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Adjacency Concept for Pairs of Natural Numbers

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Summary. First, we introduce the concept of adjacency for a pair of natural numbers. Second, we extend the concept for two pairs of natural numbers. The pairs represent points of a lattice in a plane. We show that if some property is infectious among adjacent points, and some points have the property, then all points have the property.

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The articles [9], [4], [12], [10], [1], [7], [13], [3], [2], [5], [11], [6], and [8] provide the notation and terminology for this paper.

In this paper i, j, k, k_1, k_2, n, m, i_1, i_2, j_1, j_2 denote natural numbers.

Let us consider i_1, i_2. We say that i_1 and i_2 are adjacent if and only if:

(Def. 1) $i_2 = i_1 + 1$ or $i_1 = i_2 + 1$.

Let us notice that the predicate i_1 and i_2 are adjacent is irreflexive and symmetric.

We now state two propositions:

(1) For all i_1, i_2 such that i_1 and i_2 are adjacent holds $i_1 + 1$ and $i_2 + 1$ are adjacent.

(2) For all i_1, i_2 such that i_1 and i_2 are adjacent and $1 \leq i_1$ and $1 \leq i_2$ holds $i_1 -' 1$ and $i_2 -' 1$ are adjacent.

Let us consider i_1, j_1, i_2, j_2. We say that i_1, j_1, i_2, and j_2 are adjacent if and only if:

(Def. 2) i_1 and i_2 are adjacent and $j_1 = j_2$ or $i_1 = i_2$ and j_1 and j_2 are adjacent.

One can prove the following two propositions:

(3) For all i_1, i_2, j_1, j_2 such that i_1, j_1, i_2, and j_2 are adjacent holds $i_1 + 1$, $j_1 + 1$, $i_2 + 1$, and $j_2 + 1$ are adjacent.

(4) Let given i_1, i_2, j_1, j_2. Suppose i_1, j_1, i_2, and j_2 are adjacent and $1 \leq i_1$ and $1 \leq i_2$ and $1 \leq j_1$ and $1 \leq j_2$. Then $i_1 -' 1$, $j_1 -' 1$, $i_2 -' 1$, and $j_2 -' 1$ are adjacent.

Let us consider n, i. Then $n \mapsto i$ is a finite sequence of elements of $\mathbb{N}$ and it can be characterized by the condition:

(Def. 3) $\mathrm{len}(n \mapsto i) = n$ and for every j such that $1 \leq j$ and $j \leq n$ holds $(n \mapsto i)(j) = i$.

The following propositions are true:

(6)[1] Let given n, i, j. Suppose $i \leq n$ and $j \leq n$. Then there exists a finite sequence f_1 of elements of $\mathbb{N}$ such that

(i) $f_1(1) = i$,

(ii) $f_1(\operatorname{len} f_1) = j$,

(iii) $\operatorname{len} f_1 = (i -' j) + (j -' i) + 1$,

(iv) for all k, k_1 such that $1 \leq k$ and $k \leq \operatorname{len} f_1$ and $k_1 = f_1(k)$ holds $k_1 \leq n$, and

(v) for every i_1 such that $1 \leq i_1$ and $i_1 < \operatorname{len} f_1$ holds $f_1(i_1 + 1) = (f_1)_{i_1} + 1$ or $f_1(i_1) = (f_1)_{i_1+1} + 1$.

(7) Let given n, i, j. Suppose $i \leq n$ and $j \leq n$. Then there exists a finite sequence f_1 of elements of $\mathbb{N}$ such that

(i) $f_1(1) = i$,

(ii) $f_1(\operatorname{len} f_1) = j$,

(iii) $\operatorname{len} f_1 = (i -' j) + (j -' i) + 1$,

(iv) for all k, k_1 such that $1 \leq k$ and $k \leq \operatorname{len} f_1$ and $k_1 = f_1(k)$ holds $k_1 \leq n$, and

(v) for every i_1 such that $1 \leq i_1$ and $i_1 < \operatorname{len} f_1$ holds $(f_1)_{i_1}$ and $(f_1)_{i_1+1}$ are adjacent.

(8) Let given n, m, i_1, j_1, i_2, j_2. Suppose $i_1 \leq n$ and $j_1 \leq m$ and $i_2 \leq n$ and $j_2 \leq m$. Then there exist finite sequences f_1, f_2 of elements of $\mathbb{N}$ such that

for all i, k_1, k_2 such that $i \in \operatorname{dom} f_1$ and $k_1 = f_1(i)$ and $k_2 = f_2(i)$ holds $k_1 \leq n$ and $k_2 \leq m$ and $f_1(1) = i_1$ and $f_1(\operatorname{len} f_1) = i_2$ and $f_2(1) = j_1$ and $f_2(\operatorname{len} f_2) = j_2$ and $\operatorname{len} f_1 = \operatorname{len} f_2$ and $\operatorname{len} f_1 = (i_1 -' i_2) + (i_2 -' i_1) + (j_1 -' j_2) + (j_2 -' j_1) + 1$ and for every i such that $1 \leq i$ and $i < \operatorname{len} f_1$ holds $(f_1)_i$, $(f_2)_i$, $(f_1)_{i+1}$, and $(f_2)_{i+1}$ are adjacent.

In the sequel S denotes a set.

The following proposition is true

(9) Let Y be a subset of S and F be a matrix over 2^S of dimension $n \times m$. Suppose that

(i) there exist i, j such that $i \in \operatorname{Seg} n$ and $j \in \operatorname{Seg} m$ and $F \circ (i, j) \subseteq Y$, and

(ii) for all i_1, j_1, i_2, j_2 such that $i_1 \in \operatorname{Seg} n$ and $i_2 \in \operatorname{Seg} n$ and $j_1 \in \operatorname{Seg} m$ and $j_2 \in \operatorname{Seg} m$ and i_1, j_1, i_2, and j_2 are adjacent holds $F \circ (i_1, j_1) \subseteq Y$ iff $F \circ (i_2, j_2) \subseteq Y$.

Let given i, j. If $i \in \operatorname{Seg} n$ and $j \in \operatorname{Seg} m$, then $F \circ (i, j) \subseteq Y$.

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[1] The proposition (5) has been removed.

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