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# Homotheties and Shears in Affine Planes[1]

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**Summary.** We study connections between Major Desargues Axiom and the transitivity of group of homotheties. A formal proof of the theorem which establishes an equivalence of these two properties of affine planes is given. We also study connections between trapezium version of Major Desargues Axiom and the existence of the shears in affine planes. The article contains investigations on "Scherungssatz".

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The articles [8], [2], [1], [3], [5], [6], [4], and [7] provide the notation and terminology for this paper.

For simplicity, we use the following convention: $A_1$ is an affine plane, $a$, $b$, $o$, $p$, $p'$, $q$, $q'$, $x$, $y$ are elements of $A_1$, $M$, $K$ are subsets of $A_1$, and $f$ is a permutation of the carrier of $A_1$.

One can prove the following four propositions:

(1) Suppose that not $\mathbf{L}(o,a,p)$ and $\mathbf{L}(o,a,b)$ and $\mathbf{L}(o,a,x)$ and $\mathbf{L}(o,a,y)$ and $\mathbf{L}(o,p,p')$ and $\mathbf{L}(o,p,q)$ and $\mathbf{L}(o,p,q')$ and $p \neq q$ and $a \neq x$ and $o \neq q$ and $o \neq x$ and $a,p \nparallel b,p'$ and $a,q \nparallel b,q'$ and $x,p \nparallel y,p'$ and $A_1$ satisfies **DES**. Then $x,q \nparallel y,q'$.

(2) Suppose that for all $o$, $a$, $b$ such that $o \neq a$ and $o \neq b$ and $\mathbf{L}(o,a,b)$ there exists $f$ such that $f$ is a dilatation and $f(o) = o$ and $f(a) = b$. Then $A_1$ satisfies **DES**.

(3) Suppose $A_1$ satisfies **DES**. Let given $o$, $a$, $b$. Suppose $o \neq a$ and $o \neq b$ and $\mathbf{L}(o,a,b)$. Then there exists $f$ such that $f$ is a dilatation and $f(o) = o$ and $f(a) = b$.

(4) $A_1$ satisfies **DES** if and only if for all $o$, $a$, $b$ such that $o \neq a$ and $o \neq b$ and $\mathbf{L}(o,a,b)$ there exists $f$ such that $f$ is a dilatation and $f(o) = o$ and $f(a) = b$.

Let us consider $A_1$, $f$, $K$. We say that $f$ is Sc $K$ if and only if:

(Def. 1) $f$ is a collineation and $K$ is a line and for every $x$ such that $x \in K$ holds $f(x) = x$ and for every $x$ holds $x$, $f(x) \mathbin{//} K$.

One can prove the following propositions:

(5) If $f$ is Sc $K$ and $f(p) = p$ and $p \notin K$, then $f = \mathrm{id}_{\text{the carrier of } A_1}$.

(6) If for all $a$, $b$, $K$ such that $a$, $b \mathbin{//} K$ and $a \notin K$ there exists $f$ such that $f$ is Sc $K$ and $f(a) = b$, then $A_1$ satisfies **TDES**.

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(7) Suppose that $K$ // $M$ and $p \in K$ and $q \in K$ and $p' \in K$ and $q' \in K$ and $A_1$ satisfies **TDES** and $a \in M$ and $b \in M$ and $x \in M$ and $y \in M$ and $a \neq b$ and $q \neq p$ and $p, a \nparallel p', x$ and $p, b \nparallel p', y$ and $q, a \nparallel q', x$. Then $q, b \nparallel q', y$.

(8) If $a$, $b$ // $K$ and $a \notin K$ and $A_1$ satisfies **TDES**, then there exists $f$ such that $f$ is Sc $K$ and $f(a) = b$.

(9) $A_1$ satisfies **TDES** if and only if for all $a$, $b$, $K$ such that $a$, $b$ // $K$ and $a \notin K$ there exists $f$ such that $f$ is Sc $K$ and $f(a) = b$.

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