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The Ordering of Points on a Curve. Part III[1]

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The articles [1], [13], [2], [7], [8], [11], [5], [4], [12], [6], [9], [3], and [10] provide the notation and terminology for this paper.

We use the following convention: C, P are simple closed curves and a, b, c, d, e are points of $\mathcal{E}^2_{\rm T}$.

We now state several propositions:

(1) Let n be a natural number, a, p_1, p_2 be points of $\mathcal{E}^n_{\rm T}$, and P be a subset of $\mathcal{E}^n_{\rm T}$. Suppose $a \in P$ and P is an arc from p_1 to p_2. Then there exists a map f from $\mathbb{I}$ into $(\mathcal{E}^n_{\rm T})\restriction P$ and there exists a real number r such that f is a homeomorphism and $f(0) = p_1$ and $f(1) = p_2$ and $0 \leq r$ and $r \leq 1$ and $f(r) = a$.

(2) $\mathrm{W}_{\min}(P) \leq_P \mathrm{E}_{\max}(P)$.

(3) If $a \leq_P \mathrm{E}_{\max}(P)$, then $a \in \mathrm{UpperArc}(P)$.

(4) If $\mathrm{E}_{\max}(P) \leq_P a$, then $a \in \mathrm{LowerArc}(P)$.

(5) If $a \leq_P \mathrm{W}_{\min}(P)$, then $a \in \mathrm{LowerArc}(P)$.

(6) Let P be a subset of $\mathcal{E}^2_{\rm T}$. Suppose $a \neq b$ and P is an arc from c to d and LE a, b, P, c, d. Then there exists e such that $a \neq e$ and $b \neq e$ and LE a, e, P, c, d and LE e, b, P, c, d.

(7) If $a \in P$, then there exists e such that $a \neq e$ and $a \leq_P e$.

(8) If $a \neq b$ and $a \leq_P b$, then there exists c such that $c \neq a$ and $c \neq b$ and $a \leq_P c$ and $c \leq_P b$.

Let P be a compact non empty subset of $\mathcal{E}^2_{\rm T}$ and let a, b, c, d be points of $\mathcal{E}^2_{\rm T}$. We say that a, b, c, d are in this order on P if and only if:

(Def. 1) $a \leq_P b$ and $b \leq_P c$ and $c \leq_P d$ or $b \leq_P c$ and $c \leq_P d$ and $d \leq_P a$ or $c \leq_P d$ and $d \leq_P a$ and $a \leq_P b$ or $d \leq_P a$ and $a \leq_P b$ and $b \leq_P c$.

One can prove the following propositions:

(9) If $a \in P$, then a, a, a, a are in this order on P.

(10) If a, b, c, d are in this order on P, then b, c, d, a are in this order on P.

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(11) If a, b, c, d are in this order on P, then c, d, a, b are in this order on P.

(12) If a, b, c, d are in this order on P, then d, a, b, c are in this order on P.

(13) Suppose $a \neq b$ and a, b, c, d are in this order on P. Then there exists e such that $e \neq a$ and $e \neq b$ and a, e, b, c are in this order on P.

(14) Suppose $a \neq b$ and a, b, c, d are in this order on P. Then there exists e such that $e \neq a$ and $e \neq b$ and a, e, b, d are in this order on P.

(15) Suppose $b \neq c$ and a, b, c, d are in this order on P. Then there exists e such that $e \neq b$ and $e \neq c$ and a, b, e, c are in this order on P.

(16) Suppose $b \neq c$ and a, b, c, d are in this order on P. Then there exists e such that $e \neq b$ and $e \neq c$ and b, e, c, d are in this order on P.

(17) Suppose $c \neq d$ and a, b, c, d are in this order on P. Then there exists e such that $e \neq c$ and $e \neq d$ and a, c, e, d are in this order on P.

(18) Suppose $c \neq d$ and a, b, c, d are in this order on P. Then there exists e such that $e \neq c$ and $e \neq d$ and b, c, e, d are in this order on P.

(19) Suppose $d \neq a$ and a, b, c, d are in this order on P. Then there exists e such that $e \neq d$ and $e \neq a$ and a, b, d, e are in this order on P.

(20) Suppose $d \neq a$ and a, b, c, d are in this order on P. Then there exists e such that $e \neq d$ and $e \neq a$ and a, c, d, e are in this order on P.

(21) Suppose $a \neq c$ and $a \neq d$ and $b \neq d$ and a, b, c, d are in this order on P and b, a, c, d are in this order on P. Then $a = b$.

(22) Suppose $a \neq b$ and $b \neq c$ and $c \neq d$ and a, b, c, d are in this order on P and c, b, a, d are in this order on P. Then $a = c$.

(23) Suppose $a \neq b$ and $a \neq c$ and $b \neq d$ and a, b, c, d are in this order on P and d, b, c, a are in this order on P. Then $a = d$.

(24) Suppose $a \neq c$ and $a \neq d$ and $b \neq d$ and a, b, c, d are in this order on P and a, c, b, d are in this order on P. Then $b = c$.

(25) Suppose $a \neq b$ and $b \neq c$ and $c \neq d$ and a, b, c, d are in this order on P and a, d, c, b are in this order on P. Then $b = d$.

(26) Suppose $a \neq b$ and $a \neq c$ and $b \neq d$ and a, b, c, d are in this order on P and a, b, d, c are in this order on P. Then $c = d$.

(27) Suppose $a \in C$ and $b \in C$ and $c \in C$ and $d \in C$. Then

(i) a, b, c, d are in this order on C, or

(ii) a, b, d, c are in this order on C, or

(iii) a, c, b, d are in this order on C, or

(iv) a, c, d, b are in this order on C, or

(v) a, d, b, c are in this order on C, or

(vi) a, d, c, b are in this order on C.

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