHalfline}\, a \not\subseteq \mathrm{UBD}\, P$.

(14) If $a \in \mathrm{BDD}\, P$, then $\mathrm{EastHalfline}\, a \not\subseteq \mathrm{UBD}\, P$.

(15) If $a \in \mathrm{BDD}\, P$, then $\mathrm{WestHalfline}\, a \not\subseteq \mathrm{UBD}\, P$.

(16) For every subset P of $\mathcal{E}^2_{\mathrm{T}}$ and for all points p_1, p_2, q of $\mathcal{E}^2_{\mathrm{T}}$ such that P is an arc from p_1 to p_2 and $q \neq p_2$ holds $p_2 \notin \mathrm{LSegment}(P, p_1, p_2, q)$.

(17) For every subset P of $\mathcal{E}^2_{\mathrm{T}}$ and for all points p_1, p_2, q of $\mathcal{E}^2_{\mathrm{T}}$ such that P is an arc from p_1 to p_2 and $q \neq p_1$ holds $p_1 \notin \mathrm{RSegment}(P, p_1, p_2, q)$.

(18) Let C be a simple closed curve, P be a subset of $\mathcal{E}^2_{\mathrm{T}}$, and p_1, p_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose P is an arc from p_1 to p_2 and $P \subseteq C$. Then there exists a non empty subset R of $\mathcal{E}^2_{\mathrm{T}}$ such that R is an arc from p_1 to p_2 and $P \cup R = C$ and $P \cap R = \{p_1, p_2\}$.

(19) Let P be a subset of $\mathcal{E}^2_{\mathrm{T}}$ and p_1, p_2, q_1, q_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose P is an arc from p_1 to p_2 and $q_1 \in P$ and $q_2 \in P$ and $q_1 \neq p_1$ and $q_1 \neq p_2$ and $q_2 \neq p_1$ and $q_2 \neq p_2$ and $q_1 \neq q_2$. Then there exists a non empty subset Q of $\mathcal{E}^2_{\mathrm{T}}$ such that Q is an arc from q_1 to q_2 and $Q \subseteq P$ and Q misses $\{p_1, p_2\}$.

2. Two Special Points on a Simple Closed Curve

Let us consider p, P. The functor North-Bound(p, P) yields a point of $\mathcal{E}^2_{\mathrm{T}}$ and is defined by:

(Def. 1) $\text{North-Bound}(p, P) = [p_{\mathbf{1}}, \inf(\mathrm{proj2}^\circ(P \cap \mathrm{NorthHalfline}\, p))]$.

The functor South-Bound(p, P) yields a point of $\mathcal{E}^2_{\mathrm{T}}$ and is defined as follows:

(Def. 2) $\text{South-Bound}(p, P) = [p_{\mathbf{1}}, \sup(\mathrm{proj2}^\circ(P \cap \mathrm{SouthHalfline}\, p))]$.

The following propositions are true:

(20) $(\text{North-Bound}(p, P))_{\mathbf{1}} = p_{\mathbf{1}}$ and $(\text{South-Bound}(p, P))_{\mathbf{1}} = p_{\mathbf{1}}$.

(21) $(\text{North-Bound}(p, P))_{\mathbf{2}} = \inf(\mathrm{proj2}^\circ(P \cap \mathrm{NorthHalfline}\, p))$ and $(\text{South-Bound}(p, P))_{\mathbf{2}} = \sup(\mathrm{proj2}^\circ(P \cap \mathrm{SouthHalfline}\, p))$.

(22) For every compact subset C of $\mathcal{E}^2_{\mathrm{T}}$ such that $p \in \mathrm{BDD}\, C$ holds $\text{North-Bound}(p, C) \in C$ and $\text{North-Bound}(p, C) \in \mathrm{NorthHalfline}\, p$ and $\text{South-Bound}(p, C) \in C$ and $\text{South-Bound}(p, C) \in \mathrm{SouthHalfline}\, p$.

(23) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $p \in {\rm BDD}\,C$ holds $({\rm South\text{-}Bound}(p,C))_{\bf 2} < p_{\bf 2}$ and $p_{\bf 2} < ({\rm North\text{-}Bound}(p,C))_{\bf 2}$.

(24) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $p \in {\rm BDD}\,C$ holds $\inf({\rm proj2}^\circ(C \cap {\rm NorthHalfline}\,p)) > \sup({\rm proj2}^\circ(C \cap {\rm SouthHalfline}\,p))$.

(25) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $p \in {\rm BDD}\,C$ holds ${\rm South\text{-}Bound}(p,C) \neq {\rm North\text{-}Bound}(p,C)$.

(26) For every subset C of $\mathcal{E}^2_{\rm T}$ holds $\mathcal{L}({\rm North\text{-}Bound}(p,C), {\rm South\text{-}Bound}(p,C))$ is vertical.

(27) For every compact subset C of $\mathcal{E}^2_{\rm T}$ such that $p \in {\rm BDD}\,C$ holds $\mathcal{L}({\rm North\text{-}Bound}(p,C), {\rm South\text{-}Bound}(p,C)) \cap C = \{{\rm North\text{-}Bound}(p,C), {\rm South\text{-}Bound}(p,C)\}$.

(28) Let C be a compact subset of $\mathcal{E}^2_{\rm T}$. Suppose $p \in {\rm BDD}\,C$ and $q \in {\rm BDD}\,C$ and $p_{\bf 1} \neq q_{\bf 1}$. Then ${\rm North\text{-}Bound}(p,C)$, ${\rm South\text{-}Bound}(q,C)$, ${\rm North\text{-}Bound}(q,C)$, ${\rm South\text{-}Bound}(p,C)$ are mutually different.

3. An Order of Points on a Simple Closed Curve

Let us consider n, V, s_1, s_2, t_1, t_2. We say that s_1, s_2 separate t_1, t_2 on V if and only if:

(Def. 3) For every subset A of $\mathcal{E}^n_{\rm T}$ such that A is an arc from s_1 to s_2 and $A \subseteq V$ holds A meets $\{t_1, t_2\}$.

We introduce s_1, s_2 are neighbours wrt t_1, t_2 on V as an antonym of s_1, s_2 separate t_1, t_2 on V.

The following propositions are true:

(29) t, t separate s_1, s_2 on V.

(30) If s_1, s_2 separate t_1, t_2 on V, then s_2, s_1 separate t_1, t_2 on V.

(31) If s_1, s_2 separate t_1, t_2 on V, then s_1, s_2 separate t_2, t_1 on V.

(32) s, t_1 separate s, t_2 on V.

(33) t_1, s separate t_2, s on V.

(34) t_1, s separate s, t_2 on V.

(35) s, t_1 separate t_2, s on V.

(36) Let p_1, p_2, q be points of $\mathcal{E}^2_{\rm T}$. Suppose $q \in C$ and $p_1 \in C$ and $p_2 \in C$ and $p_1 \neq p_2$ and $p_1 \neq q$ and $p_2 \neq q$. Then p_1, p_2 are neighbours wrt q, q on C.

(37) If $p_1 \neq p_2$ and $p_1 \in C$ and $p_2 \in C$, then if p_1, p_2 separate q_1, q_2 on C, then q_1, q_2 separate p_1, p_2 on C.

(38) Suppose $p_1 \in C$ and $p_2 \in C$ and $q_1 \in C$ and $p_1 \neq p_2$ and $q_1 \neq p_1$ and $q_1 \neq p_2$ and $q_2 \neq p_1$ and $q_2 \neq p_2$. Then p_1, p_2 are neighbours wrt q_1, q_2 on C or p_1, q_1 are neighbours wrt p_2, q_2 on C.

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