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Introduction to Go-Board — Part II

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Summary. In article we define Go-board determined by finite sequence of points from topological space $\mathcal{E}^2_{\mathrm{T}}$. A few facts about this notation are proved.

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The articles [14], [6], [17], [3], [15], [2], [18], [5], [4], [7], [13], [1], [11], [16], [10], [8], [9], and [12] provide the notation and terminology for this paper.

1. Real Numbers Preliminaries

For simplicity, we adopt the following convention: f, f_1, f_2, g denote finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$, v, v_1, v_2 denote finite sequences of elements of $\mathbb{R}$, n, m, i, j, k denote natural numbers, and G denotes a Go-board.

The scheme *PiLambdaD* deals with a non empty set $\mathcal{A}$, a natural number $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, and states that:

> There exists a finite sequence g of elements of $\mathcal{A}$ such that $\mathrm{len}\, g = \mathcal{B}$ and for every n such that $n \in \mathrm{dom}\, g$ holds $g_n = \mathcal{F}(n)$

for all values of the parameters.

The following proposition is true

(1) For every finite subset R of $\mathbb{R}$ such that $R \neq \emptyset$ holds R is upper bounded and $\sup R \in R$ and R is lower bounded and $\inf R \in R$.

2. Properties of Finite Sequences of Points from $\mathcal{E}^2_{\mathrm{T}}$

Next we state a number of propositions:

(3)[1] For every finite sequence f holds $1 \leq n$ and $n+1 \leq \mathrm{len}\, f$ iff $n \in \mathrm{dom}\, f$ and $n+1 \in \mathrm{dom}\, f$.

(4) For every finite sequence f holds $1 \leq n$ and $n+2 \leq \mathrm{len}\, f$ iff $n \in \mathrm{dom}\, f$ and $n+1 \in \mathrm{dom}\, f$ and $n+2 \in \mathrm{dom}\, f$.

(5) Let D be a non empty set, f_1, f_2 be finite sequences of elements of D, and given n. If $1 \leq n$ and $n \leq \mathrm{len}\, f_2$, then $(f_1 \frown f_2)_{n+\mathrm{len}\, f_1} = (f_2)_n$.

(6) If for all n, m such that $m > n+1$ and $n \in \mathrm{dom}\, f$ and $n+1 \in \mathrm{dom}\, f$ and $m \in \mathrm{dom}\, f$ and $m+1 \in \mathrm{dom}\, f$ holds $\mathcal{L}(f,n)$ misses $\mathcal{L}(f,m)$, then f is s.n.c..

[1] The proposition (2) has been removed.

(7) If f is unfolded, s.n.c., and one-to-one and $f_{\mathrm{len}\,f} \in \mathcal{L}(f,i)$ and $i \in \mathrm{dom}\, f$ and $i+1 \in \mathrm{dom}\, f$, then $i+1 = \mathrm{len}\, f$.

(8) If $k \neq 0$ and $\mathrm{len}\, f = k+1$, then $\widetilde{\mathcal{L}}(f) = \widetilde{\mathcal{L}}(f \restriction k) \cup \mathcal{L}(f,k)$.

(9) If $1 < k$ and $\mathrm{len}\, f = k+1$ and f is unfolded and s.n.c., then $\widetilde{\mathcal{L}}(f \restriction k) \cap \mathcal{L}(f,k) = \{f_k\}$.

(10) If $\mathrm{len}\, f_1 < n$ and $n+1 \leq \mathrm{len}(f_1 \frown f_2)$ and $m + \mathrm{len}\, f_1 = n$, then $\mathcal{L}(f_1 \frown f_2, n) = \mathcal{L}(f_2, m)$.

(11) $\widetilde{\mathcal{L}}(f) \subseteq \widetilde{\mathcal{L}}(f \frown g)$.

(12) If f is s.n.c., then $f \restriction i$ is s.n.c..

(13) If f_1 is special and if f_2 is special and if $((f_1)_{\mathrm{len}\, f_1})_{\mathbf{1}} = ((f_2)_1)_{\mathbf{1}}$ or $((f_1)_{\mathrm{len}\, f_1})_{\mathbf{2}} = ((f_2)_1)_{\mathbf{2}}$, then $f_1 \frown f_2$ is special.

(14) If $f \neq \emptyset$, then **X**-coordinate$(f) \neq \emptyset$.

(15) If $f \neq \emptyset$, then **Y**-coordinate$(f) \neq \emptyset$.

Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. One can check that **X**-coordinate(f) is non empty and **Y**-coordinate(f) is non empty.

Next we state several propositions:

(16) Suppose f is special. Let given n. Suppose $n \in \mathrm{dom}\, f$ and $n+1 \in \mathrm{dom}\, f$. Let given i, j, m, k. Suppose $\langle i, j \rangle \in$ the indices of G and $\langle m, k \rangle \in$ the indices of G and $f_n = G \circ (i,j)$ and $f_{n+1} = G \circ (m,k)$. Then $i = m$ or $k = j$.

(17) Suppose that

(i) for every n such that $n \in \mathrm{dom}\, f$ there exist i, j such that $\langle i, j \rangle \in$ the indices of G and $f_n = G \circ (i,j)$,

(ii) f is special, and

(iii) for every n such that $n \in \mathrm{dom}\, f$ and $n+1 \in \mathrm{dom}\, f$ holds $f_n \neq f_{n+1}$.

Then there exists g such that g is a sequence which elements belong to G and $\widetilde{\mathcal{L}}(f) = \widetilde{\mathcal{L}}(g)$ and $g_1 = f_1$ and $g_{\mathrm{len}\, g} = f_{\mathrm{len}\, f}$ and $\mathrm{len}\, f \leq \mathrm{len}\, g$.

(18) If v is increasing, then for all n, m such that $n \in \mathrm{dom}\, v$ and $m \in \mathrm{dom}\, v$ and $n \leq m$ holds $v(n) \leq v(m)$.

(19) If v is increasing, then for all n, m such that $n \in \mathrm{dom}\, v$ and $m \in \mathrm{dom}\, v$ and $n \neq m$ holds $v(n) \neq v(m)$.

(20) If v is increasing and $v_1 = v \restriction \mathrm{Seg}\, n$, then v_1 is increasing.

(21) For every v there exists v_1 such that $\mathrm{rng}\, v_1 = \mathrm{rng}\, v$ and $\mathrm{len}\, v_1 = \mathrm{card}\, \mathrm{rng}\, v$ and v_1 is increasing.

(22) For all v_1, v_2 such that $\mathrm{len}\, v_1 = \mathrm{len}\, v_2$ and $\mathrm{rng}\, v_1 = \mathrm{rng}\, v_2$ and v_1 is increasing and v_2 is increasing holds $v_1 = v_2$.

3. Go-Board Determined by Finite Sequence

Let v_1, v_2 be increasing finite sequences of elements of $\mathbb{R}$. Let us assume that $v_1 \neq \emptyset$ and $v_2 \neq \emptyset$. The Go-board of v_1, v_2 yields a matrix over $\mathcal{E}^2_{\mathrm{T}}$ and is defined by the conditions (Def. 1).

(Def. 1)(i) len the Go-board of v_1, $v_2 = \mathrm{len}\, v_1$,

(ii) width the Go-board of v_1, $v_2 = \mathrm{len}\, v_2$, and

(iii) for all n, m such that $\langle n, m \rangle \in$ the indices of the Go-board of v_1, v_2 holds the Go-board of v_1, $v_2 \circ (n,m) = [v_1(n), v_2(m)]$.

Let v_1, v_2 be non empty increasing finite sequences of elements of $\mathbb{R}$. Observe that the Go-board of v_1, v_2 is non empty yielding, line $\mathbf{X}$-constant, column $\mathbf{Y}$-constant, line $\mathbf{Y}$-increasing, and column $\mathbf{X}$-increasing.

Let us consider v. The functor $\mathrm{Inc}(v)$ yields an increasing finite sequence of elements of $\mathbb{R}$ and is defined by:

(Def. 2) $\mathrm{rng}\,\mathrm{Inc}(v) = \mathrm{rng}\,v$ and $\mathrm{len}\,\mathrm{Inc}(v) = \mathrm{card}\,\mathrm{rng}\,v$.

Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. The Go-board of f yields a matrix over $\mathcal{E}^2_{\mathrm{T}}$ and is defined by:

(Def. 3) The Go-board of $f =$ the Go-board of $\mathrm{Inc}(\mathbf{X}\text{-coordinate}(f))$, $\mathrm{Inc}(\mathbf{Y}\text{-coordinate}(f))$.

The following proposition is true

(23) If $v \neq \emptyset$, then $\mathrm{Inc}(v) \neq \emptyset$.

Let v be a non empty finite sequence of elements of $\mathbb{R}$. One can check that $\mathrm{Inc}(v)$ is non empty.

Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Note that the Go-board of f is non empty yielding, line $\mathbf{X}$-constant, column $\mathbf{Y}$-constant, line $\mathbf{Y}$-increasing, and column $\mathbf{X}$-increasing.

In the sequel f is a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$.

One can prove the following propositions:

(24) len the Go-board of $f = \mathrm{card}\,\mathrm{rng}\,\mathbf{X}\text{-coordinate}(f)$ and width the Go-board of $f = \mathrm{card}\,\mathrm{rng}\,\mathbf{Y}\text{-coordinate}(f)$.

(25) Let given n. Suppose $n \in \mathrm{dom}\,f$. Then there exist i, j such that $\langle i, j\rangle \in$ the indices of the Go-board of f and $f_n =$ the Go-board of $f \circ (i, j)$.

(26) If $n \in \mathrm{dom}\,f$ and for every m such that $m \in \mathrm{dom}\,f$ holds $(\mathbf{X}\text{-coordinate}(f))(n) \leq (\mathbf{X}\text{-coordinate}(f))(m)$, then $f_n \in \mathrm{rng}\,\mathrm{Line}(\text{the Go-board of } f, 1)$.

(27) If $n \in \mathrm{dom}\,f$ and for every m such that $m \in \mathrm{dom}\,f$ holds $(\mathbf{X}\text{-coordinate}(f))(m) \leq (\mathbf{X}\text{-coordinate}(f))(n)$, then $f_n \in \mathrm{rng}\,\mathrm{Line}(\text{the Go-board of } f, \mathrm{len}\,\text{the Go-board of } f)$.

(28) If $n \in \mathrm{dom}\,f$ and for every m such that $m \in \mathrm{dom}\,f$ holds $(\mathbf{Y}\text{-coordinate}(f))(n) \leq (\mathbf{Y}\text{-coordinate}(f))(m)$, then $f_n \in \mathrm{rng}((\text{the Go-board of } f)_{\Box,1})$.

(29) If $n \in \mathrm{dom}\,f$ and for every m such that $m \in \mathrm{dom}\,f$ holds $(\mathbf{Y}\text{-coordinate}(f))(m) \leq (\mathbf{Y}\text{-coordinate}(f))(n)$, then $f_n \in \mathrm{rng}((\text{the Go-board of } f)_{\Box,\mathrm{width}\,\text{the Go-board of } f})$.

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