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Some Topological Properties of Cells in R^2

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Summary. We examine the topological property of cells (rectangles) in a plane. First, some Fraenkel expressions of cells are shown. Second, it is proved that cells are closed. The last theorem asserts that the closure of the interior of a cell is the same as itself.

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The articles [14], [15], [5], [8], [2], [1], [13], [3], [6], [7], [16], [12], [4], [9], [10], and [11] provide the notation and terminology for this paper.

We use the following convention: i, j denote natural numbers, r, s, r_2, s_1, s_2 denote real numbers, and G_1 denotes a non empty topological space.

One can prove the following propositions:

(1) For every subset A of G_1 and for every point p of G_1 such that $p \in A$ and A is connected holds $A \subseteq \mathrm{Component}(p)$.

(2) For all subsets A, B, C of G_1 such that C is a component of G_1 and $A \subseteq C$ and B is connected and $\overline{A}$ meets $\overline{B}$ holds $B \subseteq C$.

In the sequel G_2 is a non empty topological space.

One can prove the following three propositions:

(3) Let A, B be subsets of G_2. Suppose A is a component of G_2 and B is a component of G_2. Then $A \cup B$ is a union of components of G_2.

(4) For all subsets B_1, B_2, V of G_1 holds $\mathrm{Down}(B_1 \cup B_2, V) = \mathrm{Down}(B_1, V) \cup \mathrm{Down}(B_2, V)$.

(5) For all subsets B_1, B_2, V of G_1 holds $\mathrm{Down}(B_1 \cap B_2, V) = \mathrm{Down}(B_1, V) \cap \mathrm{Down}(B_2, V)$.

In the sequel f is a non constant standard special circular sequence and G is a non empty yielding matrix over $\mathcal{E}^2_{\mathrm{T}}$.

The following proposition is true

(6) $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}} \neq \emptyset$.

Let us consider f. Note that $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ is non empty.

Next we state a number of propositions:

(7) For every f holds the carrier of $\mathcal{E}^2_{\mathrm{T}} = \bigcup\{\mathrm{cell}(\text{the Go-board of } f, i, j) : i \leq \mathrm{len}\,\text{the Go-board of } f \wedge j \leq \mathrm{width}\,\text{the Go-board of } f\}$.

(8) For all subsets P_1, P_2 of $\mathcal{E}^2_{\mathrm{T}}$ such that $P_1 = \{[r,s] : s \leq s_1\}$ and $P_2 = \{[r_2,s_2] : s_2 > s_1\}$ holds $P_1 = {P_2}^{\mathrm{c}}$.

(9) For all subsets P_1, P_2 of $\mathcal{E}^2_{\mathrm{T}}$ such that $P_1 = \{[r,s] : s \geq s_1\}$ and $P_2 = \{[r_2,s_2] : s_2 < s_1\}$ holds $P_1 = {P_2}^{\mathrm{c}}$.

(10) For all subsets P_1, P_2 of $\mathcal{E}^2_{\mathrm{T}}$ such that $P_1 = \{[s,r] : s \geq s_1\}$ and $P_2 = \{[s_2,r_2] : s_2 < s_1\}$ holds $P_1 = {P_2}^{\mathrm{c}}$.

(11) For all subsets P_1, P_2 of $\mathcal{E}^2_{\mathrm{T}}$ such that $P_1 = \{[s,r] : s \leq s_1\}$ and $P_2 = \{[s_2,r_2] : s_2 > s_1\}$ holds $P_1 = {P_2}^{\mathrm{c}}$.

(12) For every subset P of $\mathcal{E}^2_{\mathrm{T}}$ and for every s_1 such that $P = \{[r,s] : s \leq s_1\}$ holds P is closed.

(13) For every subset P of $\mathcal{E}^2_{\mathrm{T}}$ and for every s_1 such that $P = \{[r,s] : s_1 \leq s\}$ holds P is closed.

(14) For every subset P of $\mathcal{E}^2_{\mathrm{T}}$ and for every s_1 such that $P = \{[s,r] : s \leq s_1\}$ holds P is closed.

(15) For every subset P of $\mathcal{E}^2_{\mathrm{T}}$ and for every s_1 such that $P = \{[s,r] : s_1 \leq s\}$ holds P is closed.

(16) For every matrix G over $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{hstrip}(G, j)$ is closed.

(17) For every matrix G over $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{vstrip}(G, j)$ is closed.

(18) If G is line $\mathbf{X}$-constant, then $\mathrm{vstrip}(G,0) = \{[r,s] : r \leq (G \circ (1,1))_{\mathbf{1}}\}$.

(19) If G is line $\mathbf{X}$-constant, then $\mathrm{vstrip}(G,\mathrm{len}\, G) = \{[r,s] : (G \circ (\mathrm{len}\, G,1))_{\mathbf{1}} \leq r\}$.

(20) If G is line $\mathbf{X}$-constant and $1 \leq i$ and $i < \mathrm{len}\, G$, then $\mathrm{vstrip}(G,i) = \{[r,s] : (G \circ (i,1))_{\mathbf{1}} \leq r \wedge r \leq (G \circ (i+1,1))_{\mathbf{1}}\}$.

(21) If G is column $\mathbf{Y}$-constant, then $\mathrm{hstrip}(G,0) = \{[r,s] : s \leq (G \circ (1,1))_{\mathbf{2}}\}$.

(22) If G is column $\mathbf{Y}$-constant, then $\mathrm{hstrip}(G,\mathrm{width}\, G) = \{[r,s] : (G \circ (1,\mathrm{width}\, G))_{\mathbf{2}} \leq s\}$.

(23) If G is column $\mathbf{Y}$-constant and $1 \leq j$ and $j < \mathrm{width}\, G$, then $\mathrm{hstrip}(G, j) = \{[r,s] : (G \circ (1, j))_{\mathbf{2}} \leq s \wedge s \leq (G \circ (1, j+1))_{\mathbf{2}}\}$.

In the sequel G is a non empty yielding line $\mathbf{X}$-constant column $\mathbf{Y}$-constant matrix over $\mathcal{E}^2_{\mathrm{T}}$. The following propositions are true:

(24) $\mathrm{cell}(G,0,0) = \{[r,s] : r \leq (G \circ (1,1))_{\mathbf{1}} \wedge s \leq (G \circ (1,1))_{\mathbf{2}}\}$.

(25) $\mathrm{cell}(G,0,\mathrm{width}\, G) = \{[r,s] : r \leq (G \circ (1,1))_{\mathbf{1}} \wedge (G \circ (1,\mathrm{width}\, G))_{\mathbf{2}} \leq s\}$.

(26) If $1 \leq j$ and $j < \mathrm{width}\, G$, then $\mathrm{cell}(G,0, j) = \{[r,s] : r \leq (G \circ (1,1))_{\mathbf{1}} \wedge (G \circ (1, j))_{\mathbf{2}} \leq s \wedge s \leq (G \circ (1, j+1))_{\mathbf{2}}\}$.

(27) $\mathrm{cell}(G,\mathrm{len}\, G,0) = \{[r,s] : (G \circ (\mathrm{len}\, G,1))_{\mathbf{1}} \leq r \wedge s \leq (G \circ (1,1))_{\mathbf{2}}\}$.

(28) $\mathrm{cell}(G,\mathrm{len}\, G,\mathrm{width}\, G) = \{[r,s] : (G \circ (\mathrm{len}\, G,1))_{\mathbf{1}} \leq r \wedge (G \circ (1,\mathrm{width}\, G))_{\mathbf{2}} \leq s\}$.

(29) If $1 \leq j$ and $j < \mathrm{width}\, G$, then $\mathrm{cell}(G,\mathrm{len}\, G, j) = \{[r,s] : (G \circ (\mathrm{len}\, G,1))_{\mathbf{1}} \leq r \wedge (G \circ (1, j))_{\mathbf{2}} \leq s \wedge s \leq (G \circ (1, j+1))_{\mathbf{2}}\}$.

(30) If $1 \leq i$ and $i < \mathrm{len}\, G$, then $\mathrm{cell}(G,i,0) = \{[r,s] : (G \circ (i,1))_{\mathbf{1}} \leq r \wedge r \leq (G \circ (i+1,1))_{\mathbf{1}} \wedge s \leq (G \circ (1,1))_{\mathbf{2}}\}$.

(31) If $1 \leq i$ and $i < \mathrm{len}\, G$, then $\mathrm{cell}(G,i,\mathrm{width}\, G) = \{[r,s] : (G \circ (i,1))_{\mathbf{1}} \leq r \wedge r \leq (G \circ (i+1,1))_{\mathbf{1}} \wedge (G \circ (1,\mathrm{width}\, G))_{\mathbf{2}} \leq s\}$.

(32) If $1 \leq i$ and $i < \mathrm{len}\, G$ and $1 \leq j$ and $j < \mathrm{width}\, G$, then $\mathrm{cell}(G,i, j) = \{[r,s] : (G \circ (i,1))_{\mathbf{1}} \leq r \wedge r \leq (G \circ (i+1,1))_{\mathbf{1}} \wedge (G \circ (1, j))_{\mathbf{2}} \leq s \wedge s \leq (G \circ (1, j+1))_{\mathbf{2}}\}$.

(33) For every matrix G over $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{cell}(G,i, j)$ is closed.

(34) For every non empty yielding matrix G over $\mathcal{E}^2_{\mathrm{T}}$ holds $1 \leq \mathrm{len}\, G$ and $1 \leq \mathrm{width}\, G$.

(35) For every Go-board G such that $i \leq \mathrm{len}\, G$ and $j \leq \mathrm{width}\, G$ holds $\mathrm{cell}(G,i,j) = \overline{\mathrm{Int}\,\mathrm{cell}(G,i,j)}$.

REFERENCES

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/nat_1.html.

[2] Grzegorz Bancerek. The ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/ordinal1.html.

[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/finseq_1.html.

[4] Agata Darmochwał. The Euclidean space. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/euclid.html.

[5] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\mathrm{T}}$. Arcs, line segments and special polygonal arcs. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/topreal1.html.

[6] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/matrix_1.html.

[7] Stanisława Kanas, Adam Lecko, and Mariusz Startek. Metric spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/metric_1.html.

[8] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-Board — part I. *Journal of Formalized Mathematics*, 4, 1992. http://mizar.org/JFM/Vol4/goboard1.html.

[9] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-Board — part II. *Journal of Formalized Mathematics*, 4, 1992. http://mizar.org/JFM/Vol4/goboard2.html.

[10] Yatsuka Nakamura and Andrzej Trybulec. Decomposing a Go-Board into cells. *Journal of Formalized Mathematics*, 7, 1995. http://mizar.org/JFM/Vol7/goboard5.html.

[11] Yatsuka Nakamura and Andrzej Trybulec. Components and unions of components. *Journal of Formalized Mathematics*, 8, 1996. http://mizar.org/JFM/Vol8/connsp_3.html.

[12] Beata Padlewska. Connected spaces. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/connsp_1.html.

[13] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/pre_topc.html.

[14] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[15] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[16] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/tops_1.html.

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