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Properties of Left and Right Components

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The articles [28], [9], [34], [35], [7], [8], [6], [2], [16], [31], [1], [17], [24], [10], [19], [18], [5], [4], [3], [30], [27], [26], [25], [13], [14], [15], [20], [22], [29], [12], [21], [33], [32], [23], and [11] provide the notation and terminology for this paper.

1. Components

For simplicity, we follow the rules: i, j, n are natural numbers, f is a non constant standard special circular sequence, g is a clockwise oriented non constant standard special circular sequence, p, q are points of $\mathcal{E}^2_{\rm T}$, P, Q, R are subsets of $\mathcal{E}^2_{\rm T}$, C is a compact non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$, and G is a Go-board.

The following propositions are true:

(1) Let T be a topological space, A be a subset of T, and B be a subset of T. If B is a component of A, then B is connected.

(2) For every subset A of $\mathcal{E}^n_{\rm T}$ and for every subset B of $\mathcal{E}^n_{\rm T}$ such that B is inside component of A holds B is connected.

(3) Let A be a subset of $\mathcal{E}^n_{\rm T}$ and B be a subset of $\mathcal{E}^n_{\rm T}$. If B is outside component of A, then B is connected.

(4) For every subset A of $\mathcal{E}^n_{\rm T}$ and for every subset B of $\mathcal{E}^n_{\rm T}$ such that B is a component of $A^{\rm c}$ holds A misses B.

(5) If P is outside component of Q and R is inside component of Q, then P misses R.

(6) Let A, B be subsets of $\mathcal{E}^2_{\rm T}$. Suppose A is outside component of $\widetilde{\mathcal{L}}(f)$ and B is outside component of $\widetilde{\mathcal{L}}(f)$. Then $A = B$.

(7) Let p be a point of $\mathcal{E}^2$. Suppose $p = 0_{\mathcal{E}^2_{\rm T}}$ and P is outside component of $\widetilde{\mathcal{L}}(f)$. Then there exists a real number r such that $r > 0$ and $\mathrm{Ball}(p,r)^{\rm c} \subseteq P$.

Let C be a closed subset of $\mathcal{E}^2_{\rm T}$. One can check that $\mathrm{BDD}\,C$ is open and $\mathrm{UBD}\,C$ is open.
Let C be a compact subset of $\mathcal{E}^2_{\rm T}$. Observe that $\mathrm{UBD}\,C$ is connected.

2. Go-Boards

The following proposition is true

(8) For every finite sequence f of elements of $\mathcal{E}^n_{\mathrm{T}}$ such that $\widetilde{\mathcal{L}}(f) \neq \emptyset$ holds $2 \leq \operatorname{len} f$.

Let n be a natural number and let a, b be points of $\mathcal{E}^n_{\mathrm{T}}$. The functor $\rho(a,b)$ yields a real number and is defined by:

(Def. 1) There exist points p, q of $\mathcal{E}^n$ such that $p = a$ and $q = b$ and $\rho(a,b) = \rho(p,q)$.

Let us note that the functor $\rho(a,b)$ is commutative.

The following propositions are true:

(9) $\rho(p,q) = \sqrt{(p_{\mathbf{1}} - q_{\mathbf{1}})^{\mathbf{2}} + (p_{\mathbf{2}} - q_{\mathbf{2}})^{\mathbf{2}}}$.

(10) For every point p of $\mathcal{E}^n_{\mathrm{T}}$ holds $\rho(p,p) = 0$.

(11) For all points p, q, r of $\mathcal{E}^n_{\mathrm{T}}$ holds $\rho(p,r) \leq \rho(p,q) + \rho(q,r)$.

(12) Let x_1, x_2, y_1, y_2 be real numbers and a, b be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $x_1 \leq a_{\mathbf{1}}$ and $a_{\mathbf{1}} \leq x_2$ and $y_1 \leq a_{\mathbf{2}}$ and $a_{\mathbf{2}} \leq y_2$ and $x_1 \leq b_{\mathbf{1}}$ and $b_{\mathbf{1}} \leq x_2$ and $y_1 \leq b_{\mathbf{2}}$ and $b_{\mathbf{2}} \leq y_2$. Then $\rho(a,b) \leq (x_2 - x_1) + (y_2 - y_1)$.

(13) If $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$, then $\operatorname{cell}(G,i,j) = \prod[1 \longmapsto [(G \circ (i,1))_{\mathbf{1}}, (G \circ (i+1,1))_{\mathbf{1}}], 2 \longmapsto [(G \circ (1,j))_{\mathbf{2}}, (G \circ (1,j+1))_{\mathbf{2}}]]$.

(14) If $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$, then $\operatorname{cell}(G,i,j)$ is compact.

(15) If $\langle i, j \rangle \in$ the indices of G and $\langle i+n, j \rangle \in$ the indices of G, then $\rho(G \circ (i,j), G \circ (i+n,j)) = (G \circ (i+n,j))_{\mathbf{1}} - (G \circ (i,j))_{\mathbf{1}}$.

(16) If $\langle i, j \rangle \in$ the indices of G and $\langle i, j+n \rangle \in$ the indices of G, then $\rho(G \circ (i,j), G \circ (i,j+n)) = (G \circ (i,j+n))_{\mathbf{2}} - (G \circ (i,j))_{\mathbf{2}}$.

(17) $3 \leq \operatorname{len} \operatorname{Gauge}(C,n) -' 1$.

(18) Suppose $i \leq j$. Let a, b be natural numbers. Suppose $2 \leq a$ and $a \leq \operatorname{len} \operatorname{Gauge}(C,i) - 1$ and $2 \leq b$ and $b \leq \operatorname{len} \operatorname{Gauge}(C,i) - 1$. Then there exist natural numbers c, d such that

$2 \leq c$ and $c \leq \operatorname{len} \operatorname{Gauge}(C,j) - 1$ and $2 \leq d$ and $d \leq \operatorname{len} \operatorname{Gauge}(C,j) - 1$ and $\langle c, d \rangle \in$ the indices of $\operatorname{Gauge}(C,j)$ and $\operatorname{Gauge}(C,i) \circ (a,b) = \operatorname{Gauge}(C,j) \circ (c,d)$ and $c = 2 + 2^{j -' i} \cdot (a -' 2)$ and $d = 2 + 2^{j -' i} \cdot (b -' 2)$.

(19) If $\langle i, j \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$ and $\langle i, j+1 \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$, then $\rho(\operatorname{Gauge}(C,n) \circ (i,j), \operatorname{Gauge}(C,n) \circ (i,j+1)) = \frac{\text{N-bound}(C) - \text{S-bound}(C)}{2^n}$.

(20) If $\langle i, j \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$ and $\langle i+1, j \rangle \in$ the indices of $\operatorname{Gauge}(C,n)$, then $\rho(\operatorname{Gauge}(C,n) \circ (i,j), \operatorname{Gauge}(C,n) \circ (i+1,j)) = \frac{\text{E-bound}(C) - \text{W-bound}(C)}{2^n}$.

(21) Let r, t be real numbers. Suppose $r > 0$ and $t > 0$. Then there exists a natural number n such that $1 < n$ and $\rho(\operatorname{Gauge}(C,n) \circ (1,1), \operatorname{Gauge}(C,n) \circ (1,2)) < r$ and $\rho(\operatorname{Gauge}(C,n) \circ (1,1), \operatorname{Gauge}(C,n) \circ (2,1)) < t$.

3. LeftComp and RightComp

We now state a number of propositions:

(22) For every subset P of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ such that P is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ holds $P = \operatorname{RightComp}(f)$ or $P = \operatorname{LeftComp}(f)$.

(23) Let A_1, A_2 be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that

(i) $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}} = A_1 \cup A_2$,

(ii) A_1 misses A_2, and

(iii) for all subsets C_1, C_2 of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ such that $C_1 = A_1$ and $C_2 = A_2$ holds C_1 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and C_2 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$.

Then $A_1 = \mathrm{RightComp}(f)$ and $A_2 = \mathrm{LeftComp}(f)$ or $A_1 = \mathrm{LeftComp}(f)$ and $A_2 = \mathrm{RightComp}(f)$.

(24) $\mathrm{LeftComp}(f)$ misses $\mathrm{RightComp}(f)$.

(25) $\widetilde{\mathcal{L}}(f) \cup \mathrm{RightComp}(f) \cup \mathrm{LeftComp}(f) =$ the carrier of $\mathcal{E}^2_{\mathrm{T}}$.

(26) $p \in \widetilde{\mathcal{L}}(f)$ iff $p \notin \mathrm{LeftComp}(f)$ and $p \notin \mathrm{RightComp}(f)$.

(27) $p \in \mathrm{LeftComp}(f)$ iff $p \notin \widetilde{\mathcal{L}}(f)$ and $p \notin \mathrm{RightComp}(f)$.

(28) $p \in \mathrm{RightComp}(f)$ iff $p \notin \widetilde{\mathcal{L}}(f)$ and $p \notin \mathrm{LeftComp}(f)$.

(29) $\widetilde{\mathcal{L}}(f) = \overline{\mathrm{RightComp}(f)} \setminus \mathrm{RightComp}(f)$.

(30) $\widetilde{\mathcal{L}}(f) = \overline{\mathrm{LeftComp}(f)} \setminus \mathrm{LeftComp}(f)$.

(31) $\overline{\mathrm{RightComp}(f)} = \mathrm{RightComp}(f) \cup \widetilde{\mathcal{L}}(f)$.

(32) $\overline{\mathrm{LeftComp}(f)} = \mathrm{LeftComp}(f) \cup \widetilde{\mathcal{L}}(f)$.

Let f be a non constant standard special circular sequence. One can verify that $\widetilde{\mathcal{L}}(f)$ is Jordan.

In the sequel f is a clockwise oriented non constant standard special circular sequence.

The following propositions are true:

(33) If $p \in \mathrm{RightComp}(f)$, then $\text{W-bound}(\widetilde{\mathcal{L}}(f)) < p_{\mathbf{1}}$.

(34) If $p \in \mathrm{RightComp}(f)$, then $\text{E-bound}(\widetilde{\mathcal{L}}(f)) > p_{\mathbf{1}}$.

(35) If $p \in \mathrm{RightComp}(f)$, then $\text{N-bound}(\widetilde{\mathcal{L}}(f)) > p_{\mathbf{2}}$.

(36) If $p \in \mathrm{RightComp}(f)$, then $\text{S-bound}(\widetilde{\mathcal{L}}(f)) < p_{\mathbf{2}}$.

(37) If $p \in \mathrm{RightComp}(f)$ and $q \in \mathrm{LeftComp}(f)$, then $\mathcal{L}(p,q)$ meets $\widetilde{\mathcal{L}}(f)$.

(38) $\overline{\mathrm{RightComp}(\mathrm{SpStSeq}\,C)} = \prod[1 \longmapsto [\text{W-bound}(\widetilde{\mathcal{L}}(\mathrm{SpStSeq}\,C)), \text{E-bound}(\widetilde{\mathcal{L}}(\mathrm{SpStSeq}\,C))], 2 \longmapsto [\text{S-bound}(\widetilde{\mathcal{L}}(\mathrm{SpStSeq}\,C)), \text{N-bound}(\widetilde{\mathcal{L}}(\mathrm{SpStSeq}\,C))]]$.

(39) $\mathrm{proj1}^\circ \overline{\mathrm{RightComp}(f)} = \mathrm{proj1}^\circ \widetilde{\mathcal{L}}(f)$.

(40) $\mathrm{proj2}^\circ \overline{\mathrm{RightComp}(f)} = \mathrm{proj2}^\circ \widetilde{\mathcal{L}}(f)$.

(41) $\mathrm{RightComp}(g) \subseteq \mathrm{RightComp}(\mathrm{SpStSeq}\,\widetilde{\mathcal{L}}(g))$.

(42) $\overline{\mathrm{RightComp}(g)}$ is compact.

(43) $\mathrm{LeftComp}(g)$ is non Bounded.

(44) $\mathrm{LeftComp}(g)$ is outside component of $\widetilde{\mathcal{L}}(g)$.

(45) $\mathrm{RightComp}(g)$ is inside component of $\widetilde{\mathcal{L}}(g)$.

(46) $\mathrm{UBD}\,\widetilde{\mathcal{L}}(g) = \mathrm{LeftComp}(g)$.

(47) $\mathrm{BDD}\,\widetilde{\mathcal{L}}(g) = \mathrm{RightComp}(g)$.

(48) If P is outside component of $\widetilde{\mathcal{L}}(g)$, then $P = \mathrm{LeftComp}(g)$.

(49) If P is inside component of $\widetilde{\mathcal{L}}(g)$, then P meets $\mathrm{RightComp}(g)$.

(50) If P is inside component of $\widetilde{\mathcal{L}}(g)$, then $P = \mathrm{BDD}\,\widetilde{\mathcal{L}}(g)$.

(51) $\text{W-bound}(\widetilde{\mathcal{L}}(g)) = \text{W-bound}(\mathrm{RightComp}(g))$.

(52) $\text{E-bound}(\widetilde{\mathcal{L}}(g)) = \text{E-bound}(\mathrm{RightComp}(g))$.

(53) $\text{N-bound}(\widetilde{\mathcal{L}}(g)) = \text{N-bound}(\mathrm{RightComp}(g))$.

(54) $\text{S-bound}(\widetilde{\mathcal{L}}(g)) = \text{S-bound}(\mathrm{RightComp}(g))$.

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