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Basic Properties of Genetic Algorithm

Akihiko Uchibori
Yamaguchi University
Ube

Noboru Endou
Shinshu University
Nagano

Summary. We defined the set of the gene, the space treated by the genetic algorithm and the individual of the space. Moreover, we defined some genetic operators such as one point crossover and two points crossover, and the validity of many characters were proven.

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The articles [7], [8], [1], [3], [9], [2], [6], [5], and [4] provide the notation and terminology for this paper.

1. Definitions of Gene-Set, GA-Space and Individual

We follow the rules: D is a non empty set, f_1, f_2 are finite sequences of elements of D, and i, n, n_1, n_2, n_3, n_4, n_5, n_6 are natural numbers.

One can prove the following two propositions:

(1) If $n \leq \operatorname{len} f_1$, then $(f_1 \frown f_2)_{\restriction n} = ((f_1)_{\restriction n}) \frown f_2$.

(2) $(f_1 \frown f_2) \restriction (\operatorname{len} f_1 + i) = f_1 \frown (f_2 \restriction i)$.

A Gene-Set is a non-empty non empty finite sequence.

Let S be a Gene-Set. We introduce $\mathrm{GA} - \mathrm{Space} S$ as a synonym of $\bigcup S$.

Let f be a non-empty non empty function. Note that $\bigcup f$ is non empty.

Let S be a Gene-Set. A finite sequence of elements of $\mathrm{GA} - \mathrm{Space} S$ is said to be an Individual of S if:

(Def. 1) $\operatorname{len} \mathrm{it} = \operatorname{len} S$ and for every i such that $i \in \operatorname{dom} \mathrm{it}$ holds $\mathrm{it}(i) \in S(i)$.

2. Definitions of Several Genetic Operators

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA} - \mathrm{Space} S$, and let us consider n. The functor $\mathrm{crossover}(p_1, p_2, n)$ yields a finite sequence of elements of $\mathrm{GA} - \mathrm{Space} S$ and is defined by:

(Def. 2) $\mathrm{crossover}(p_1, p_2, n) = (p_1 \restriction n) \frown ((p_2)_{\restriction n})$.

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA} - \mathrm{Space} S$, and let us consider n_1, n_2. The functor $\mathrm{crossover}(p_1, p_2, n_1, n_2)$ yielding a finite sequence of elements of $\mathrm{GA} - \mathrm{Space} S$ is defined by:

(Def. 3) $\mathrm{crossover}(p_1, p_2, n_1, n_2) = \mathrm{crossover}(\mathrm{crossover}(p_1, p_2, n_1), \mathrm{crossover}(p_2, p_1, n_1), n_2)$.

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA}-\mathrm{Space}S$, and let us consider n_1, n_2, n_3. The functor $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3)$ yields a finite sequence of elements of $\mathrm{GA}-\mathrm{Space}S$ and is defined by:

(Def. 4) $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3)=\mathrm{crossover}(\mathrm{crossover}(p_1,p_2,n_1,n_2),\mathrm{crossover}(p_2,p_1,n_1,n_2),n_3)$.

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA}-\mathrm{Space}S$, and let us consider n_1, n_2, n_3, n_4. The functor $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4)$ yields a finite sequence of elements of $\mathrm{GA}-\mathrm{Space}S$ and is defined by:

(Def. 5) $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4)=\mathrm{crossover}(\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3),\mathrm{crossover}(p_2,p_1,n_1,n_2,n_3),n_4)$.

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA}-\mathrm{Space}S$, and let us consider n_1, n_2, n_3, n_4, n_5. The functor $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4,n_5)$ yields a finite sequence of elements of $\mathrm{GA}-\mathrm{Space}S$ and is defined as follows:

(Def. 6) $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4,n_5)=\mathrm{crossover}(\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4),\mathrm{crossover}(p_2,p_1,n_1,n_2,n_3,n_4),n_5)$.

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA}-\mathrm{Space}S$, and let us consider n_1, n_2, n_3, n_4, n_5, n_6. The functor $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4,n_5,n_6)$ yielding a finite sequence of elements of $\mathrm{GA}-\mathrm{Space}S$ is defined by:

(Def. 7) $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4,n_5,n_6)=\mathrm{crossover}(\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4,n_5),\mathrm{crossover}(p_2,p_1,n_1,n_2,n_3,n_4,n_5),$

3. Properties of 1-point Crossover

In the sequel S is a Gene-Set and p_1, p_2 are Individuals of S.

Next we state the proposition

(3) $\mathrm{crossover}(p_1,p_2,n)$ is an Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individuals of S, and let us consider n. Then $\mathrm{crossover}(p_1,p_2,n)$ is an Individual of S.

One can prove the following two propositions:

(4) $\mathrm{crossover}(p_1,p_2,0)=p_2$.

(5) If $n \geq \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1,p_2,n)=p_1$.

4. Properties of 2-points Crossover

Next we state the proposition

(6) $\mathrm{crossover}(p_1,p_2,n_1,n_2)$ is an Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individuals of S, and let us consider n_1, n_2. Then $\mathrm{crossover}(p_1,p_2,n_1,n_2)$ is an Individual of S.

One can prove the following propositions:

(7) $\mathrm{crossover}(p_1,p_2,0,n)=\mathrm{crossover}(p_2,p_1,n)$.

(8) $\mathrm{crossover}(p_1,p_2,n,0)=\mathrm{crossover}(p_2,p_1,n)$.

(9) If $n_1 \geq \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1,p_2,n_1,n_2)=\mathrm{crossover}(p_1,p_2,n_2)$.

(10) If $n_2 \geq \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1,p_2,n_1,n_2)=\mathrm{crossover}(p_1,p_2,n_1)$.

(11) If $n_1 \geq \mathrm{len}\, p_1$ and $n_2 \geq \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1,p_2,n_1,n_2)=p_1$.

(12) $\mathrm{crossover}(p_1,p_2,n_1,n_1)=p_1$.

(13) $\mathrm{crossover}(p_1,p_2,n_1,n_2)=\mathrm{crossover}(p_1,p_2,n_2,n_1)$.

5. Properties of 3-points Crossover

The following proposition is true

(14) crossover$(p_1, p_2, n_1, n_2, n_3)$ is an Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individuals of S, and let us consider n_1, n_2, n_3. Then crossover$(p_1, p_2, n_1, n_2, n_3)$ is an Individual of S.

We now state a number of propositions:

(15) crossover$(p_1, p_2, 0, n_2, n_3)$ = crossover(p_2, p_1, n_2, n_3) and crossover$(p_1, p_2, n_1, 0, n_3)$ = crossover(p_2, p_1, n_1, n_3) and crossover$(p_1, p_2, n_1, n_2, 0)$ = crossover(p_2, p_1, n_1, n_2).

(16) crossover$(p_1, p_2, 0, 0, n_3)$ = crossover(p_1, p_2, n_3) and crossover$(p_1, p_2, n_1, 0, 0)$ = crossover(p_1, p_2, n_1) and crossover$(p_1, p_2, 0, n_2, 0)$ = crossover(p_1, p_2, n_2).

(17) crossover$(p_1, p_2, 0, 0, 0) = p_2$.

(18) If $n_1 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3)$ = crossover(p_1, p_2, n_2, n_3).

(19) If $n_2 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3)$ = crossover(p_1, p_2, n_1, n_3).

(20) If $n_3 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3)$ = crossover(p_1, p_2, n_1, n_2).

(21) If $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3)$ = crossover(p_1, p_2, n_3).

(22) If $n_1 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3)$ = crossover(p_1, p_2, n_2).

(23) If $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3)$ = crossover(p_1, p_2, n_1).

(24) If $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3) = p_1$.

(25) crossover$(p_1, p_2, n_1, n_2, n_3)$ = crossover$(p_1, p_2, n_2, n_1, n_3)$ and crossover$(p_1, p_2, n_1, n_2, n_3)$ = crossover$(p_1, p_2, n_1, n_3, n_2)$.

(26) crossover$(p_1, p_2, n_1, n_2, n_3)$ = crossover$(p_1, p_2, n_3, n_1, n_2)$.

(27) crossover$(p_1, p_2, n_1, n_1, n_3)$ = crossover(p_1, p_2, n_3) and crossover$(p_1, p_2, n_1, n_2, n_1)$ = crossover(p_1, p_2, n_2) and crossover$(p_1, p_2, n_1, n_2, n_2)$ = crossover(p_1, p_2, n_1).

6. Properties of 4-points Crossover

Next we state the proposition

(28) crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ is an Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individuals of S, and let us consider n_1, n_2, n_3, n_4. Then crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ is an Individual of S.

One can prove the following propositions:

(29) crossover$(p_1, p_2, 0, n_2, n_3, n_4)$ = crossover$(p_2, p_1, n_2, n_3, n_4)$ and crossover$(p_1, p_2, n_1, 0, n_3, n_4)$ = crossover$(p_2, p_1, n_1, n_3, n_4)$ and crossover$(p_1, p_2, n_1, n_2, 0, n_4)$ = crossover$(p_2, p_1, n_1, n_2, n_4)$ and crossover$(p_1, p_2, n_1, n_2, n_3, 0)$ = crossover$(p_2, p_1, n_1, n_2, n_3)$.

(30) crossover$(p_1, p_2, 0, 0, n_3, n_4)$ = crossover(p_1, p_2, n_3, n_4) and crossover$(p_1, p_2, 0, n_2, 0, n_4)$ = crossover(p_1, p_2, n_2, n_4) and crossover$(p_1, p_2, 0, n_2, n_3, 0)$ = crossover(p_1, p_2, n_2, n_3) and crossover$(p_1, p_2, n_1, 0, n_3, 0)$ = crossover(p_1, p_2, n_1, n_3) and crossover$(p_1, p_2, n_1, 0, 0, n_4)$ = crossover(p_1, p_2, n_1, n_4) and crossover$(p_1, p_2, n_1, n_2, 0, 0)$ = crossover(p_1, p_2, n_1, n_2).

(31) crossover$(p_1, p_2, n_1, 0, 0, 0)$ = crossover(p_2, p_1, n_1) and crossover$(p_1, p_2, 0, n_2, 0, 0)$ = crossover(p_2, p_1, n_2) and crossover$(p_1, p_2, 0, 0, n_3, 0)$ = crossover(p_2, p_1, n_3) and crossover$(p_1, p_2, 0, 0, 0, n_4)$ = crossover(p_2, p_1, n_4).

(32) crossover$(p_1, p_2, 0, 0, 0, 0) = p_1$.

(33)(i) If $n_1 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_2, n_3, n_4)$,

(ii) if $n_2 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_1, n_3, n_4)$,

(iii) if $n_3 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_1, n_2, n_4)$, and

(iv) if $n_4 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_1, n_2, n_3)$.

(34)(i) If $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover(p_1, p_2, n_3, n_4),

(ii) if $n_1 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover(p_1, p_2, n_2, n_4),

(iii) if $n_1 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover(p_1, p_2, n_2, n_3),

(iv) if $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover(p_1, p_2, n_1, n_4),

(v) if $n_2 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover(p_1, p_2, n_1, n_3), and

(vi) if $n_3 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover(p_1, p_2, n_1, n_2).

(35)(i) If $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover(p_1, p_2, n_4),

(ii) if $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover(p_1, p_2, n_3),

(iii) if $n_1 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover(p_1, p_2, n_2), and

(iv) if $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover(p_1, p_2, n_1).

(36) If $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2, n_3, n_4) = p_1$.

(37) crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_1, n_2, n_4, n_3)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_1, n_3, n_2, n_4)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_1, n_3, n_4, n_2)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_1, n_4, n_2, n_3)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_1, n_4, n_3, n_2)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_2, n_1, n_3, n_4)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_2, n_1, n_4, n_3)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_2, n_3, n_1, n_4)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_2, n_3, n_4, n_1)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_2, n_4, n_1, n_3)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_2, n_4, n_3, n_1)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_3, n_1, n_2, n_4)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_3, n_1, n_4, n_2)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_3, n_2, n_1, n_4)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_3, n_2, n_4, n_1)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_3, n_4, n_1, n_2)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_3, n_4, n_2, n_1)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_4, n_1, n_2, n_3)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_4, n_1, n_3, n_2)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_4, n_2, n_1, n_3)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_4, n_2, n_3, n_1)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_4, n_3, n_1, n_2)$ and crossover$(p_1, p_2, n_1, n_2, n_3, n_4) =$ crossover$(p_1, p_2, n_4, n_3, n_2, n_1)$.

(38) crossover$(p_1, p_2, n_1, n_1, n_3, n_4) =$ crossover(p_1, p_2, n_3, n_4) and crossover$(p_1, p_2, n_1, n_2, n_1, n_4) =$ crossover(p_1, p_2, n_2, n_4) and crossover$(p_1, p_2, n_1, n_2, n_3, n_1) =$ crossover(p_1, p_2, n_2, n_3) and crossover$(p_1, p_2, n_1, n_2, n_2, n_4) =$ crossover(p_1, p_2, n_1, n_4) and crossover$(p_1, p_2, n_1, n_2, n_3, n_2) =$ crossover(p_1, p_2, n_1, n_3) and crossover$(p_1, p_2, n_1, n_2, n_3, n_3) =$ crossover(p_1, p_2, n_1, n_2).

(39) crossover$(p_1, p_2, n_1, n_1, n_3, n_3) = p_1$ and crossover$(p_1, p_2, n_1, n_2, n_1, n_2) = p_1$ and crossover$(p_1, p_2, n_1, n_2, n_2, n_1) = p_1$.

7. Properties of 5-points Crossover

One can prove the following proposition

(40) $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5)$ is an Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individuals of S, and let us consider n_1, n_2, n_3, n_4, n_5. Then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5)$ is an Individual of S.

Next we state a number of propositions:

(41) $\text{crossover}(p_1, p_2, 0, n_2, n_3, n_4, n_5) = \text{crossover}(p_2, p_1, n_2, n_3, n_4, n_5)$ and $\text{crossover}(p_1, p_2, n_1, 0, n_3, n_4, n_5) = \text{crossover}(p_2, p_1, n_1, n_3, n_4, n_5)$ and $\text{crossover}(p_1, p_2, n_1, n_2, 0, n_4, n_5) = \text{crossover}(p_2, p_1, n_1, n_2, n_4, n_5)$ and $\text{crossover}(p_1, p_2, n_1, n_2, n_3, 0, n_5) = \text{crossover}(p_2, p_1, n_1, n_2, n_3, n_5)$ and $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, 0) = \text{crossover}(p_2, p_1, n_1, n_2, n_3, n_4)$.

(42) $\text{crossover}(p_1, p_2, 0, 0, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_3, n_4, n_5)$ and $\text{crossover}(p_1, p_2, 0, n_2, 0, n_4, n_5) = \text{crossover}(p_1, p_2, n_2, n_4, n_5)$ and $\text{crossover}(p_1, p_2, 0, n_2, n_3, 0, n_5) = \text{crossover}(p_1, p_2, n_2, n_3, n_5)$ and $\text{crossover}(p_1, p_2, 0, n_2, n_3, n_4, 0) = \text{crossover}(p_1, p_2, n_2, n_3, n_4)$ and $\text{crossover}(p_1, p_2, n_1, 0, 0, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_4, n_5)$ and $\text{crossover}(p_1, p_2, n_1, 0, n_3, 0, n_5) = \text{crossover}(p_1, p_2, n_1, n_3, n_5)$ and $\text{crossover}(p_1, p_2, n_1, 0, n_3, n_4, 0) = \text{crossover}(p_1, p_2, n_1, n_3, n_4)$ and $\text{crossover}(p_1, p_2, n_1, n_2, 0, 0, n_5) = \text{crossover}(p_1, p_2, n_1, n_2, n_5)$ and $\text{crossover}(p_1, p_2, n_1, n_2, 0, n_4, 0) = \text{crossover}(p_1, p_2, n_1, n_2, n_4)$ and $\text{crossover}(p_1, p_2, n_1, n_2, n_3, 0, 0) = \text{crossover}(p_1, p_2, n_1, n_2, n_3)$.

(43) $\text{crossover}(p_1, p_2, 0, 0, 0, n_4, n_5) = \text{crossover}(p_2, p_1, n_4, n_5)$ and $\text{crossover}(p_1, p_2, 0, 0, n_3, 0, n_5) = \text{crossover}(p_2, p_1, n_3, n_5)$ and $\text{crossover}(p_1, p_2, 0, 0, n_3, n_4, 0) = \text{crossover}(p_2, p_1, n_3, n_4)$ and $\text{crossover}(p_1, p_2, 0, n_2, 0, 0, n_5) = \text{crossover}(p_2, p_1, n_2, n_5)$ and $\text{crossover}(p_1, p_2, 0, n_2, 0, n_4, 0) = \text{crossover}(p_2, p_1, n_2, n_4)$ and $\text{crossover}(p_1, p_2, 0, n_2, n_3, 0, 0) = \text{crossover}(p_2, p_1, n_2, n_3)$ and $\text{crossover}(p_1, p_2, n_1, 0, 0, 0, n_5) = \text{crossover}(p_2, p_1, n_1, n_5)$ and $\text{crossover}(p_1, p_2, n_1, 0, 0, n_4, 0) = \text{crossover}(p_2, p_1, n_1, n_4)$ and $\text{crossover}(p_1, p_2, n_1, 0, n_3, 0, 0) = \text{crossover}(p_2, p_1, n_1, n_3)$ and $\text{crossover}(p_1, p_2, n_1, n_2, 0, 0, 0) = \text{crossover}(p_2, p_1, n_1, n_2)$.

(44) $\text{crossover}(p_1, p_2, 0, 0, 0, 0, n_5) = \text{crossover}(p_1, p_2, n_5)$ and $\text{crossover}(p_1, p_2, 0, 0, 0, n_4, 0) = \text{crossover}(p_1, p_2, n_4)$ and $\text{crossover}(p_1, p_2, 0, 0, n_3, 0, 0) = \text{crossover}(p_1, p_2, n_3)$ and $\text{crossover}(p_1, p_2, 0, n_2, 0, 0, 0) = \text{crossover}(p_1, p_2, n_2)$ and $\text{crossover}(p_1, p_2, n_1, 0, 0, 0, 0) = \text{crossover}(p_1, p_2, n_1)$.

(45) $\text{crossover}(p_1, p_2, 0, 0, 0, 0, 0) = p_2$.

(46)(i) If $n_1 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_2, n_3, n_4, n_5)$,

(ii) if $n_2 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_3, n_4, n_5)$,

(iii) if $n_3 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_2, n_4, n_5)$,

(iv) if $n_4 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_2, n_3, n_5)$, and

(v) if $n_5 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4)$.

(47)(i) If $n_1 \geq \text{len}\, p_1$ and $n_2 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_3, n_4, n_5)$,

(ii) if $n_1 \geq \text{len}\, p_1$ and $n_3 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_2, n_4, n_5)$,

(iii) if $n_1 \geq \text{len}\, p_1$ and $n_4 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_2, n_3, n_5)$,

(iv) if $n_1 \geq \text{len}\, p_1$ and $n_5 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_2, n_3, n_4)$,

(v) if $n_2 \geq \text{len}\, p_1$ and $n_3 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_4, n_5)$,

(vi) if $n_2 \geq \text{len}\, p_1$ and $n_4 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_3, n_5)$,

(vii) if $n_2 \geq \text{len}\, p_1$ and $n_5 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_3, n_4)$,

(viii) if $n_3 \geq \text{len}\, p_1$ and $n_4 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_2, n_5)$,

(ix) if $n_3 \geq \text{len}\, p_1$ and $n_5 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_2, n_4)$, and

(x) if $n_4 \geq \text{len}\, p_1$ and $n_5 \geq \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_2, n_3)$.

(48)(i) If $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_4, n_5)$,

(ii) if $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_3, n_5)$,

(iii) if $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$ and $n_5 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_3, n_4)$,

(iv) if $n_1 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_5)$,

(v) if $n_1 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_5 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_4)$,

(vi) if $n_1 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$ and $n_5 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_3)$,

(vii) if $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_5)$,

(viii) if $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_5 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_4)$,

(ix) if $n_2 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$ and $n_5 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_3)$, and

(x) if $n_3 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$ and $n_5 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_2)$.

(49)(i) If $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_5)$,

(ii) if $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_5 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_4)$,

(iii) if $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$ and $n_5 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_3)$,

(iv) if $n_1 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$ and $n_5 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2)$, and

(v) if $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$ and $n_5 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1)$.

(50) If $n_1 \geq \operatorname{len} p_1$ and $n_2 \geq \operatorname{len} p_1$ and $n_3 \geq \operatorname{len} p_1$ and $n_4 \geq \operatorname{len} p_1$ and $n_5 \geq \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = p_1$.

(51) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_1, n_3, n_4, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_3, n_2, n_1, n_4, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_4, n_2, n_3, n_1, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_5, n_2, n_3, n_4, n_1)$.

(52) $\operatorname{crossover}(p_1, p_2, n_1, n_1, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_3, n_4, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_1, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_4, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_1, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_3, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_1) = \operatorname{crossover}(p_1, p_2, n_2, n_3, n_4)$.

8. PROPERTIES OF 6-POINTS CROSSOVER

We now state the proposition

(53) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6)$ is an Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individuals of S, and let us consider n_1, n_2, n_3, n_4, n_5, n_6. Then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6)$ is an Individual of S.

We now state four propositions:

(54)(i) $\mathrm{crossover}(p_1, p_2, 0, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_2, p_1, n_2, n_3, n_4, n_5, n_6)$,
(ii) $\mathrm{crossover}(p_1, p_2, n_1, 0, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_2, p_1, n_1, n_3, n_4, n_5, n_6)$,
(iii) $\mathrm{crossover}(p_1, p_2, n_1, n_2, 0, n_4, n_5, n_6) = \mathrm{crossover}(p_2, p_1, n_1, n_2, n_4, n_5, n_6)$,
(iv) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, 0, n_5, n_6) = \mathrm{crossover}(p_2, p_1, n_1, n_2, n_3, n_5, n_6)$,
(v) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, 0, n_6) = \mathrm{crossover}(p_2, p_1, n_1, n_2, n_3, n_4, n_6)$, and
(vi) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, 0) = \mathrm{crossover}(p_2, p_1, n_1, n_2, n_3, n_4, n_5)$.

(55)(i) If $n_1 \geq \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_2, n_3, n_4, n_5, n_6)$,
(ii) if $n_2 \geq \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_1, n_3, n_4, n_5, n_6)$,
(iii) if $n_3 \geq \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_1, n_2, n_4, n_5, n_6)$,
(iv) if $n_4 \geq \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_5, n_6)$,
(v) if $n_5 \geq \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_6)$, and
(vi) if $n_6 \geq \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5)$.

(56)(i) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_2, n_1, n_3, n_4, n_5, n_6)$,
(ii) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_3, n_2, n_1, n_4, n_5, n_6)$,
(iii) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_4, n_2, n_3, n_1, n_5, n_6)$,
(iv) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_5, n_2, n_3, n_4, n_1, n_6)$, and
(v) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_6, n_2, n_3, n_4, n_5, n_1)$.

(57)(i) $\mathrm{crossover}(p_1, p_2, n_1, n_1, n_3, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_3, n_4, n_5, n_6)$,
(ii) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_1, n_4, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_2, n_4, n_5, n_6)$,
(iii) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_1, n_5, n_6) = \mathrm{crossover}(p_1, p_2, n_2, n_3, n_5, n_6)$,
(iv) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_1, n_6) = \mathrm{crossover}(p_1, p_2, n_2, n_3, n_4, n_6)$, and
(v) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_1) = \mathrm{crossover}(p_1, p_2, n_2, n_3, n_4, n_5)$.

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