JOURNAL OF FORMALIZED MATHEMATICS
Volume 3, Released 1991, Published 2003
Inst. of Computer Science, Univ. of Białystok

# Categories of Groups

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**Summary.** We define the category of groups and its subcategories: category of Abelian groups and category of groups with the operator of $1/2$. The carriers of the groups are included in a universum. The universum is a parameter of the categories.

MML Identifier: GRCAT_1.

WWW: http://mizar.org/JFM/Vol3/grcat_1.html

The articles [14], [6], [17], [18], [2], [15], [1], [16], [8], [10], [5], [4], [13], [3], [7], [12], [9], and [11] provide the notation and terminology for this paper.

In this paper $x$, $y$ denote sets, $D$ denotes a non empty set, and $U_1$ denotes a universal class.

One can prove the following three propositions:

(2)[1] Let $X$, $Y$, $A$ be sets and $z$ be a set. Suppose $z \in A$ and $A \subseteq [: X, Y :]$. Then there exists an element $x$ of $X$ and there exists an element $y$ of $Y$ such that $z = \langle x, y \rangle$.

(3) For all elements $u_1$, $u_2$, $u_3$, $u_4$ of $U_1$ holds $\langle u_1, u_2, u_3 \rangle$ is an element of $U_1$ and $\langle u_1, u_2, u_3, u_4 \rangle$ is an element of $U_1$.

(4) For all $x$, $y$ such that $x \in y$ and $y \in U_1$ holds $x \in U_1$.

In this article we present several logical schemes. The scheme *PartLambda2* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding a set, and a binary predicate $\mathcal{P}$, and states that:

There exists a partial function $f$ from $[: \mathcal{A}, \mathcal{B} :]$ to $\mathcal{C}$ such that for all $x$, $y$ holds $\langle x, y \rangle \in \operatorname{dom} f$ iff $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $\mathcal{P}[x, y]$ and for all $x$, $y$ such that $\langle x, y \rangle \in \operatorname{dom} f$ holds $f(\langle x, y \rangle) = \mathcal{F}(x, y)$

provided the following requirement is met:

- For all $x$, $y$ such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $\mathcal{P}[x, y]$ holds $\mathcal{F}(x, y) \in \mathcal{C}$.

The scheme *PartLambda2D* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, a set $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding a set, and a binary predicate $\mathcal{P}$, and states that:

There exists a partial function $f$ from $[: \mathcal{A}, \mathcal{B} :]$ to $\mathcal{C}$ such that

(i) for every element $x$ of $\mathcal{A}$ and for every element $y$ of $\mathcal{B}$ holds $\langle x, y \rangle \in \operatorname{dom} f$ iff $\mathcal{P}[x, y]$, and

(ii) for every element $x$ of $\mathcal{A}$ and for every element $y$ of $\mathcal{B}$ such that $\langle x, y \rangle \in \operatorname{dom} f$ holds $f(\langle x, y \rangle) = \mathcal{F}(x, y)$

provided the following condition is satisfied:

- For every element $x$ of $\mathcal{A}$ and for every element $y$ of $\mathcal{B}$ such that $\mathcal{P}[x, y]$ holds $\mathcal{F}(x, y) \in \mathcal{C}$.

One can prove the following propositions:

[1] The proposition (1) has been removed.

(5) $\mathrm{op}_2(\emptyset, \emptyset) = \emptyset$ and $\mathrm{op}_1(\emptyset) = \emptyset$ and $\mathrm{op}_0 = \emptyset$.

(6) $\{\emptyset\} \in U_1$ and $\langle \{\emptyset\}, \{\emptyset\} \rangle \in U_1$ and $[: \{\emptyset\}, \{\emptyset\} :] \in U_1$ and $\mathrm{op}_2 \in U_1$ and $\mathrm{op}_1 \in U_1$.

(7) $\langle \{\emptyset\}, \mathrm{op}_2, \mathrm{Extract}(\emptyset) \rangle$ is midpoint operator.

Let us note that the trivial loop is midpoint operator.
One can prove the following proposition

(8)(i) For every element $x$ of the trivial loop holds $x = \emptyset$,

(ii) for all elements $x$, $y$ of the trivial loop holds $x + y = \emptyset$,

(iii) for every element $x$ of the trivial loop holds $-x = \emptyset$, and

(iv) $0_{\text{the trivial loop}} = \emptyset$.

In the sequel $C$ is a category and $O$ is a non empty subset of the objects of $C$.
Let us consider $C$, $O$. The functor $\mathrm{Morphs}\,O$ yields a subset of the morphisms of $C$ and is defined by:

(Def. 5)[2] $\mathrm{Morphs}\,O = \bigcup\{\hom(a, b); a$ ranges over objects of $C$, $b$ ranges over objects of $C$: $a \in O \wedge b \in O\}$.

Let us consider $C$, $O$. Observe that $\mathrm{Morphs}\,O$ is non empty.
Let us consider $C$, $O$. The functor $\mathrm{dom}\,O$ yields a function from $\mathrm{Morphs}\,O$ into $O$ and is defined by:

(Def. 6) $\mathrm{dom}\,O = (\text{the dom-map of } C) \restriction \mathrm{Morphs}\,O$.

The functor $\mathrm{cod}\,O$ yielding a function from $\mathrm{Morphs}\,O$ into $O$ is defined by:

(Def. 7) $\mathrm{cod}\,O = (\text{the cod-map of } C) \restriction \mathrm{Morphs}\,O$.

The functor $\mathrm{comp}\,O$ yielding a partial function from $[: \mathrm{Morphs}\,O, \mathrm{Morphs}\,O :]$ to $\mathrm{Morphs}\,O$ is defined as follows:

(Def. 8) $\mathrm{comp}\,O = (\text{the composition of } C) \restriction ([: \mathrm{Morphs}\,O, \mathrm{Morphs}\,O :]$ **qua** set$)$.

The functor $\mathrm{I}_O$ yields a function from $O$ into $\mathrm{Morphs}\,O$ and is defined as follows:

(Def. 9) $\mathrm{I}_O = (\text{the id-map of } C) \restriction O$.

Next we state the proposition

(9) $\langle O, \mathrm{Morphs}\,O, \mathrm{dom}\,O, \mathrm{cod}\,O, \mathrm{comp}\,O, \mathrm{I}_O \rangle$ is full subcategory of $C$.

Let us consider $C$, $O$. The functor $\mathrm{cat}\,O$ yields a subcategory of $C$ and is defined as follows:

(Def. 10) $\mathrm{cat}\,O = \langle O, \mathrm{Morphs}\,O, \mathrm{dom}\,O, \mathrm{cod}\,O, \mathrm{comp}\,O, \mathrm{I}_O \rangle$.

Let us consider $C$, $O$. One can verify that $\mathrm{cat}\,O$ is strict.
Next we state the proposition

(10) The objects of $\mathrm{cat}\,O = O$.

Let $G$ be a 1-sorted structure. The functor $\mathrm{id}_G$ yielding a map from $G$ into $G$ is defined as follows:

(Def. 11) $\mathrm{id}_G = \mathrm{id}_{\text{the carrier of } G}$.

Next we state two propositions:

(11) For every non empty 1-sorted structure $G$ and for every element $x$ of $G$ holds $\mathrm{id}_G(x) = x$.

[2] The definitions (Def. 1)–(Def. 4) have been removed.

(12) Let $G$ be a 1-sorted structure, $H$ be a non empty 1-sorted structure, and $f$ be a map from $G$ into $H$. Then $f \cdot \mathrm{id}_G = f$ and $\mathrm{id}_H \cdot f = f$.

Let $G$, $H$ be non empty zero structures. The functor $\mathrm{ZeroMap}(G,H)$ yields a map from $G$ into $H$ and is defined as follows:

(Def. 12) $\mathrm{ZeroMap}(G,H) = (\text{the carrier of } G) \longmapsto 0_H$.

Let $G$, $H$ be non empty loop structures and let $f$ be a map from $G$ into $H$. We say that $f$ is additive if and only if:

(Def. 13) For all elements $x$, $y$ of $G$ holds $f(x+y) = f(x) + f(y)$.

One can prove the following four propositions:

(13) $\mathrm{comp}(\text{the trivial loop}) = \mathrm{op}_1$.

(14) Let $G_1$, $G_2$, $G_3$ be non empty loop structures, $f$ be a map from $G_1$ into $G_2$, and $g$ be a map from $G_2$ into $G_3$. If $f$ is additive and $g$ is additive, then $g \cdot f$ is additive.

(15) For every non empty zero structure $G$ and for every non empty loop structure $H$ and for every element $x$ of $G$ holds $(\mathrm{ZeroMap}(G,H))(x) = 0_H$.

(16) For every non empty loop structure $G$ and for every right zeroed non empty loop structure $H$ holds $\mathrm{ZeroMap}(G,H)$ is additive.

In the sequel $G$, $H$ denote groups.

We consider group morphism structures as systems

$\langle$ a dom-map, a cod-map, a `Fun` $\rangle$,

where the dom-map and the cod-map are groups and the `Fun` is a map from the dom-map into the cod-map.

Let $f$ be a group morphism structure. The functor $\mathrm{dom}\, f$ yielding a group is defined by:

(Def. 14) $\mathrm{dom}\, f =$ the dom-map of $f$.

The functor $\mathrm{cod}\, f$ yields a group and is defined as follows:

(Def. 15) $\mathrm{cod}\, f =$ the cod-map of $f$.

Let $f$ be a group morphism structure. The functor $\mathrm{fun}\, f$ yields a map from $\mathrm{dom}\, f$ into $\mathrm{cod}\, f$ and is defined by:

(Def. 16) $\mathrm{fun}\, f =$ the `Fun` of $f$.

We now state the proposition

(17) Let $f$ be a group morphism structure, $G_1$, $G_2$ be groups, and $f_0$ be a map from $G_1$ into $G_2$. If $f = \langle G_1, G_2, f_0 \rangle$, then $\mathrm{dom}\, f = G_1$ and $\mathrm{cod}\, f = G_2$ and $\mathrm{fun}\, f = f_0$.

Let us consider $G$, $H$. The functor $\mathrm{ZERO}(G,H)$ yielding a group morphism structure is defined as follows:

(Def. 17) $\mathrm{ZERO}(G,H) = \langle G, H, \mathrm{ZeroMap}(G,H) \rangle$.

Let us consider $G$, $H$. One can verify that $\mathrm{ZERO}(G,H)$ is strict.

Let $I_1$ be a group morphism structure. We say that $I_1$ is morphism of groups-like if and only if:

(Def. 18) $\mathrm{fun}\, I_1$ is additive.

One can check that there exists a group morphism structure which is strict and morphism of groups-like.

A morphism of groups is a morphism of groups-like group morphism structure.

Next we state the proposition

(18) For every morphism $F$ of groups holds the `Fun` of $F$ is additive.

Let us consider $G$, $H$. Observe that $\mathrm{ZERO}(G,H)$ is morphism of groups-like.
Let us consider $G$, $H$. A morphism of groups is said to be a morphism from $G$ to $H$ if:

(Def. 19) $\mathrm{dom}\,\mathrm{it} = G$ and $\mathrm{cod}\,\mathrm{it} = H$.

Let us consider $G$, $H$. Note that there exists a morphism from $G$ to $H$ which is strict.
One can prove the following three propositions:

(19) Let $f$ be a strict group morphism structure. Suppose $\mathrm{dom}\, f = G$ and $\mathrm{cod}\, f = H$ and $\mathrm{fun}\, f$ is additive. Then $f$ is a strict morphism from $G$ to $H$.

(20) For every map $f$ from $G$ into $H$ such that $f$ is additive holds $\langle G, H, f\rangle$ is a strict morphism from $G$ to $H$.

(21) For every non empty loop structure $G$ holds $\mathrm{id}_G$ is additive.

Let us consider $G$. The functor $\mathrm{I}_G$ yielding a morphism from $G$ to $G$ is defined by:

(Def. 20) $\mathrm{I}_G = \langle G, G, \mathrm{id}_G\rangle$.

Let us consider $G$. Note that $\mathrm{I}_G$ is strict.
Let us consider $G$, $H$. Then $\mathrm{ZERO}(G,H)$ is a strict morphism from $G$ to $H$.
One can prove the following propositions:

(22) Let $F$ be a morphism from $G$ to $H$. Then there exists a map $f$ from $G$ into $H$ such that the group morphism structure of $F = \langle G, H, f\rangle$ and $f$ is additive.

(23) For every strict morphism $F$ from $G$ to $H$ there exists a map $f$ from $G$ into $H$ such that $F = \langle G, H, f\rangle$.

(24) For every morphism $F$ of groups there exist $G$, $H$ such that $F$ is a morphism from $G$ to $H$.

(25) Let $F$ be a strict morphism of groups. Then there exist groups $G$, $H$ and there exists a map $f$ from $G$ into $H$ such that $F$ is a morphism from $G$ to $H$ and $F = \langle G, H, f\rangle$ and $f$ is additive.

(26) Let $g$, $f$ be morphisms of groups. Suppose $\mathrm{dom}\, g = \mathrm{cod}\, f$. Then there exist groups $G_1$, $G_2$, $G_3$ such that $g$ is a morphism from $G_2$ to $G_3$ and $f$ is a morphism from $G_1$ to $G_2$.

Let $G$, $F$ be morphisms of groups. Let us assume that $\mathrm{dom}\, G = \mathrm{cod}\, F$. The functor $G \cdot F$ yielding a strict morphism of groups is defined by the condition (Def. 21).

(Def. 21) Let $G_1$, $G_2$, $G_3$ be groups, $g$ be a map from $G_2$ into $G_3$, and $f$ be a map from $G_1$ into $G_2$. Suppose the group morphism structure of $G = \langle G_2, G_3, g\rangle$ and the group morphism structure of $F = \langle G_1, G_2, f\rangle$. Then $G \cdot F = \langle G_1, G_3, g \cdot f\rangle$.

Next we state the proposition

(28)[3] Let $G_1$, $G_2$, $G_3$ be groups, $G$ be a morphism from $G_2$ to $G_3$, and $F$ be a morphism from $G_1$ to $G_2$. Then $G \cdot F$ is a morphism from $G_1$ to $G_3$.

Let $G_1$, $G_2$, $G_3$ be groups, let $G$ be a morphism from $G_2$ to $G_3$, and let $F$ be a morphism from $G_1$ to $G_2$. Then $G \cdot F$ is a strict morphism from $G_1$ to $G_3$.
The following propositions are true:

(29) Let $G_1$, $G_2$, $G_3$ be groups, $G$ be a morphism from $G_2$ to $G_3$, $F$ be a morphism from $G_1$ to $G_2$, $g$ be a map from $G_2$ into $G_3$, and $f$ be a map from $G_1$ into $G_2$. If $G = \langle G_2, G_3, g\rangle$ and $F = \langle G_1, G_2, f\rangle$, then $G \cdot F = \langle G_1, G_3, g \cdot f\rangle$.

[3] The proposition (27) has been removed.

(30) Let $f$, $g$ be strict morphisms of groups. Suppose $\mathrm{dom}\, g = \mathrm{cod}\, f$. Then there exist groups $G_1$, $G_2$, $G_3$ and there exists a map $f_0$ from $G_1$ into $G_2$ and there exists a map $g_0$ from $G_2$ into $G_3$ such that $f = \langle G_1, G_2, f_0 \rangle$ and $g = \langle G_2, G_3, g_0 \rangle$ and $g \cdot f = \langle G_1, G_3, g_0 \cdot f_0 \rangle$.

(31) For all strict morphisms $f$, $g$ of groups such that $\mathrm{dom}\, g = \mathrm{cod}\, f$ holds $\mathrm{dom}(g \cdot f) = \mathrm{dom}\, f$ and $\mathrm{cod}(g \cdot f) = \mathrm{cod}\, g$.

(32) Let $G_1$, $G_2$, $G_3$, $G_4$ be groups, $f$ be a strict morphism from $G_1$ to $G_2$, $g$ be a strict morphism from $G_2$ to $G_3$, and $h$ be a strict morphism from $G_3$ to $G_4$. Then $h \cdot (g \cdot f) = (h \cdot g) \cdot f$.

(33) For all strict morphisms $f$, $g$, $h$ of groups such that $\mathrm{dom}\, h = \mathrm{cod}\, g$ and $\mathrm{dom}\, g = \mathrm{cod}\, f$ holds $h \cdot (g \cdot f) = (h \cdot g) \cdot f$.

(34)(i) $\mathrm{dom}(\mathrm{I}_G) = G$,

(ii) $\mathrm{cod}(\mathrm{I}_G) = G$,

(iii) for every strict morphism $f$ of groups such that $\mathrm{cod}\, f = G$ holds $\mathrm{I}_G \cdot f = f$, and

(iv) for every strict morphism $g$ of groups such that $\mathrm{dom}\, g = G$ holds $g \cdot \mathrm{I}_G = g$.

Let $I_1$ be a set. We say that $I_1$ is non empty set of groups-like if and only if:

(Def. 22) For every set $x$ such that $x \in I_1$ holds $x$ is a strict group.

Let us note that there exists a set which is non empty set of groups-like and non empty.
A non empty set of groups is a non empty set of groups-like non empty set.
In the sequel $V$ is a non empty set of groups.
Let us consider $V$. We see that the element of $V$ is a group.
Let us consider $V$. One can verify that there exists an element of $V$ which is strict.
Let $I_1$ be a set. We say that $I_1$ is non empty set of morphisms of groups-like if and only if:

(Def. 23) For every set $x$ such that $x \in I_1$ holds $x$ is a strict morphism of groups.

Let us mention that there exists a set which is non empty set of morphisms of groups-like and non empty.

A non empty set of morphisms of groups is a non empty set of morphisms of groups-like non empty set.

Let $M$ be a non empty set of morphisms of groups. We see that the element of $M$ is a morphism of groups.

Let $M$ be a non empty set of morphisms of groups. Note that there exists an element of $M$ which is strict.

Next we state the proposition

(37)[4] For every strict morphism $f$ of groups holds $\{f\}$ is a non empty set of morphisms of groups.

Let us consider $G$, $H$. A non empty set of morphisms of groups is said to be a non empty set of morphisms from $G$ into $H$ if:

(Def. 24) Every element of it is a strict morphism from $G$ to $H$.

The following propositions are true:

(38) $D$ is a non empty set of morphisms from $G$ into $H$ iff every element of $D$ is a strict morphism from $G$ to $H$.

(39) For every strict morphism $f$ from $G$ to $H$ holds $\{f\}$ is a non empty set of morphisms from $G$ into $H$.

Let $G$, $H$ be 1-sorted structures. Set of maps from $G$ into $H$ is defined by:

[4] The propositions (35) and (36) have been removed.

(Def. 25) For every set $x$ such that $x \in$ it holds $x$ is a map from $G$ into $H$.

Let $G$, $H$ be 1-sorted structures. The functor $\mathrm{Maps}(G,H)$ yielding a set of maps from $G$ into $H$ is defined by:

(Def. 26) $\mathrm{Maps}(G,H) =$ (the carrier of $H)^{\text{the carrier of } G}$.

Let $G$ be a 1-sorted structure and let $H$ be a non empty 1-sorted structure. Observe that $\mathrm{Maps}(G,H)$ is non empty.

Let $G$ be a 1-sorted structure and let $H$ be a non empty 1-sorted structure. Observe that there exists a set of maps from $G$ into $H$ which is non empty.

Let $G$ be a 1-sorted structure, let $H$ be a non empty 1-sorted structure, and let $M$ be a non empty set of maps from $G$ into $H$. We see that the element of $M$ is a map from $G$ into $H$.

Let us consider $G$, $H$. The functor $\mathrm{Morphs}(G,H)$ yielding a non empty set of morphisms from $G$ into $H$ is defined as follows:

(Def. 27) $x \in \mathrm{Morphs}(G,H)$ iff $x$ is a strict morphism from $G$ to $H$.

Let us consider $G$, $H$ and let $M$ be a non empty set of morphisms from $G$ into $H$. We see that the element of $M$ is a morphism from $G$ to $H$.

Let us consider $G$, $H$ and let $M$ be a non empty set of morphisms from $G$ into $H$. Note that there exists an element of $M$ which is strict.

Let us consider $x$, $y$. The predicate $\mathrm{P}_{\mathrm{ob}}\,x,y$ is defined by the condition (Def. 28).

(Def. 28) There exist sets $x_1$, $x_2$, $x_3$, $x_4$ such that

(i) $x = \langle x_1, x_2, x_3, x_4 \rangle$, and

(ii) there exists a strict group $G$ such that $y = G$ and $x_1 =$ the carrier of $G$ and $x_2 =$ the addition of $G$ and $x_3 = \mathrm{comp}\,G$ and $x_4 =$ the zero of $G$.

One can prove the following two propositions:

(40) For all sets $x$, $y_1$, $y_2$ such that $\mathrm{P}_{\mathrm{ob}}\,x,y_1$ and $\mathrm{P}_{\mathrm{ob}}\,x,y_2$ holds $y_1 = y_2$.

(41) There exists $x$ such that $x \in U_1$ and $\mathrm{P}_{\mathrm{ob}}\,x,$ the trivial loop.

Let us consider $U_1$. The functor $\mathrm{GroupObj}(U_1)$ yielding a set is defined as follows:

(Def. 29) For every $y$ holds $y \in \mathrm{GroupObj}(U_1)$ iff there exists $x$ such that $x \in U_1$ and $\mathrm{P}_{\mathrm{ob}}\,x,y$.

We now state the proposition

(42) The trivial loop $\in \mathrm{GroupObj}(U_1)$.

Let us consider $U_1$. One can check that $\mathrm{GroupObj}(U_1)$ is non empty.

One can prove the following proposition

(43) Every element of $\mathrm{GroupObj}(U_1)$ is a strict group.

Let us consider $U_1$. Note that $\mathrm{GroupObj}(U_1)$ is non empty set of groups-like.

Let us consider $V$. The functor $\mathrm{Morphs}\,V$ yielding a non empty set of morphisms of groups is defined as follows:

(Def. 30) For every $x$ holds $x \in \mathrm{Morphs}\,V$ iff there exist strict elements $G$, $H$ of $V$ such that $x$ is a strict morphism from $G$ to $H$.

Let us consider $V$ and let $F$ be an element of $\mathrm{Morphs}\,V$. Then $\mathrm{dom}\,F$ is a strict element of $V$. Then $\mathrm{cod}\,F$ is a strict element of $V$.

Let us consider $V$ and let $G$ be an element of $V$. The functor $\mathrm{I}_G$ yielding a strict element of $\mathrm{Morphs}\,V$ is defined by:

(Def. 31) $\mathrm{I}_G = \mathrm{I}_G$.

Let us consider $V$. The functor $\operatorname{dom} V$ yields a function from $\operatorname{Morphs} V$ into $V$ and is defined by:

(Def. 32) For every element $f$ of $\operatorname{Morphs} V$ holds $(\operatorname{dom} V)(f) = \operatorname{dom} f$.

The functor $\operatorname{cod} V$ yielding a function from $\operatorname{Morphs} V$ into $V$ is defined by:

(Def. 33) For every element $f$ of $\operatorname{Morphs} V$ holds $(\operatorname{cod} V)(f) = \operatorname{cod} f$.

The functor $\mathrm{I}_V$ yields a function from $V$ into $\operatorname{Morphs} V$ and is defined as follows:

(Def. 34) For every element $G$ of $V$ holds $\mathrm{I}_V(G) = \mathrm{I}_G$.

The following two propositions are true:

(44) Let $g$, $f$ be elements of $\operatorname{Morphs} V$. Suppose $\operatorname{dom} g = \operatorname{cod} f$. Then there exist strict elements $G_1$, $G_2$, $G_3$ of $V$ such that $g$ is a morphism from $G_2$ to $G_3$ and $f$ is a morphism from $G_1$ to $G_2$.

(45) For all elements $g$, $f$ of $\operatorname{Morphs} V$ such that $\operatorname{dom} g = \operatorname{cod} f$ holds $g \cdot f \in \operatorname{Morphs} V$.

Let us consider $V$. The functor $\operatorname{comp} V$ yields a partial function from $[:\operatorname{Morphs} V, \operatorname{Morphs} V:]$ to $\operatorname{Morphs} V$ and is defined by the conditions (Def. 35).

(Def. 35)(i) For all elements $g$, $f$ of $\operatorname{Morphs} V$ holds $\langle g, f \rangle \in \operatorname{dom} \operatorname{comp} V$ iff $\operatorname{dom} g = \operatorname{cod} f$, and

(ii) for all elements $g$, $f$ of $\operatorname{Morphs} V$ such that $\langle g, f \rangle \in \operatorname{dom} \operatorname{comp} V$ holds $(\operatorname{comp} V)(\langle g, f \rangle) = g \cdot f$.

Let us consider $U_1$. The functor $\operatorname{GroupCat}(U_1)$ yields a category structure and is defined as follows:

(Def. 36) $\operatorname{GroupCat}(U_1) = \langle \operatorname{GroupObj}(U_1), \operatorname{Morphs} \operatorname{GroupObj}(U_1), \operatorname{dom} \operatorname{GroupObj}(U_1), \operatorname{cod} \operatorname{GroupObj}(U_1), \operatorname{comp} \operatorname{GroupObj}(U_1), \mathrm{I}_{\operatorname{GroupObj}(U_1)} \rangle$.

Let us consider $U_1$. Observe that $\operatorname{GroupCat}(U_1)$ is strict.
The following propositions are true:

(46) For all morphisms $f$, $g$ of $\operatorname{GroupCat}(U_1)$ holds $\langle g, f \rangle \in \operatorname{dom}(\text{the composition of } \operatorname{GroupCat}(U_1))$ iff $\operatorname{dom} g = \operatorname{cod} f$.

(47) Let $f$ be a morphism of $\operatorname{GroupCat}(U_1)$, $f'$ be an element of $\operatorname{Morphs} \operatorname{GroupObj}(U_1)$, $b$ be an object of $\operatorname{GroupCat}(U_1)$, and $b'$ be an element of $\operatorname{GroupObj}(U_1)$. Then

(i) $f$ is a strict element of $\operatorname{Morphs} \operatorname{GroupObj}(U_1)$,

(ii) $f'$ is a morphism of $\operatorname{GroupCat}(U_1)$,

(iii) $b$ is a strict element of $\operatorname{GroupObj}(U_1)$, and

(iv) $b'$ is an object of $\operatorname{GroupCat}(U_1)$.

(48) For every object $b$ of $\operatorname{GroupCat}(U_1)$ and for every element $b'$ of $\operatorname{GroupObj}(U_1)$ such that $b = b'$ holds $\mathrm{id}_b = \mathrm{I}_{b'}$.

(49) For every morphism $f$ of $\operatorname{GroupCat}(U_1)$ and for every element $f'$ of $\operatorname{Morphs} \operatorname{GroupObj}(U_1)$ such that $f = f'$ holds $\operatorname{dom} f = \operatorname{dom} f'$ and $\operatorname{cod} f = \operatorname{cod} f'$.

(50) Let $f$, $g$ be morphisms of $\operatorname{GroupCat}(U_1)$ and $f'$, $g'$ be elements of $\operatorname{Morphs} \operatorname{GroupObj}(U_1)$ such that $f = f'$ and $g = g'$. Then

(i) $\operatorname{dom} g = \operatorname{cod} f$ iff $\operatorname{dom} g' = \operatorname{cod} f'$,

(ii) $\operatorname{dom} g = \operatorname{cod} f$ iff $\langle g', f' \rangle \in \operatorname{dom} \operatorname{comp} \operatorname{GroupObj}(U_1)$,

(iii) if $\operatorname{dom} g = \operatorname{cod} f$, then $g \cdot f = g' \cdot f'$,

(iv) $\operatorname{dom} f = \operatorname{dom} g$ iff $\operatorname{dom} f' = \operatorname{dom} g'$, and

(v) $\operatorname{cod} f = \operatorname{cod} g$ iff $\operatorname{cod} f' = \operatorname{cod} g'$.

Let us consider $U_1$. One can check that GroupCat($U_1$) is category-like.

Let us consider $U_1$. The functor AbGroupObj($U_1$) yields a subset of the objects of GroupCat($U_1$) and is defined by:

(Def. 37) AbGroupObj($U_1$) $= \{G; G$ ranges over elements of the objects of GroupCat($U_1$): $\bigvee_{H:\text{Abelian group}} G = H\}$.

One can prove the following proposition

(51) The trivial loop $\in$ AbGroupObj($U_1$).

Let us consider $U_1$. One can check that AbGroupObj($U_1$) is non empty.

Let us consider $U_1$. The functor AbGroupCat($U_1$) yielding a subcategory of GroupCat($U_1$) is defined as follows:

(Def. 38) AbGroupCat($U_1$) $=$ cat AbGroupObj($U_1$).

Let us consider $U_1$. Observe that AbGroupCat($U_1$) is strict.

One can prove the following proposition

(52) The objects of AbGroupCat($U_1$) $=$ AbGroupObj($U_1$).

Let us consider $U_1$. The functor $\frac{1}{2}$GroupObj($U_1$) yielding a subset of the objects of AbGroupCat($U_1$) is defined by:

(Def. 39) $\frac{1}{2}$GroupObj($U_1$) $= \{G; G$ ranges over elements of the objects of AbGroupCat($U_1$): $\bigvee_{H:\text{ midpoint operator Abelian group}} G = H\}$.

Let us consider $U_1$. Observe that $\frac{1}{2}$GroupObj($U_1$) is non empty.

Let us consider $U_1$. The functor $\frac{1}{2}$GroupCat($U_1$) yielding a subcategory of AbGroupCat($U_1$) is defined by:

(Def. 40) $\frac{1}{2}$GroupCat($U_1$) $=$ cat $\frac{1}{2}$GroupObj($U_1$).

Let us consider $U_1$. Note that $\frac{1}{2}$GroupCat($U_1$) is strict.

One can prove the following propositions:

(53) The objects of $\frac{1}{2}$GroupCat($U_1$) $= \frac{1}{2}$GroupObj($U_1$).

(54) The trivial loop $\in \frac{1}{2}$GroupObj($U_1$).

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*Received October 3, 1991*

*Published January 2, 2004*