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The First Part of Jordan's Theorem for Special Polygons

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Summary. We prove here the first part of Jordan's theorem for special polygons, i.e., the complement of a special polygon is the union of two components (a left component and a right component). At this stage, we do not know if the two components are different from each other.

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The articles [15], [4], [19], [6], [8], [17], [1], [14], [3], [2], [18], [7], [20], [13], [5], [9], [10], [16], [12], and [11] provide the notation and terminology for this paper.

We adopt the following rules: i, j, k_1, k_2, i_1, i_2, j_1, j_2 are natural numbers and f is a non constant standard special circular sequence.

We now state a number of propositions:

(2)[1] For all i, j such that $i \leq \mathrm{len}$ the Go-board of f and $j \leq \mathrm{width}$ the Go-board of f holds $\mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, i, j) \subseteq (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$.

(3) Let given i, j. Suppose $i \leq \mathrm{len}$ the Go-board of f and $j \leq \mathrm{width}$ the Go-board of f. Then $\overline{\mathrm{Down}(\mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, i, j), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}})} = \mathrm{cell}(\text{the Go-board of } f, i, j) \cap (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$.

(4) Let given i, j. Suppose $i \leq \mathrm{len}$ the Go-board of f and $j \leq \mathrm{width}$ the Go-board of f. Then $\overline{\mathrm{Down}(\mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, i, j), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}})}$ is connected and $\mathrm{Down}(\mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, i, j), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}) = \mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, i, j)$.

(5) $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}} = \bigcup\{\overline{\mathrm{Down}(\mathrm{Int}\,\mathrm{cell}(\text{the Go-board of } f, i, j), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}})} : i \leq \mathrm{len}\text{ the Go-board of } f \wedge j \leq \mathrm{width}\text{ the Go-board of } f\}$.

(6) $\mathrm{Down}(\mathrm{LeftComp}(f), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}) \cup \mathrm{Down}(\mathrm{RightComp}(f), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}})$ is a union of components of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and $\mathrm{Down}(\mathrm{LeftComp}(f), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}) = \mathrm{LeftComp}(f)$ and $\mathrm{Down}(\mathrm{RightComp}(f), (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}) = \mathrm{RightComp}(f)$.

(7) Let given i_1, j_1, i_2, j_2. Suppose that

(i) $i_1 \leq \mathrm{len}$ the Go-board of f,

(ii) $j_1 \leq \mathrm{width}$ the Go-board of f,

(iii) $i_2 \leq \mathrm{len}$ the Go-board of f,

[1] The proposition (1) has been removed.

(iv) $j_2 \leq \mathrm{width}\,$the Go-board of f, and

(v) i_1, j_1, i_2, and j_2 are adjacent.

Then $\mathrm{Intcell}($the Go-board of f, $i_1, j_1) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$ if and only if $\mathrm{Intcell}($the Go-board of f, $i_2, j_2) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$.

(8) Let F_1, F_2 be finite sequences of elements of $\mathbb{N}$. Suppose that

(i) $\mathrm{len}\, F_1 = \mathrm{len}\, F_2$,

(ii) there exists i such that $i \in \mathrm{dom}\, F_1$ and $\mathrm{Intcell}($the Go-board of f, $(F_1)_i, (F_2)_i) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$,

(iii) for every i such that $1 \leq i$ and $i < \mathrm{len}\, F_1$ holds $(F_1)_i$, $(F_2)_i$, $(F_1)_{i+1}$, and $(F_2)_{i+1}$ are adjacent, and

(iv) for all i, k_1, k_2 such that $i \in \mathrm{dom}\, F_1$ and $k_1 = F_1(i)$ and $k_2 = F_2(i)$ holds $k_1 \leq \mathrm{len}\,$the Go-board of f and $k_2 \leq \mathrm{width}\,$the Go-board of f.

Let given i. If $i \in \mathrm{dom}\, F_1$, then $\mathrm{Intcell}($the Go-board of f, $(F_1)_i, (F_2)_i) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$.

(9) There exist i, j such that $i \leq \mathrm{len}\,$the Go-board of f and $j \leq \mathrm{width}\,$the Go-board of f and $\mathrm{Intcell}($the Go-board of f, $i, j) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$.

(10) For all i, j such that $i \leq \mathrm{len}\,$the Go-board of f and $j \leq \mathrm{width}\,$the Go-board of f holds $\mathrm{Intcell}($the Go-board of f, $i, j) \subseteq \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$.

(11) $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}} = \mathrm{LeftComp}(f) \cup \mathrm{RightComp}(f)$.

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