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The Jordan's Property for Certain Subsets of the Plane

Yatsuka Nakamura
Shinshu University
Nagano

Jarosław Kotowicz
Warsaw University
Białystok

Summary. Let S be a subset of the topological Euclidean plane $\mathcal{E}^2_{\mathrm{T}}$. We say that S has Jordan's property if there exist two non-empty, disjoint and connected subsets G_1 and G_2 of $\mathcal{E}^2_{\mathrm{T}}$ such that $S^{\mathrm{c}} = G_1 \cup G_2$ and $\overline{G_1} \setminus G_1 = \overline{G_2} \setminus G_2$ (see [13], [8]). The aim is to prove that the boundaries of some special polygons in $\mathcal{E}^2_{\mathrm{T}}$ have this property (see Section 3). Moreover, it is proved that both the interior and the exterior of the boundary of any rectangle in $\mathcal{E}^2_{\mathrm{T}}$ is open and connected.

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The articles [14], [16], [1], [9], [17], [4], [5], [3], [12], [11], [10], [2], [15], [7], and [6] provide the notation and terminology for this paper.

1. Selected Theorems on Connected Spaces

In this paper G_1, G_2 are non empty topological spaces and x, y are points of G_1.

One can prove the following propositions:

(1) For every topological structure G_1 and for every subset A of G_1 holds the carrier of $G_1 \restriction A = A$.

(2) Let G_1 be a non empty topological space. Suppose that for all points x, y of G_1 there exists G_2 such that G_2 is connected and there exists a map f from G_2 into G_1 such that f is continuous and $x \in \operatorname{rng} f$ and $y \in \operatorname{rng} f$. Then G_1 is connected.

(4)[1] Let G_1 be a non empty topological space. Suppose that for all points x, y of G_1 there exists a map h from $\mathbb{I}$ into G_1 such that h is continuous and $x = h(0)$ and $y = h(1)$. Then G_1 is connected.

(5) Let A be a subset of G_1. Suppose that for all points x_1, y_1 of G_1 such that $x_1 \in A$ and $y_1 \in A$ and $x_1 \neq y_1$ there exists a map h from $\mathbb{I}$ into $G_1 \restriction A$ such that h is continuous and $x_1 = h(0)$ and $y_1 = h(1)$. Then A is connected.

(6) Let A_0 be a subset of G_1 and A_1 be a subset of G_1. Suppose A_0 is connected and A_1 is connected and A_0 meets A_1. Then $A_0 \cup A_1$ is connected.

(7) Let A_0, A_1, A_2 be subsets of G_1. Suppose A_0 is connected and A_1 is connected and A_2 is connected and A_0 meets A_1 and A_1 meets A_2. Then $A_0 \cup A_1 \cup A_2$ is connected.

[1] The proposition (3) has been removed.

(8) Let A_0, A_1, A_2, A_3 be subsets of G_1. Suppose that A_0 is connected and A_1 is connected and A_2 is connected and A_3 is connected and A_0 meets A_1 and A_1 meets A_2 and A_2 meets A_3. Then $A_0 \cup A_1 \cup A_2 \cup A_3$ is connected.

2. Certain connected and open subsets in the Euclidean plane

In the sequel Q, P_1, P_2 denote subsets of $\mathcal{E}^2_{\mathrm{T}}$ and P denotes a subset of $\mathcal{E}^2_{\mathrm{T}}$.

Let n be a natural number and let P be a subset of $\mathcal{E}^n_{\mathrm{T}}$. We say that P is convex if and only if:

(Def. 1) For all points w_1, w_2 of $\mathcal{E}^n_{\mathrm{T}}$ such that $w_1 \in P$ and $w_2 \in P$ holds $\mathcal{L}(w_1, w_2) \subseteq P$.

We now state the proposition

(9) For every natural number n and for every subset P of $\mathcal{E}^n_{\mathrm{T}}$ such that P is convex holds P is connected.

In the sequel s_1, t_1, s_2, t_2, s, t, s_3, t_3, s_4, t_4, s_5, t_5, s_6, t_6, l, s_7, t_7 denote real numbers.

The following propositions are true:

(10) If $s_1 < s_3$ and $s_1 < s_4$ and $0 \leq l$ and $l \leq 1$, then $s_1 < (1-l) \cdot s_3 + l \cdot s_4$.

(11) If $s_3 < s_1$ and $s_4 < s_1$ and $0 \leq l$ and $l \leq 1$, then $(1-l) \cdot s_3 + l \cdot s_4 < s_1$.

In the sequel s_8, t_8 are real numbers.

One can prove the following propositions:

(12) $\{[s,t] : s_1 < s \wedge s < s_2 \wedge t_1 < t \wedge t < t_2\} = \{[s_3,t_3] : s_1 < s_3\} \cap \{[s_4,t_4] : s_4 < s_2\} \cap \{[s_5, t_5] : t_1 < t_5\} \cap \{[s_6,t_6] : t_6 < t_2\}$.

(13) $\{[s,t] : s_1 \not\leq s \vee s \not\leq s_2 \vee t_1 \not\leq t \vee t \not\leq t_2\} = \{[s_3,t_3] : s_3 < s_1\} \cup \{[s_4,t_4] : t_4 < t_1\} \cup \{[s_5, t_5] : s_2 < s_5\} \cup \{[s_6,t_6] : t_2 < t_6\}$.

(14) For all s_1, t_1, s_2, t_2, P such that $P = \{[s,t] : s_1 < s \wedge s < s_2 \wedge t_1 < t \wedge t < t_2\}$ holds P is convex.

(15) For all s_1, t_1, s_2, t_2, P such that $P = \{[s,t] : s_1 < s \wedge s < s_2 \wedge t_1 < t \wedge t < t_2\}$ holds P is connected.

(16) For all s_1, P such that $P = \{[s,t] : s_1 < s\}$ holds P is convex.

(17) For all s_1, P such that $P = \{[s,t] : s_1 < s\}$ holds P is connected.

(18) For all s_2, P such that $P = \{[s,t] : s < s_2\}$ holds P is convex.

(19) For all s_2, P such that $P = \{[s,t] : s < s_2\}$ holds P is connected.

(20) For all t_1, P such that $P = \{[s,t] : t_1 < t\}$ holds P is convex.

(21) For all t_1, P such that $P = \{[s,t] : t_1 < t\}$ holds P is connected.

(22) For all t_2, P such that $P = \{[s,t] : t < t_2\}$ holds P is convex.

(23) For all t_2, P such that $P = \{[s,t] : t < t_2\}$ holds P is connected.

(24) For all s_1, t_1, s_2, t_2, P such that $P = \{[s,t] : s_1 \not\leq s \vee s \not\leq s_2 \vee t_1 \not\leq t \vee t \not\leq t_2\}$ holds P is connected.

(25) For every s_1 and for every subset P of $\mathcal{E}^2_{\mathrm{T}}$ such that $P = \{[s,t] : s_1 < s\}$ holds P is open.

(26) For every s_1 and for every subset P of $\mathcal{E}^2_{\mathrm{T}}$ such that $P = \{[s,t] : s_1 > s\}$ holds P is open.

(27) For every s_1 and for every subset P of $\mathcal{E}^2_{\mathrm{T}}$ such that $P = \{[s,t] : s_1 < t\}$ holds P is open.

(28) For every s_1 and for every subset P of $\mathcal{E}^2_{\mathrm{T}}$ such that $P = \{[s,t] : s_1 > t\}$ holds P is open.

(29) For all s_1, t_1, s_2, t_2 and for every subset P of $\mathcal{E}^2_{\mathrm{T}}$ such that $P = \{[s,t] : s_1 < s \wedge s < s_2 \wedge t_1 < t \wedge t < t_2\}$ holds P is open.

(30) For all s_1, t_1, s_2, t_2 and for every subset P of $\mathcal{E}^2_{\mathrm{T}}$ such that $P = \{[s,t] : s_1 \not\leq s \vee s \not\leq s_2 \vee t_1 \not\leq t \vee t \not\leq t_2\}$ holds P is open.

(31) Let given s_1, t_1, s_2, t_2, P, Q. Suppose $P = \{[s_7,t_7] : s_1 < s_7 \wedge s_7 < s_2 \wedge t_1 < t_7 \wedge t_7 < t_2\}$ and $Q = \{[s_8,t_8] : s_1 \not\leq s_8 \vee s_8 \not\leq s_2 \vee t_1 \not\leq t_8 \vee t_8 \not\leq t_2\}$. Then P misses Q.

(32) Let s_1, s_2, t_1, t_2 be real numbers. Then $\{p; p$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 < p_{\mathbf{1}} \wedge p_{\mathbf{1}} < s_2 \wedge t_1 < p_{\mathbf{2}} \wedge p_{\mathbf{2}} < t_2\} = \{[s_7,t_7] : s_1 < s_7 \wedge s_7 < s_2 \wedge t_1 < t_7 \wedge t_7 < t_2\}$.

(33) For all s_1, s_2, t_1, t_2 holds $\{q_1; q_1$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 \not\leq (q_1)_{\mathbf{1}} \vee (q_1)_{\mathbf{1}} \not\leq s_2 \vee t_1 \not\leq (q_1)_{\mathbf{2}} \vee (q_1)_{\mathbf{2}} \not\leq t_2\} = \{[s_8,t_8] : s_1 \not\leq s_8 \vee s_8 \not\leq s_2 \vee t_1 \not\leq t_8 \vee t_8 \not\leq t_2\}$.

(34) For all s_1, s_2, t_1, t_2 holds $\{p_0; p_0$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 < (p_0)_{\mathbf{1}} \wedge (p_0)_{\mathbf{1}} < s_2 \wedge t_1 < (p_0)_{\mathbf{2}} \wedge (p_0)_{\mathbf{2}} < t_2\}$ is a subset of $\mathcal{E}^2_{\mathrm{T}}$.

(35) For all s_1, s_2, t_1, t_2 holds $\{p_1; p_1$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 \not\leq (p_1)_{\mathbf{1}} \vee (p_1)_{\mathbf{1}} \not\leq s_2 \vee t_1 \not\leq (p_1)_{\mathbf{2}} \vee (p_1)_{\mathbf{2}} \not\leq t_2\}$ is a subset of $\mathcal{E}^2_{\mathrm{T}}$.

(36) For all s_1, t_1, s_2, t_2, P such that $P = \{p_0; p_0$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 < (p_0)_{\mathbf{1}} \wedge (p_0)_{\mathbf{1}} < s_2 \wedge t_1 < (p_0)_{\mathbf{2}} \wedge (p_0)_{\mathbf{2}} < t_2\}$ holds P is connected.

(37) For all s_1, t_1, s_2, t_2, P such that $P = \{p_1; p_1$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 \not\leq (p_1)_{\mathbf{1}} \vee (p_1)_{\mathbf{1}} \not\leq s_2 \vee t_1 \not\leq (p_1)_{\mathbf{2}} \vee (p_1)_{\mathbf{2}} \not\leq t_2\}$ holds P is connected.

(38) Let given s_1, t_1, s_2, t_2 and P be a subset of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $P = \{p_0; p_0$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 < (p_0)_{\mathbf{1}} \wedge (p_0)_{\mathbf{1}} < s_2 \wedge t_1 < (p_0)_{\mathbf{2}} \wedge (p_0)_{\mathbf{2}} < t_2\}$. Then P is open.

(39) Let given s_1, t_1, s_2, t_2 and P be a subset of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $P = \{p_1; p_1$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 \not\leq (p_1)_{\mathbf{1}} \vee (p_1)_{\mathbf{1}} \not\leq s_2 \vee t_1 \not\leq (p_1)_{\mathbf{2}} \vee (p_1)_{\mathbf{2}} \not\leq t_2\}$. Then P is open.

(40) Let given s_1, t_1, s_2, t_2, P, Q. Suppose that

(i) $P = \{p; p$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 < p_{\mathbf{1}} \wedge p_{\mathbf{1}} < s_2 \wedge t_1 < p_{\mathbf{2}} \wedge p_{\mathbf{2}} < t_2\}$, and

(ii) $Q = \{q_1; q_1$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 \not\leq (q_1)_{\mathbf{1}} \vee (q_1)_{\mathbf{1}} \not\leq s_2 \vee t_1 \not\leq (q_1)_{\mathbf{2}} \vee (q_1)_{\mathbf{2}} \not\leq t_2\}$.

Then P misses Q.

(41) Let given s_1, t_1, s_2, t_2 and P, P_1, P_2 be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that

(i) $s_1 < s_2$,

(ii) $t_1 < t_2$,

(iii) $P = \{p; p$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $p_{\mathbf{1}} = s_1 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_2 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_1 \vee p_{\mathbf{1}} = s_2 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1\}$,

(iv) $P_1 = \{p_2; p_2$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 < (p_2)_{\mathbf{1}} \wedge (p_2)_{\mathbf{1}} < s_2 \wedge t_1 < (p_2)_{\mathbf{2}} \wedge (p_2)_{\mathbf{2}} < t_2\}$, and

(v) $P_2 = \{p_3; p_3$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 \not\leq (p_3)_{\mathbf{1}} \vee (p_3)_{\mathbf{1}} \not\leq s_2 \vee t_1 \not\leq (p_3)_{\mathbf{2}} \vee (p_3)_{\mathbf{2}} \not\leq t_2\}$.

Then

(vi) $P^{\mathrm{c}} = P_1 \cup P_2$,

(vii) $P^{\mathrm{c}} \neq \emptyset$,

(viii) P_1 misses P_2, and

(ix) for all subsets P_3, P_4 of $(\mathcal{E}^2_{\mathrm{T}}) \restriction P^{\mathrm{c}}$ such that $P_3 = P_1$ and $P_4 = P_2$ holds P_3 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction P^{\mathrm{c}}$ and P_4 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction P^{\mathrm{c}}$.

(42) Let given s_1, t_1, s_2, t_2 and P, P_1, P_2 be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that

(i) $s_1 < s_2$,

(ii) $t_1 < t_2$,

(iii) $P = \{p; p$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $p_{\mathbf{1}} = s_1 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_2 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_1 \vee p_{\mathbf{1}} = s_2 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1\}$,

(iv) $P_1 = \{p_2; p_2$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 < (p_2)_{\mathbf{1}} \wedge (p_2)_{\mathbf{1}} < s_2 \wedge t_1 < (p_2)_{\mathbf{2}} \wedge (p_2)_{\mathbf{2}} < t_2\}$, and

(v) $P_2 = \{p_3; p_3$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 \not\leq (p_3)_{\mathbf{1}} \vee (p_3)_{\mathbf{1}} \not\leq s_2 \vee t_1 \not\leq (p_3)_{\mathbf{2}} \vee (p_3)_{\mathbf{2}} \not\leq t_2\}$.

Then $P = \overline{P_1} \setminus P_1$ and $P = \overline{P_2} \setminus P_2$.

(43) Let given s_1, s_2, t_1, t_2, P, P_1. Suppose that

(i) $P = \{p; p$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $p_{\mathbf{1}} = s_1 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_2 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_1 \vee p_{\mathbf{1}} = s_2 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1\}$, and

(ii) $P_1 = \{p_2; p_2$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 < (p_2)_{\mathbf{1}} \wedge (p_2)_{\mathbf{1}} < s_2 \wedge t_1 < (p_2)_{\mathbf{2}} \wedge (p_2)_{\mathbf{2}} < t_2\}$.

Then $P_1 \subseteq \Omega_{(\mathcal{E}^2_{\mathrm{T}}) \upharpoonright P^{\mathrm{c}}}$.

(44) Let given s_1, s_2, t_1, t_2, P, P_1. Suppose that

(i) $P = \{p; p$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $p_{\mathbf{1}} = s_1 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_2 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_1 \vee p_{\mathbf{1}} = s_2 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1\}$, and

(ii) $P_1 = \{p_2; p_2$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 < (p_2)_{\mathbf{1}} \wedge (p_2)_{\mathbf{1}} < s_2 \wedge t_1 < (p_2)_{\mathbf{2}} \wedge (p_2)_{\mathbf{2}} < t_2\}$.

Then P_1 is a subset of $(\mathcal{E}^2_{\mathrm{T}}) \upharpoonright P^{\mathrm{c}}$.

(45) Let given s_1, s_2, t_1, t_2, P, P_2. Suppose that

(i) $s_1 < s_2$,

(ii) $t_1 < t_2$,

(iii) $P = \{p; p$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $p_{\mathbf{1}} = s_1 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_2 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_1 \vee p_{\mathbf{1}} = s_2 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1\}$, and

(iv) $P_2 = \{p_3; p_3$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 \not\leq (p_3)_{\mathbf{1}} \vee (p_3)_{\mathbf{1}} \not\leq s_2 \vee t_1 \not\leq (p_3)_{\mathbf{2}} \vee (p_3)_{\mathbf{2}} \not\leq t_2\}$.

Then $P_2 \subseteq \Omega_{(\mathcal{E}^2_{\mathrm{T}}) \upharpoonright P^{\mathrm{c}}}$.

(46) Let given s_1, s_2, t_1, t_2, P, P_2. Suppose that

(i) $s_1 < s_2$,

(ii) $t_1 < t_2$,

(iii) $P = \{p; p$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $p_{\mathbf{1}} = s_1 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_2 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_1 \vee p_{\mathbf{1}} = s_2 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1\}$, and

(iv) $P_2 = \{p_3; p_3$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $s_1 \not\leq (p_3)_{\mathbf{1}} \vee (p_3)_{\mathbf{1}} \not\leq s_2 \vee t_1 \not\leq (p_3)_{\mathbf{2}} \vee (p_3)_{\mathbf{2}} \not\leq t_2\}$.

Then P_2 is a subset of $(\mathcal{E}^2_{\mathrm{T}}) \upharpoonright P^{\mathrm{c}}$.

3. Jordan's Property

Let S be a subset of $\mathcal{E}^2_{\mathrm{T}}$. We say that S is Jordan if and only if the conditions (Def. 2) are satisfied.

(Def. 2)(i) $S^{\mathrm{c}} \neq \emptyset$, and

(ii) there exist subsets A_1, A_2 of $\mathcal{E}^2_{\mathrm{T}}$ such that $S^{\mathrm{c}} = A_1 \cup A_2$ and A_1 misses A_2 and $\overline{A_1} \setminus A_1 = \overline{A_2} \setminus A_2$ and for all subsets C_1, C_2 of $(\mathcal{E}^2_{\mathrm{T}}) \upharpoonright S^{\mathrm{c}}$ such that $C_1 = A_1$ and $C_2 = A_2$ holds C_1 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \upharpoonright S^{\mathrm{c}}$ and C_2 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \upharpoonright S^{\mathrm{c}}$.

We introduce S has Jordan's property as a synonym of S is Jordan.

Next we state two propositions:

(47) Let S be a subset of $\mathcal{E}^2_{\mathrm{T}}$. Suppose S has Jordan's property. Then

(i) $S^{\mathrm{c}} \neq \emptyset$, and

(ii) there exist subsets A_1, A_2 of $\mathcal{E}^2_{\mathrm{T}}$ and there exist subsets C_1, C_2 of $(\mathcal{E}^2_{\mathrm{T}})\restriction S^{\mathrm{c}}$ such that $S^{\mathrm{c}} = A_1 \cup A_2$ and A_1 misses A_2 and $\overline{A_1} \setminus A_1 = \overline{A_2} \setminus A_2$ and $C_1 = A_1$ and $C_2 = A_2$ and C_1 is a component of $(\mathcal{E}^2_{\mathrm{T}})\restriction S^{\mathrm{c}}$ and C_2 is a component of $(\mathcal{E}^2_{\mathrm{T}})\restriction S^{\mathrm{c}}$ and for every subset C_3 of $(\mathcal{E}^2_{\mathrm{T}})\restriction S^{\mathrm{c}}$ such that C_3 is a component of $(\mathcal{E}^2_{\mathrm{T}})\restriction S^{\mathrm{c}}$ holds $C_3 = C_1$ or $C_3 = C_2$.

(48) Let given s_1, s_2, t_1, t_2 and P be a subset of $\mathcal{E}^2_{\mathrm{T}}$ such that $s_1 < s_2$ and $t_1 < t_2$ and $P = \{p; p$ ranges over points of $\mathcal{E}^2_{\mathrm{T}}$: $p_{\mathbf{1}} = s_1 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_2 \vee p_{\mathbf{1}} \leq s_2 \wedge p_{\mathbf{1}} \geq s_1 \wedge p_{\mathbf{2}} = t_1 \vee p_{\mathbf{1}} = s_2 \wedge p_{\mathbf{2}} \leq t_2 \wedge p_{\mathbf{2}} \geq t_1\}$. Then P has Jordan's property.

REFERENCES

[1] Grzegorz Bancerek. The ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/ordinal1.html`.

[2] Leszek Borys. Paracompact and metrizable spaces. *Journal of Formalized Mathematics*, 3, 1991. `http://mizar.org/JFM/Vol3/pcomps_1.html`.

[3] Czesław Byliński. Binary operations. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/binop_1.html`.

[4] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_1.html`.

[5] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_2.html`.

[6] Agata Darmochwał. The Euclidean space. *Journal of Formalized Mathematics*, 3, 1991. `http://mizar.org/JFM/Vol3/euclid.html`.

[7] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\mathrm{T}}$. Arcs, line segments and special polygonal arcs. *Journal of Formalized Mathematics*, 3, 1991. `http://mizar.org/JFM/Vol3/topreal1.html`.

[8] Dick Wick Hall and Guilford L.Spencer II. *Elementary Topology*. John Wiley and Sons Inc., 1955.

[9] Krzysztof Hryniewiecki. Basic properties of real numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/real_1.html`.

[10] Stanisława Kanas, Adam Lecko, and Mariusz Startek. Metric spaces. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/metric_1.html`.

[11] Beata Padlewska. Connected spaces. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/connsp_1.html`.

[12] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/pre_topc.html`.

[13] Yukio Takeuchi and Yatsuka Nakamura. On the Jordan curve theorem. Technical Report 19804, Dept. of Information Eng., Shinshu University, 500 Wakasato, Nagano city, Japan, April 1980.

[14] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[15] Andrzej Trybulec. A Borsuk theorem on homotopy types. *Journal of Formalized Mathematics*, 3, 1991. `http://mizar.org/JFM/Vol3/borsuk_1.html`.

[16] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

[17] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

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