JOURNAL OF FORMALIZED MATHEMATICS
Volume 4, Released 1992, Published 2003
Inst. of Computer Science, Univ. of Białystok

Definitions of Petri Net. Part I

Waldemar Korczyński
Pedagogical University
Kielce

Summary. In the paper the classical definition of Petri net is described. The article also contains some theorems needed for proving equivalences of these definitions with other definitions of Petri net as relational algebras. See [3], [4] and other.

MML Identifier: `FF_SIEC`.

WWW: `http://mizar.org/JFM/Vol4/ff_siec.html`

The articles [5], [1], [6], and [2] provide the notation and terminology for this paper.

In this paper x, y, X, Y denote sets.

Let N be a net. The functor chaos(N) yields a set and is defined by:

(Def. 2)[1] $\mathrm{chaos}(N) = \mathrm{Elements}(N) \cup \{\mathrm{Elements}(N)\}$.

In the sequel M is a Petri net.

Let us consider X, Y. Let us assume that X misses Y. The functor $\mathrm{PTempty}_f(X,Y)$ yielding a strict Petri net is defined by:

(Def. 4)[2] $\mathrm{PTempty}_f(X,Y) = \langle X, Y, \emptyset \rangle$.

Let us consider X. The functor $\mathrm{Tempty}_f(X)$ yields a strict Petri net and is defined by:

(Def. 5) $\mathrm{Tempty}_f(X) = \mathrm{PTempty}_f(X, \emptyset)$.

The functor $\mathrm{Pempty}_f(X)$ yields a strict Petri net and is defined by:

(Def. 6) $\mathrm{Pempty}_f(X) = \mathrm{PTempty}_f(\emptyset, X)$.

Let us consider x. The functor $\mathrm{Tsingle}_f(x)$ yields a strict Petri net and is defined as follows:

(Def. 7) $\mathrm{Tsingle}_f(x) = \mathrm{PTempty}_f(\emptyset, \{x\})$.

The functor $\mathrm{Psingle}_f(x)$ yields a strict Petri net and is defined by:

(Def. 8) $\mathrm{Psingle}_f(x) = \mathrm{PTempty}_f(\{x\}, \emptyset)$.

The strict Petri net empty_f is defined as follows:

(Def. 9) $\mathrm{empty}_f = \mathrm{PTempty}_f(\emptyset, \emptyset)$.

One can prove the following propositions:

---

[1] The definition (Def. 1) has been removed.
[2] The definition (Def. 3) has been removed.

(2)[3] Suppose X misses Y. Then the places of $\mathrm{PTempty}_f(X,Y) = X$ and the transitions of $\mathrm{PTempty}_f(X,Y) = Y$ and the flow relation of $\mathrm{PTempty}_f(X,Y) = \emptyset$.

(3) The places of $\mathrm{Tempty}_f(X) = X$ and the transitions of $\mathrm{Tempty}_f(X) = \emptyset$ and the flow relation of $\mathrm{Tempty}_f(X) = \emptyset$.

(4) For every X holds the places of $\mathrm{Pempty}_f(X) = \emptyset$ and the transitions of $\mathrm{Pempty}_f(X) = X$ and the flow relation of $\mathrm{Pempty}_f(X) = \emptyset$.

(5) For every x holds the places of $\mathrm{Tsingle}_f(x) = \emptyset$ and the transitions of $\mathrm{Tsingle}_f(x) = \{x\}$ and the flow relation of $\mathrm{Tsingle}_f(x) = \emptyset$.

(6) For every x holds the places of $\mathrm{Psingle}_f(x) = \{x\}$ and the transitions of $\mathrm{Psingle}_f(x) = \emptyset$ and the flow relation of $\mathrm{Psingle}_f(x) = \emptyset$.

(7) The places of $\mathrm{empty}_f = \emptyset$ and the transitions of $\mathrm{empty}_f = \emptyset$ and the flow relation of $\mathrm{empty}_f = \emptyset$.

(8) The places of $M \subseteq \mathrm{Elements}(M)$ and the transitions of $M \subseteq \mathrm{Elements}(M)$.

(11)[4](i) If $\langle x, y\rangle \in$ the flow relation of M and $x \in$ the transitions of M, then $x \notin$ the places of M and $y \notin$ the transitions of M and $y \in$ the places of M,

(ii) if $\langle x, y\rangle \in$ the flow relation of M and $y \in$ the transitions of M, then $y \notin$ the places of M and $x \notin$ the transitions of M and $x \in$ the places of M,

(iii) if $\langle x, y\rangle \in$ the flow relation of M and $x \in$ the places of M, then $y \notin$ the places of M and $x \notin$ the transitions of M and $y \in$ the transitions of M, and

(iv) if $\langle x, y\rangle \in$ the flow relation of M and $y \in$ the places of M, then $x \notin$ the places of M and $y \notin$ the transitions of M and $x \in$ the transitions of M.

(12) $\mathrm{chaos}(M) \neq \emptyset$.

(13) The flow relation of $M \subseteq [:\mathrm{Elements}(M), \mathrm{Elements}(M):]$ and (the flow relation of $M)^{\smile} \subseteq [:\mathrm{Elements}(M), \mathrm{Elements}(M):]$.

(14) rng((the flow relation of M)$\restriction$(the transitions of M)) $\subseteq$ the places of M and rng((the flow relation of $M)^{\smile}\restriction$the transitions of M) $\subseteq$ the places of M and dom((the flow relation of M)$\restriction$(the transitions of M)) $\subseteq$ the transitions of M and dom((the flow relation of $M)^{\smile}\restriction$the transitions of M) $\subseteq$ the transitions of M and rng((the flow relation of M)$\restriction$(the places of M)) $\subseteq$ the transitions of M and rng((the flow relation of $M)^{\smile}\restriction$the places of M) $\subseteq$ the transitions of M and dom((the flow relation of M)$\restriction$(the places of M)) $\subseteq$ the places of M and dom((the flow relation of $M)^{\smile}\restriction$the places of M) $\subseteq$ the places of M and $\mathrm{rng}(\mathrm{id}_{\text{the transitions of } M}) \subseteq$ the transitions of M and $\mathrm{dom}(\mathrm{id}_{\text{the transitions of } M}) \subseteq$ the transitions of M and $\mathrm{rng}(\mathrm{id}_{\text{the places of } M}) \subseteq$ the places of M and $\mathrm{dom}(\mathrm{id}_{\text{the places of } M}) \subseteq$ the places of M.

(15) rng((the flow relation of M)$\restriction$(the transitions of M)) misses dom((the flow relation of M)$\restriction$(the transitions of M)) and rng((the flow relation of M)$\restriction$(the transitions of M)) misses dom((the flow relation of $M)^{\smile}\restriction$the transitions of M) and rng((the flow relation of M)$\restriction$(the transitions of M)) misses $\mathrm{dom}(\mathrm{id}_{\text{the transitions of } M})$ and rng((the flow relation of $M)^{\smile}\restriction$the transitions of M) misses dom((the flow relation of M)$\restriction$(the transitions of M)) and rng((the flow relation of $M)^{\smile}\restriction$the transitions of M) misses dom((the flow relation of $M)^{\smile}\restriction$the transitions of M) and rng((the flow relation of $M)^{\smile}\restriction$the transitions of M) misses $\mathrm{dom}(\mathrm{id}_{\text{the transitions of } M})$ and dom((the flow relation of M)$\restriction$(the transitions of M)) misses rng((the flow relation of M)$\restriction$(the transitions of M)) and dom((the flow relation of M)$\restriction$(the transitions of M)) misses rng((the flow relation of $M)^{\smile}\restriction$the transitions of M) and dom((the flow relation of M)$\restriction$(the transitions of M)) misses $\mathrm{rng}(\mathrm{id}_{\text{the places of } M})$ and dom((the flow relation of $M)^{\smile}\restriction$the transitions of M) misses rng((the flow relation of M)$\restriction$(the transitions of M)) and dom((the flow

[3] The proposition (1) has been removed.

[4] The propositions (9) and (10) have been removed.

relation of $M)^{\smile}\restriction$the transitions of M) misses rng((the flow relation of $M)^{\smile}\restriction$the transitions of M) and dom((the flow relation of $M)^{\smile}\restriction$the transitions of M) misses rng($\mathrm{id}_{\text{the places of } M}$) and rng((the flow relation of $M)\restriction$(the places of M)) misses dom((the flow relation of $M)\restriction$(the places of M)) and rng((the flow relation of $M)\restriction$(the places of M)) misses dom((the flow relation of $M)^{\smile}\restriction$the places of M) and rng((the flow relation of $M)\restriction$(the places of M)) misses dom($\mathrm{id}_{\text{the places of } M}$) and rng((the flow relation of $M)^{\smile}\restriction$the places of M) misses dom((the flow relation of $M)\restriction$(the places of M)) and rng((the flow relation of $M)^{\smile}\restriction$the places of M) misses dom((the flow relation of $M)^{\smile}\restriction$the places of M) and rng((the flow relation of $M)^{\smile}\restriction$the places of M) misses dom($\mathrm{id}_{\text{the places of } M}$) and dom((the flow relation of $M)\restriction$(the places of M)) misses rng((the flow relation of $M)\restriction$(the places of M)) and dom((the flow relation of $M)\restriction$(the places of M)) misses rng((the flow relation of $M)^{\smile}\restriction$the places of M) and dom((the flow relation of $M)\restriction$(the places of M)) misses rng($\mathrm{id}_{\text{the transitions of } M}$) and dom((the flow relation of $M)^{\smile}\restriction$the places of M) misses rng((the flow relation of $M)\restriction$(the places of M)) and dom((the flow relation of $M)^{\smile}\restriction$the places of M) misses rng((the flow relation of $M)^{\smile}\restriction$the places of M) and dom((the flow relation of $M)^{\smile}\restriction$the places of M) misses rng($\mathrm{id}_{\text{the transitions of } M}$).

(16) ((The flow relation of $M)\restriction$(the transitions of M)) $\cdot$ ((the flow relation of $M)\restriction$(the transitions of M)) $= \emptyset$ and ((the flow relation of $M)^{\smile}\restriction$the transitions of M) $\cdot$ ((the flow relation of $M)^{\smile}\restriction$the transitions of M) $= \emptyset$ and ((the flow relation of $M)\restriction$(the transitions of M)) $\cdot$ ((the flow relation of $M)^{\smile}\restriction$the transitions of M) $= \emptyset$ and ((the flow relation of $M)^{\smile}\restriction$the transitions of M) $\cdot$ ((the flow relation of $M)\restriction$(the transitions of M)) $= \emptyset$ and ((the flow relation of $M)\restriction$(the places of M)) $\cdot$ ((the flow relation of $M)\restriction$(the places of M)) $= \emptyset$ and ((the flow relation of $M)^{\smile}\restriction$the places of M) $\cdot$ ((the flow relation of $M)^{\smile}\restriction$the places of M) $= \emptyset$ and ((the flow relation of $M)\restriction$(the places of M)) $\cdot$ ((the flow relation of $M)^{\smile}\restriction$the places of M) $= \emptyset$ and ((the flow relation of $M)^{\smile}\restriction$the places of M) $\cdot$ ((the flow relation of $M)\restriction$(the places of M)) $= \emptyset$.

(17) ((The flow relation of $M)\restriction$(the transitions of M)) $\cdot\, \mathrm{id}_{\text{the places of } M} =$ (the flow relation of $M)\restriction$(the transitions of M) and ((the flow relation of $M)^{\smile}\restriction$the transitions of M) $\cdot\, \mathrm{id}_{\text{the places of } M} =$ (the flow relation of $M)^{\smile}\restriction$the transitions of M and $\mathrm{id}_{\text{the transitions of } M} \cdot$ ((the flow relation of $M)\restriction$(the transitions of M)) $=$ (the flow relation of $M)\restriction$(the transitions of M) and $\mathrm{id}_{\text{the transitions of } M} \cdot$ ((the flow relation of $M)^{\smile}\restriction$the transitions of M) $=$ (the flow relation of $M)^{\smile}\restriction$the transitions of M and ((the flow relation of $M)\restriction$(the places of M)) $\cdot\, \mathrm{id}_{\text{the transitions of } M} =$ (the flow relation of $M)\restriction$(the places of M) and ((the flow relation of $M)^{\smile}\restriction$the places of M) $\cdot\, \mathrm{id}_{\text{the transitions of } M} =$ (the flow relation of $M)^{\smile}\restriction$the places of M and $\mathrm{id}_{\text{the places of } M} \cdot$ ((the flow relation of $M)\restriction$(the places of M)) $=$ (the flow relation of $M)\restriction$(the places of M) and $\mathrm{id}_{\text{the places of } M} \cdot$ ((the flow relation of $M)^{\smile}\restriction$the places of M) $=$ (the flow relation of $M)^{\smile}\restriction$the places of M and ((the flow relation of $M)\restriction$(the places of M)) $\cdot\, \mathrm{id}_{\text{the transitions of } M} =$ (the flow relation of $M)\restriction$(the places of M) and ((the flow relation of $M)^{\smile}\restriction$the places of M) $\cdot\, \mathrm{id}_{\text{the transitions of } M} =$ (the flow relation of $M)^{\smile}\restriction$the places of M and $\mathrm{id}_{\text{the transitions of } M} \cdot$ ((the flow relation of $M)\restriction$(the places of M)) $= \emptyset$ and $\mathrm{id}_{\text{the transitions of } M} \cdot$ ((the flow relation of $M)^{\smile}\restriction$the places of M) $= \emptyset$ and ((the flow relation of $M)\restriction$(the places of M)) $\cdot\, \mathrm{id}_{\text{the places of } M} = \emptyset$ and ((the flow relation of $M)^{\smile}\restriction$the places of M) $\cdot\, \mathrm{id}_{\text{the places of } M} = \emptyset$ and $\mathrm{id}_{\text{the places of } M} \cdot$ ((the flow relation of $M)\restriction$(the transitions of M)) $= \emptyset$ and $\mathrm{id}_{\text{the places of } M} \cdot$ ((the flow relation of $M)^{\smile}\restriction$the transitions of M) $= \emptyset$ and ((the flow relation of $M)\restriction$(the transitions of M)) $\cdot\, \mathrm{id}_{\text{the transitions of } M} = \emptyset$ and ((the flow relation of $M)^{\smile}\restriction$the transitions of M) $\cdot\, \mathrm{id}_{\text{the transitions of } M} = \emptyset$.

(18)(i) (The flow relation of $M)^{\smile}\restriction$the transitions of M misses $\mathrm{id}_{\mathrm{Elements}(M)}$,

(ii) (the flow relation of $M)\restriction$(the transitions of M) misses $\mathrm{id}_{\mathrm{Elements}(M)}$,

(iii) (the flow relation of $M)^{\smile}\restriction$the places of M misses $\mathrm{id}_{\mathrm{Elements}(M)}$, and

(iv) (the flow relation of $M)\restriction$(the places of M) misses $\mathrm{id}_{\mathrm{Elements}(M)}$.

(19) ((The flow relation of $M)^{\smile}\restriction$the transitions of $M \cup \mathrm{id}_{\text{the places of } M}) \setminus \mathrm{id}_{\mathrm{Elements}(M)} =$ (the flow relation of $M)^{\smile}\restriction$the transitions of M and ((the flow relation of $M)\restriction$(the transitions of $M) \cup \mathrm{id}_{\text{the places of } M}) \setminus \mathrm{id}_{\mathrm{Elements}(M)} =$ (the flow relation of $M)\restriction$(the transitions of M) and ((the flow relation of $M)^{\smile}\restriction$the places of $M \cup \mathrm{id}_{\text{the places of } M}) \setminus \mathrm{id}_{\mathrm{Elements}(M)} =$ (the flow relation of $M)^{\smile}\restriction$the places of M and ((the flow relation of $M)\restriction$(the places of $M) \cup \mathrm{id}_{\text{the places of } M}) \setminus$

$\mathrm{id}_{\mathrm{Elements}(M)}$ = (the flow relation of M)$\restriction$(the places of M) and ((the flow relation of M)$^{\smile}\restriction$the places of $M \cup \mathrm{id}_{\text{the transitions of } M}) \setminus \mathrm{id}_{\mathrm{Elements}(M)}$ = (the flow relation of M)$^{\smile}\restriction$the places of M and ((the flow relation of M)$\restriction$(the places of M) $\cup\, \mathrm{id}_{\text{the transitions of } M}) \setminus \mathrm{id}_{\mathrm{Elements}(M)}$ = (the flow relation of M)$\restriction$(the places of M) and ((the flow relation of M)$^{\smile}\restriction$the transitions of $M \cup \mathrm{id}_{\text{the transitions of } M}) \setminus \mathrm{id}_{\mathrm{Elements}(M)}$ = (the flow relation of M)$^{\smile}\restriction$the transitions of M and ((the flow relation of M)$\restriction$(the transitions of M) $\cup\, \mathrm{id}_{\text{the transitions of } M}) \setminus \mathrm{id}_{\mathrm{Elements}(M)}$ = (the flow relation of M)$\restriction$(the transitions of M).

(20)(i) ((The flow relation of M)$\restriction$(the places of M))$^{\smile}$ = (the flow relation of M)$^{\smile}\restriction$the transitions of M, and

(ii) ((the flow relation of M)$\restriction$(the transitions of M))$^{\smile}$ = (the flow relation of M)$^{\smile}\restriction$the places of M.

(21)(i) (The flow relation of M)$\restriction$(the places of M) $\cup$ (the flow relation of M)$\restriction$(the transitions of M) = the flow relation of M,

(ii) (the flow relation of M)$\restriction$(the transitions of M) $\cup$ (the flow relation of M)$\restriction$(the places of M) = the flow relation of M,

(iii) ((the flow relation of M)$\restriction$(the places of M))$^{\smile}$ $\cup$ ((the flow relation of M)$\restriction$(the transitions of M))$^{\smile}$ = (the flow relation of M)$^{\smile}$, and

(iv) ((the flow relation of M)$\restriction$(the transitions of M))$^{\smile}$ $\cup$ ((the flow relation of M)$\restriction$(the places of M))$^{\smile}$ = (the flow relation of M)$^{\smile}$.

Let us consider M. The functor $\mathrm{enter}_f(M)$ yields a binary relation and is defined by:

(Def. 10) $\mathrm{enter}_f(M)$ = (the flow relation of M)$^{\smile}\restriction$the transitions of $M \cup \mathrm{id}_{\text{the places of } M}$.

The functor $\mathrm{exit}_f(M)$ yields a binary relation and is defined as follows:

(Def. 11) $\mathrm{exit}_f(M)$ = (the flow relation of M)$\restriction$(the transitions of M) $\cup\, \mathrm{id}_{\text{the places of } M}$.

The following four propositions are true:

(22) $\mathrm{exit}_f(M) \subseteq [:\mathrm{Elements}(M), \mathrm{Elements}(M):]$ and $\mathrm{enter}_f(M) \subseteq [:\mathrm{Elements}(M), \mathrm{Elements}(M):]$.

(23) $\mathrm{dom}\,\mathrm{exit}_f(M) \subseteq \mathrm{Elements}(M)$ and $\mathrm{rng}\,\mathrm{exit}_f(M) \subseteq \mathrm{Elements}(M)$ and $\mathrm{dom}\,\mathrm{enter}_f(M) \subseteq \mathrm{Elements}(M)$ and $\mathrm{rng}\,\mathrm{enter}_f(M) \subseteq \mathrm{Elements}(M)$.

(24) $\mathrm{exit}_f(M) \cdot \mathrm{exit}_f(M) = \mathrm{exit}_f(M)$ and $\mathrm{exit}_f(M) \cdot \mathrm{enter}_f(M) = \mathrm{exit}_f(M)$ and $\mathrm{enter}_f(M) \cdot \mathrm{enter}_f(M) = \mathrm{enter}_f(M)$ and $\mathrm{enter}_f(M) \cdot \mathrm{exit}_f(M) = \mathrm{enter}_f(M)$.

(25) $\mathrm{exit}_f(M) \cdot (\mathrm{exit}_f(M) \setminus \mathrm{id}_{\mathrm{Elements}(M)}) = \emptyset$ and $\mathrm{enter}_f(M) \cdot (\mathrm{enter}_f(M) \setminus \mathrm{id}_{\mathrm{Elements}(M)}) = \emptyset$.

Let us consider M. The functor $\mathrm{prox}_f(M)$ yielding a binary relation is defined by the condition (Def. 12).

(Def. 12) $\mathrm{prox}_f(M)$ = (the flow relation of M)$\restriction$(the places of M) $\cup$ (the flow relation of M)$^{\smile}\restriction$the places of $M \cup \mathrm{id}_{\text{the places of } M}$.

The functor $\mathrm{flow}_f(M)$ yielding a binary relation is defined by:

(Def. 13) $\mathrm{flow}_f(M)$ = (the flow relation of M) $\cup\, \mathrm{id}_{\mathrm{Elements}(M)}$.

We now state the proposition

(26) $\mathrm{prox}_f(M) \cdot \mathrm{prox}_f(M) = \mathrm{prox}_f(M)$ and $(\mathrm{prox}_f(M) \setminus \mathrm{id}_{\mathrm{Elements}(M)}) \cdot \mathrm{prox}_f(M) = \emptyset$ and $\mathrm{prox}_f(M) \cup (\mathrm{prox}_f(M))^{\smile} \cup \mathrm{id}_{\mathrm{Elements}(M)} = \mathrm{flow}_f(M) \cup (\mathrm{flow}_f(M))^{\smile}$.

Let us consider M. The functor $\mathrm{places}_f(M)$ yielding a set is defined as follows:

(Def. 14) $\mathrm{places}_f(M)$ = the places of M.

The functor $\text{transitions}_f(M)$ yielding a set is defined as follows:

(Def. 15) $\text{transitions}_f(M) =$ the transitions of M.

The functor $\text{pre}_f(M)$ yielding a binary relation is defined by:

(Def. 16) $\text{pre}_f(M) =$ (the flow relation of M)$\restriction$(the transitions of M).

The functor $\text{post}_f(M)$ yielding a binary relation is defined by:

(Def. 17) $\text{post}_f(M) =$ (the flow relation of M)$^{\smile}\restriction$the transitions of M.

Next we state three propositions:

(27) $\text{pre}_f(M) \subseteq [:\text{transitions}_f(M), \text{places}_f(M):]$ and $\text{post}_f(M) \subseteq [:\text{transitions}_f(M), \text{places}_f(M):]$.

(28) $\text{places}_f(M)$ misses $\text{transitions}_f(M)$.

(29) $\text{prox}_f(M) \subseteq [:\text{Elements}(M), \text{Elements}(M):]$ and $\text{flow}_f(M) \subseteq [:\text{Elements}(M), \text{Elements}(M):]$.

Let us consider M. The functor $\text{entrance}_f(M)$ yielding a binary relation is defined as follows:

(Def. 18) $\text{entrance}_f(M) =$ (the flow relation of M)$^{\smile}\restriction$the places of $M \cup \text{id}_{\text{the transitions of } M}$.

The functor $\text{escape}_f(M)$ yielding a binary relation is defined as follows:

(Def. 19) $\text{escape}_f(M) =$ (the flow relation of M)$\restriction$(the places of M) $\cup\, \text{id}_{\text{the transitions of } M}$.

Next we state four propositions:

(30) $\text{escape}_f(M) \subseteq [:\text{Elements}(M), \text{Elements}(M):]$ and $\text{entrance}_f(M) \subseteq [:\text{Elements}(M), \text{Elements}(M):]$.

(31) $\text{dom}\,\text{escape}_f(M) \subseteq \text{Elements}(M)$ and $\text{rng}\,\text{escape}_f(M) \subseteq \text{Elements}(M)$ and $\text{dom}\,\text{entrance}_f(M) \subseteq \text{Elements}(M)$ and $\text{rng}\,\text{entrance}_f(M) \subseteq \text{Elements}(M)$.

(32) $\text{escape}_f(M) \cdot \text{escape}_f(M) = \text{escape}_f(M)$ and $\text{escape}_f(M) \cdot \text{entrance}_f(M) = \text{escape}_f(M)$ and $\text{entrance}_f(M) \cdot \text{entrance}_f(M) = \text{entrance}_f(M)$ and $\text{entrance}_f(M) \cdot \text{escape}_f(M) = \text{entrance}_f(M)$.

(33) $\text{escape}_f(M) \cdot (\text{escape}_f(M) \setminus \text{id}_{\text{Elements}(M)}) = \emptyset$ and $\text{entrance}_f(M) \cdot (\text{entrance}_f(M) \setminus \text{id}_{\text{Elements}(M)}) = \emptyset$.

Let us consider M. The functor $\text{adjac}_f(M)$ yielding a binary relation is defined by the condition (Def. 20).

(Def. 20) $\text{adjac}_f(M) =$ (the flow relation of M)$\restriction$(the transitions of M) $\cup$ (the flow relation of M)$^{\smile}\restriction$the transitions of $M \cup \text{id}_{\text{the transitions of } M}$.

We introduce $\text{circulation}_f(M)$ as a synonym of $\text{flow}_f(M)$.

The following proposition is true

(34) $\text{adjac}_f(M) \cdot \text{adjac}_f(M) = \text{adjac}_f(M)$ and $(\text{adjac}_f(M) \setminus \text{id}_{\text{Elements}(M)}) \cdot \text{adjac}_f(M) = \emptyset$ and $\text{adjac}_f(M) \cup (\text{adjac}_f(M))^{\smile} \cup \text{id}_{\text{Elements}(M)} = \text{circulation}_f(M) \cup (\text{circulation}_f(M))^{\smile}$.

REFERENCES

[1] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[2] Waldemar Korczyński. Some elementary notions of the theory of Petri nets. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/net_1.html.

[3] Waldemar Korczyński. Definitions of Petri net. Part II. *Journal of Formalized Mathematics*, 4, 1992. http://mizar.org/JFM/Vol4/e_siec.html.

[4] Waldemar Korczyński. Definitions of Petri net. Part III. *Journal of Formalized Mathematics*, 4, 1992. http://mizar.org/JFM/Vol4/s_siec.html.

[5] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[6] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

Received January 31, 1992

Published January 2, 2004