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Properties of the Upper and Lower Sequence on the Cage[1]

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The articles [25], [29], [2], [27], [1], [4], [5], [3], [15], [13], [26], [23], [24], [12], [22], [9], [10], [7], [28], [16], [11], [19], [18], [6], [21], [20], [8], [14], and [17] provide the notation and terminology for this paper.

In this paper n is a natural number.

We now state a number of propositions:

(1) For all subsets A, B of $\mathcal{E}^2_{\rm T}$ such that A meets B holds $\mathrm{proj1}^\circ A$ meets $\mathrm{proj1}^\circ B$.

(2) Let A, B be subsets of $\mathcal{E}^2_{\rm T}$ and s be a real number. If A misses B and $A \subseteq \mathrm{HorizontalLine}(s)$ and $B \subseteq \mathrm{HorizontalLine}(s)$, then $\mathrm{proj1}^\circ A$ misses $\mathrm{proj1}^\circ B$.

(3) For every closed subset S of $\mathcal{E}^2_{\rm T}$ such that S is Bounded holds $\mathrm{proj1}^\circ S$ is closed.

(4) For every compact subset S of $\mathcal{E}^2_{\rm T}$ holds $\mathrm{proj1}^\circ S$ is compact.

(5) Let p, q, p_1, q_1 be points of $\mathcal{E}^2_{\rm T}$. Suppose $\mathcal{L}(p,q)$ is vertical and $\mathcal{L}(p_1,q_1)$ is vertical and $p_{\bf 1} = (p_1)_{\bf 1}$ and $p_{\bf 2} \leq (p_1)_{\bf 2}$ and $(p_1)_{\bf 2} \leq (q_1)_{\bf 2}$ and $(q_1)_{\bf 2} \leq q_{\bf 2}$. Then $\mathcal{L}(p_1,q_1) \subseteq \mathcal{L}(p,q)$.

(6) Let p, q, p_1, q_1 be points of $\mathcal{E}^2_{\rm T}$. Suppose $\mathcal{L}(p,q)$ is horizontal and $\mathcal{L}(p_1,q_1)$ is horizontal and $p_{\bf 2} = (p_1)_{\bf 2}$ and $p_{\bf 1} \leq (p_1)_{\bf 1}$ and $(p_1)_{\bf 1} \leq (q_1)_{\bf 1}$ and $(q_1)_{\bf 1} \leq q_{\bf 1}$. Then $\mathcal{L}(p_1,q_1) \subseteq \mathcal{L}(p,q)$.

(7) Let G be a Go-board and i, j, k, j_1, k_1 be natural numbers. Suppose $1 \leq i$ and $i \leq \mathrm{len}\, G$ and $1 \leq j$ and $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $k \leq \mathrm{width}\, G$. Then $\mathcal{L}(G \circ (i,j_1), G \circ (i,k_1)) \subseteq \mathcal{L}(G \circ (i,j), G \circ (i,k))$.

(8) Let G be a Go-board and i, j, k, j_1, k_1 be natural numbers. Suppose $1 \leq i$ and $i \leq \mathrm{width}\, G$ and $1 \leq j$ and $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $k \leq \mathrm{len}\, G$. Then $\mathcal{L}(G \circ (j_1,i), G \circ (k_1,i)) \subseteq \mathcal{L}(G \circ (j,i), G \circ (k,i))$.

(9) Let G be a Go-board and j, k, j_1, k_1 be natural numbers. Suppose $1 \leq j$ and $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $k \leq \mathrm{width}\, G$. Then $\mathcal{L}(G \circ (\mathrm{Center}\, G, j_1), G \circ (\mathrm{Center}\, G, k_1)) \subseteq \mathcal{L}(G \circ (\mathrm{Center}\, G, j), G \circ (\mathrm{Center}\, G, k))$.

(10) Let G be a Go-board. Suppose $\mathrm{len}\, G = \mathrm{width}\, G$. Let j, k, j_1, k_1 be natural numbers. Suppose $1 \leq j$ and $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $k \leq \mathrm{len}\, G$. Then $\mathcal{L}(G \circ (j_1, \mathrm{Center}\, G), G \circ (k_1, \mathrm{Center}\, G)) \subseteq \mathcal{L}(G \circ (j, \mathrm{Center}\, G), G \circ (k, \mathrm{Center}\, G))$.

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(11) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$. Then there exists a natural number j_1 such that $j \leq j_1$ and $j_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j_1), \mathrm{Gauge}(C,n) \circ (i,k)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,j_1)\}$.

(12) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$. Then there exists a natural number k_1 such that $j \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k_1)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,k_1)\}$.

(13) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$. Then there exist natural numbers j_1, k_1 such that $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j_1), \mathrm{Gauge}(C,n) \circ (i,k_1)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,j_1)\}$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j_1), \mathrm{Gauge}(C,n) \circ (i,k_1)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,k_1)\}$.

(14) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (j,i) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$. Then there exists a natural number j_1 such that $j \leq j_1$ and $j_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j_1,i), \mathrm{Gauge}(C,n) \circ (k,i)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (j_1,i)\}$.

(15) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (k,i) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$. Then there exists a natural number k_1 such that $j \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k_1,i)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (k_1,i)\}$.

(16) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (j,i) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$ and $\mathrm{Gauge}(C,n) \circ (k,i) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$. Then there exist natural numbers j_1, k_1 such that $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j_1,i), \mathrm{Gauge}(C,n) \circ (k_1,i)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (j_1,i)\}$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j_1,i), \mathrm{Gauge}(C,n) \circ (k_1,i)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (k_1,i)\}$.

(17) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$. Then there exists a natural number j_1 such that $j \leq j_1$ and $j_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j_1), \mathrm{Gauge}(C,n) \circ (i,k)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,j_1)\}$.

(18) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$. Then there exists a natural number k_1 such that $j \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k_1)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,k_1)\}$.

(19) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq i$ and $i \leq \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$. Then there exist natural numbers j_1, k_1 such

that $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j_1), \mathrm{Gauge}(C,n) \circ (i,k_1)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,j_1)\}$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j_1), \mathrm{Gauge}(C,n) \circ (i,k_1)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,k_1)\}$.

(20) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (j,i) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$. Then there exists a natural number j_1 such that $j \leq j_1$ and $j_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j_1,i), \mathrm{Gauge}(C,n) \circ (k,i)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (j_1,i)\}$.

(21) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (k,i) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$. Then there exists a natural number k_1 such that $j \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k_1,i)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (k_1,i)\}$.

(22) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (j,i) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$ and $\mathrm{Gauge}(C,n) \circ (k,i) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$. Then there exist natural numbers j_1, k_1 such that $j \leq j_1$ and $j_1 \leq k_1$ and $k_1 \leq k$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j_1,i), \mathrm{Gauge}(C,n) \circ (k_1,i)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (j_1,i)\}$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j_1,i), \mathrm{Gauge}(C,n) \circ (k_1,i)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (k_1,i)\}$.

(23) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k))$ meets $\mathrm{LowerArc}(C)$.

(24) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k))$ meets $\mathrm{UpperArc}(C)$.

(25) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $n > 0$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k))$ meets $\mathrm{LowerArc}(C)$.

(26) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $n > 0$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k))$ meets $\mathrm{UpperArc}(C)$.

(27) Let C be a simple closed curve and j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \mathrm{width}\,\mathrm{Gauge}(C,n+1)$ and $\mathrm{Gauge}(C,n+1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,n+1),k) \in \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$ and $\mathrm{Gauge}(C,n+1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,n+1),j) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n+1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,n+1),j), \mathrm{Gauge}(C,n+1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,n+1),k))$ meets $\mathrm{LowerArc}(C)$.

(28) Let C be a simple closed curve and j, k be natural numbers. Suppose $1 \leq j$ and $j \leq k$ and $k \leq \mathrm{width}\,\mathrm{Gauge}(C,n+1)$ and $\mathrm{Gauge}(C,n+1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,n+1),k) \in \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$ and $\mathrm{Gauge}(C,n+1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,n+1),j) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n+1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,n+1),j), \mathrm{Gauge}(C,n+1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,n+1),k))$ meets $\mathrm{UpperArc}(C)$.

(29) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j, k be natural numbers. Suppose $1 < j$ and $k < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (k,i) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$ and $\mathrm{Gauge}(C,n) \circ (j,i) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$. Then $j \neq k$.

(30) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < j$ and $j \leq k$ and $k < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (k,i)\}$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (j,i)\}$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i))$ meets $\mathrm{LowerArc}(C)$.

(31) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < j$ and $j \leq k$ and $k < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (k,i)\}$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (j,i)\}$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i))$ meets $\mathrm{UpperArc}(C)$.

(32) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < j$ and $j \leq k$ and $k < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (k,i) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$ and $\mathrm{Gauge}(C,n) \circ (j,i) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i))$ meets $\mathrm{LowerArc}(C)$.

(33) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < j$ and $j \leq k$ and $k < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (k,i) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$ and $\mathrm{Gauge}(C,n) \circ (j,i) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i))$ meets $\mathrm{UpperArc}(C)$.

(34) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < j$ and $j \leq k$ and $k < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $n > 0$ and $\mathrm{Gauge}(C,n) \circ (k,i) \in \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$ and $\mathrm{Gauge}(C,n) \circ (j,i) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i))$ meets $\mathrm{LowerArc}(C)$.

(35) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < j$ and $j \leq k$ and $k < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $n > 0$ and $\mathrm{Gauge}(C,n) \circ (k,i) \in \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$ and $\mathrm{Gauge}(C,n) \circ (j,i) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i))$ meets $\mathrm{UpperArc}(C)$.

(36) Let C be a simple closed curve and j, k be natural numbers. Suppose $1 < j$ and $j \leq k$ and $k < \mathrm{len}\,\mathrm{Gauge}(C,n+1)$ and $\mathrm{Gauge}(C,n+1) \circ (k, \mathrm{Center}\,\mathrm{Gauge}(C,n+1)) \in \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$ and $\mathrm{Gauge}(C,n+1) \circ (j, \mathrm{Center}\,\mathrm{Gauge}(C,n+1)) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n+1) \circ (j, \mathrm{Center}\,\mathrm{Gauge}(C,n+1)), \mathrm{Gauge}(C,n+1) \circ (k, \mathrm{Center}\,\mathrm{Gauge}(C,n+1)))$ meets $\mathrm{LowerArc}(C)$.

(37) Let C be a simple closed curve and j, k be natural numbers. Suppose $1 < j$ and $j \leq k$ and $k < \mathrm{len}\,\mathrm{Gauge}(C,n+1)$ and $\mathrm{Gauge}(C,n+1) \circ (k, \mathrm{Center}\,\mathrm{Gauge}(C,n+1)) \in \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$ and $\mathrm{Gauge}(C,n+1) \circ (j, \mathrm{Center}\,\mathrm{Gauge}(C,n+1)) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n+1) \circ (j, \mathrm{Center}\,\mathrm{Gauge}(C,n+1)), \mathrm{Gauge}(C,n+1) \circ (k, \mathrm{Center}\,\mathrm{Gauge}(C,n+1)))$ meets $\mathrm{UpperArc}(C)$.

(38) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < j$ and $j \leq k$ and $k < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (j,i)\}$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (k,i)\}$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i))$ meets $\mathrm{LowerArc}(C)$.

(39) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < j$ and $j \leq k$ and $k < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq i$ and $i \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (j,i), \mathrm{Gauge}(C,n) \circ (k,i)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (j,i)\}$ and

$\mathcal{L}(\text{Gauge}(C,n)\circ(j,i),\text{Gauge}(C,n)\circ(k,i))\cap\widetilde{\mathcal{L}}(\text{LowerSeq}(C,n))=\{\text{Gauge}(C,n)\circ(k,i)\}$. Then $\mathcal{L}(\text{Gauge}(C,n)\circ(j,i),\text{Gauge}(C,n)\circ(k,i))$ meets $\text{UpperArc}(C)$.

(40) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1<j$ and $j\leq k$ and $k<\text{len}\,\text{Gauge}(C,n)$ and $1\leq i$ and $i\leq\text{width}\,\text{Gauge}(C,n)$ and $\text{Gauge}(C,n)\circ(j,i)\in\widetilde{\mathcal{L}}(\text{UpperSeq}(C,n))$ and $\text{Gauge}(C,n)\circ(k,i)\in\widetilde{\mathcal{L}}(\text{LowerSeq}(C,n))$. Then $\mathcal{L}(\text{Gauge}(C,n)\circ(j,i),\text{Gauge}(C,n)\circ(k,i))$ meets $\text{LowerArc}(C)$.

(41) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1<j$ and $j\leq k$ and $k<\text{len}\,\text{Gauge}(C,n)$ and $1\leq i$ and $i\leq\text{width}\,\text{Gauge}(C,n)$ and $\text{Gauge}(C,n)\circ(j,i)\in\widetilde{\mathcal{L}}(\text{UpperSeq}(C,n))$ and $\text{Gauge}(C,n)\circ(k,i)\in\widetilde{\mathcal{L}}(\text{LowerSeq}(C,n))$. Then $\mathcal{L}(\text{Gauge}(C,n)\circ(j,i),\text{Gauge}(C,n)\circ(k,i))$ meets $\text{UpperArc}(C)$.

(42) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1<j$ and $j\leq k$ and $k<\text{len}\,\text{Gauge}(C,n)$ and $1\leq i$ and $i\leq\text{width}\,\text{Gauge}(C,n)$ and $n>0$ and $\text{Gauge}(C,n)\circ(j,i)\in\text{UpperArc}(\widetilde{\mathcal{L}}(\text{Cage}(C,n)))$ and $\text{Gauge}(C,n)\circ(k,i)\in\text{LowerArc}(\widetilde{\mathcal{L}}(\text{Cage}(C,n)))$. Then $\mathcal{L}(\text{Gauge}(C,n)\circ(j,i),\text{Gauge}(C,n)\circ(k,i))$ meets $\text{LowerArc}(C)$.

(43) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1<j$ and $j\leq k$ and $k<\text{len}\,\text{Gauge}(C,n)$ and $1\leq i$ and $i\leq\text{width}\,\text{Gauge}(C,n)$ and $n>0$ and $\text{Gauge}(C,n)\circ(j,i)\in\text{UpperArc}(\widetilde{\mathcal{L}}(\text{Cage}(C,n)))$ and $\text{Gauge}(C,n)\circ(k,i)\in\text{LowerArc}(\widetilde{\mathcal{L}}(\text{Cage}(C,n)))$. Then $\mathcal{L}(\text{Gauge}(C,n)\circ(j,i),\text{Gauge}(C,n)\circ(k,i))$ meets $\text{UpperArc}(C)$.

(44) Let C be a simple closed curve and j, k be natural numbers. Suppose $1<j$ and $j\leq k$ and $k<\text{len}\,\text{Gauge}(C,n+1)$ and $\text{Gauge}(C,n+1)\circ(j,\text{Center}\,\text{Gauge}(C,n+1))\in\text{UpperArc}(\widetilde{\mathcal{L}}(\text{Cage}(C,n+1)))$ and $\text{Gauge}(C,n+1)\circ(k,\text{Center}\,\text{Gauge}(C,n+1))\in\text{LowerArc}(\widetilde{\mathcal{L}}(\text{Cage}(C,n+1)))$. Then $\mathcal{L}(\text{Gauge}(C,n+1)\circ(j,\text{Center}\,\text{Gauge}(C,n+1)),\text{Gauge}(C,n+1)\circ(k,\text{Center}\,\text{Gauge}(C,n+1)))$ meets $\text{LowerArc}(C)$.

(45) Let C be a simple closed curve and j, k be natural numbers. Suppose $1<j$ and $j\leq k$ and $k<\text{len}\,\text{Gauge}(C,n+1)$ and $\text{Gauge}(C,n+1)\circ(j,\text{Center}\,\text{Gauge}(C,n+1))\in\text{UpperArc}(\widetilde{\mathcal{L}}(\text{Cage}(C,n+1)))$ and $\text{Gauge}(C,n+1)\circ(k,\text{Center}\,\text{Gauge}(C,n+1))\in\text{LowerArc}(\widetilde{\mathcal{L}}(\text{Cage}(C,n+1)))$. Then $\mathcal{L}(\text{Gauge}(C,n+1)\circ(j,\text{Center}\,\text{Gauge}(C,n+1)),\text{Gauge}(C,n+1)\circ(k,\text{Center}\,\text{Gauge}(C,n+1)))$ meets $\text{UpperArc}(C)$.

(46) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1<i_1$ and $i_1\leq i_2$ and $i_2<\text{len}\,\text{Gauge}(C,n)$ and $1\leq j$ and $j\leq k$ and $k\leq\text{width}\,\text{Gauge}(C,n)$ and $(\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,j),\text{Gauge}(C,n)\circ(i_1,k))\cup\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,k),\text{Gauge}(C,n)\circ(i_2,k)))\cap\widetilde{\mathcal{L}}(\text{UpperSeq}(C,n))=\{\text{Gauge}(C,n)\circ(i_2,k)\}$ and $(\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,j),\text{Gauge}(C,n)\circ(i_1,k))\cup\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,k),\text{Gauge}(C,n)\circ(i_2,k)))\cap\widetilde{\mathcal{L}}(\text{LowerSeq}(C,n))=\{\text{Gauge}(C,n)\circ(i_1,j)\}$. Then $\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,j),\text{Gauge}(C,n)\circ(i_1,k))\cup\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,k),\text{Gauge}(C,n)\circ(i_2,k))$ meets $\text{UpperArc}(C)$.

(47) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1<i_1$ and $i_1\leq i_2$ and $i_2<\text{len}\,\text{Gauge}(C,n)$ and $1\leq j$ and $j\leq k$ and $k\leq\text{width}\,\text{Gauge}(C,n)$ and $(\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,j),\text{Gauge}(C,n)\circ(i_1,k))\cup\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,k),\text{Gauge}(C,n)\circ(i_2,k)))\cap\widetilde{\mathcal{L}}(\text{UpperSeq}(C,n))=\{\text{Gauge}(C,n)\circ(i_2,k)\}$ and $(\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,j),\text{Gauge}(C,n)\circ(i_1,k))\cup\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,k),\text{Gauge}(C,n)\circ(i_2,k)))\cap\widetilde{\mathcal{L}}(\text{LowerSeq}(C,n))=\{\text{Gauge}(C,n)\circ(i_1,j)\}$. Then $\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,j),\text{Gauge}(C,n)\circ(i_1,k))\cup\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,k),\text{Gauge}(C,n)\circ(i_2,k))$ meets $\text{LowerArc}(C)$.

(48) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1<i_2$ and $i_2\leq i_1$ and $i_1<\text{len}\,\text{Gauge}(C,n)$ and $1\leq j$ and $j\leq k$ and $k\leq\text{width}\,\text{Gauge}(C,n)$ and $(\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,j),\text{Gauge}(C,n)\circ(i_1,k))\cup\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,k),\text{Gauge}(C,n)\circ(i_2,k)))\cap\widetilde{\mathcal{L}}(\text{UpperSeq}(C,n))=\{\text{Gauge}(C,n)\circ(i_2,k)\}$ and $(\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,j),\text{Gauge}(C,n)\circ(i_1,k))\cup\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,k),\text{Gauge}(C,n)\circ(i_2,k)))\cap\widetilde{\mathcal{L}}(\text{LowerSeq}(C,n))=\{\text{Gauge}(C,n)\circ(i_1,j)\}$. Then $\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,j),\text{Gauge}(C,n)\circ(i_1,k))\cup\mathcal{L}(\text{Gauge}(C,n)\circ(i_1,k),\text{Gauge}(C,n)\circ(i_2,k))$ meets $\text{UpperArc}(C)$.

(49) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_2$ and $i_2 \leq i_1$ and $i_1 < \operatorname{len}\operatorname{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\operatorname{Gauge}(C,n)$ and $(\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i_1,j), \operatorname{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\operatorname{Gauge}(C,n) \circ (i_1,k), \operatorname{Gauge}(C,n) \circ (i_2,k))) \cap \widetilde{\mathcal{L}}(\operatorname{UpperSeq}(C,n)) = \{\operatorname{Gauge}(C,n) \circ (i_2,k)\}$ and $(\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i_1,j), \operatorname{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\operatorname{Gauge}(C,n) \circ (i_1,k), \operatorname{Gauge}(C,n) \circ (i_2,k))) \cap \widetilde{\mathcal{L}}(\operatorname{LowerSeq}(C,n)) = \{\operatorname{Gauge}(C,n) \circ (i_1,j)\}$. Then $\mathcal{L}(\operatorname{Gauge}(C,n) \circ (i_1,j), \operatorname{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\operatorname{Gauge}(C,n) \circ (i_1,k), \operatorname{Gauge}(C,n) \circ (i_2,k))$ meets $\operatorname{LowerArc}(C)$.

(50) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_1$ and $i_1 < \operatorname{len}\operatorname{Gauge}(C,n+1)$ and $1 < i_2$ and $i_2 < \operatorname{len}\operatorname{Gauge}(C,n+1)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\operatorname{Gauge}(C,n+1)$ and $\operatorname{Gauge}(C,n+1) \circ (i_1,k) \in \operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1)))$ and $\operatorname{Gauge}(C,n+1) \circ (i_2,j) \in \operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1)))$. Then $\mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (i_2,j), \operatorname{Gauge}(C,n+1) \circ (i_2,k)) \cup \mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (i_2,k), \operatorname{Gauge}(C,n+1) \circ (i_1,k))$ meets $\operatorname{UpperArc}(C)$.

(51) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_1$ and $i_1 < \operatorname{len}\operatorname{Gauge}(C,n+1)$ and $1 < i_2$ and $i_2 < \operatorname{len}\operatorname{Gauge}(C,n+1)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\operatorname{Gauge}(C,n+1)$ and $\operatorname{Gauge}(C,n+1) \circ (i_1,k) \in \operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1)))$ and $\operatorname{Gauge}(C,n+1) \circ (i_2,j) \in \operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1)))$. Then $\mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (i_2,j), \operatorname{Gauge}(C,n+1) \circ (i_2,k)) \cup \mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (i_2,k), \operatorname{Gauge}(C,n+1) \circ (i_1,k))$ meets $\operatorname{LowerArc}(C)$.

(52) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \operatorname{len}\operatorname{Gauge}(C,n+1)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\operatorname{Gauge}(C,n+1)$ and $\operatorname{Gauge}(C,n+1) \circ (i,k) \in \operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1)))$ and $\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center}\operatorname{Gauge}(C,n+1), j) \in \operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1)))$. Then $\mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center}\operatorname{Gauge}(C,n+1), j), \operatorname{Gauge}(C,n+1) \circ (\operatorname{Center}\operatorname{Gauge}(C,n+1), k)) \cup \mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center}\operatorname{Gauge}(C,n+1), k), \operatorname{Gauge}(C,n+1) \circ (i,k))$ meets $\operatorname{UpperArc}(C)$.

(53) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \operatorname{len}\operatorname{Gauge}(C,n+1)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\operatorname{Gauge}(C,n+1)$ and $\operatorname{Gauge}(C,n+1) \circ (i,k) \in \operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1)))$ and $\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center}\operatorname{Gauge}(C,n+1), j) \in \operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n+1)))$. Then $\mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center}\operatorname{Gauge}(C,n+1), j), \operatorname{Gauge}(C,n+1) \circ (\operatorname{Center}\operatorname{Gauge}(C,n+1), k)) \cup \mathcal{L}(\operatorname{Gauge}(C,n+1) \circ (\operatorname{Center}\operatorname{Gauge}(C,n+1), k), \operatorname{Gauge}(C,n+1) \circ (i,k))$ meets $\operatorname{LowerArc}(C)$.

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